Problem 2.3

Refering to Fig 1 below, at an arbitrary point Q we must have

$$r_1 + r_c = \frac{\sigma}{P_B - P_\infty} \qquad (1)$$

where r_1 & r_c are the principle radii of curvature. Taking r_1 as the radius in the plane of the page, we can write

$$r_1 = \left[\frac{(1+y'^2)^{3/2}}{|y''|} \right]_Q \qquad (2)$$

where $y' = dy/dx$ on the bubble surface.
The radius of curvature in the plane perpendicular to the page and passing through the point Q is r_c as shown in the figure (note that the bubble is axi-symmetric), and it can be easily shown from geometry that

$$r_c = \left\{ |x| \left[1 + \frac{1}{y'^2} \right]^{1/2} \right\} \qquad (3)$$

Combining Eqs. (1) ~ (3), the equation

defining the shape of the bubble's profile is derived as

$$\frac{(1+y'^2)^{\frac{3}{2}}}{|y''|} + |x|\left[1+\frac{1}{y'^2}\right]^{1/2} = \frac{\sigma}{P_B - P_\infty} \qquad (4)$$

To find $P_B - P_\infty$, we note that it is a constant, and must apply to point D as well. At point D, we can assume the bubble's surface is spherical with a radius of curvature $r_{c0}/\sin\theta_0$. Then

$$P_B - P_\infty = \frac{2\sigma_w}{r_{c0}/\sin\theta_0} \qquad (5)$$

The right side of Eq. (4) then becomes

$$\frac{\sigma\, r_{c0}/\sin\theta_0}{2\sigma_w} \qquad (6)$$

Substitution of Eq.(6) into Eq.(4) then gives a second order ordinary differential equation. The boundary (initial) conditions for the equation will be:

At $x = r_{co}$

$$y = 0$$

$$\frac{dy}{dx} \hat{=} y' = \tan\theta_o$$

Now, non-dimensionalization using the definitions of X and Y, will give the desired dimensionless second-order ordinary differential equation. The initial conditions will then be:

at $X = 0$

$$Y = 0$$

$$\frac{dY}{dX} = \tan\theta_o$$

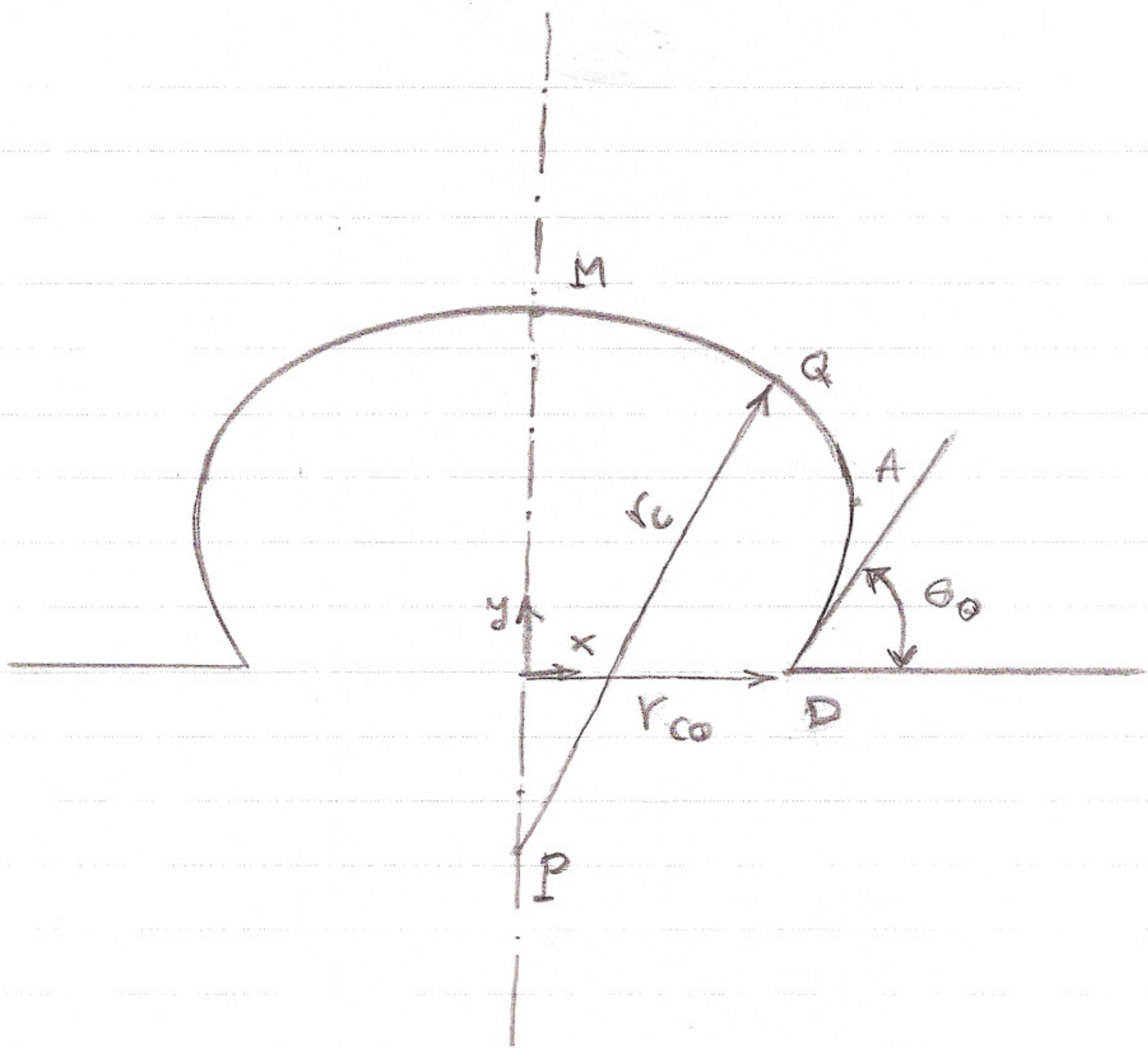

Figure 1

(Problem 2.3)