Problem 2.7. For a microbubble surrounded and at thermal and mechanical equilibrium with liquid, prove that the liquid must be superheated according to:

$$P_{sat} - P_L \approx \frac{2\sigma}{R}\frac{\rho_L + \rho_v}{\rho_L}$$

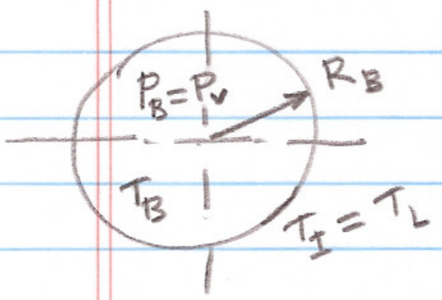
where P_L is the pressure in the liquid phase and P_{sat} is the saturation pressure corresponding to the temperature of the temperature of the bubble and its surroundings.

Also, for a micro droplet surrounded and at thermal and mechanical equilibrium with its vapor, prove that the vapor must be supercooled according to

$$P_{sat} - P_v \approx -\frac{2\sigma}{R}\frac{\rho_v}{\rho_L}$$

where P_v is the pressure in the vapor phase and P_{sat} is the saturation pressure corresponding to the temperature of the temperature of the bubble and its surroundings.

Problem 2.7



P_L, T_L

$T_B = T_L$ (from thermal equilibrium)

From Eq. (2.50)

$$P_B = P_{sat}(T_L) \exp\left(-\frac{2\sigma}{\rho_L \frac{R_u}{M} T_L R}\right) \qquad (1)$$

Since $\dfrac{2\sigma}{\rho_L \frac{R_u}{M} T_L R} << 1$, then

$$P_B \approx P_{sat}\big|_{T_L}\left[1 - \frac{2\sigma}{\rho_L \frac{R_u}{M} T_L R}\right]$$

$$P_{sat}\big|_{T_L} - P_B \approx P_{sat}\big|_{T_L} \frac{2\sigma}{\rho_L \frac{R_u}{M} T_L R} \approx \frac{2\sigma \rho_g}{\rho_L R} \qquad (2)$$

Mechanical equilibrium requires that

$$P_B - P_\infty = \frac{2\sigma}{R} \qquad (3)$$

$$(2) \,\&\, (3) \Rightarrow P_{sat}\big|_{T_L} - P_\infty = \frac{2\sigma(\rho_L - \rho_g)}{\rho_L R} \approx \frac{2\sigma}{R} \qquad (4)$$

Now, from Clapeyron's relation:

$$\frac{dP}{dT} = \frac{h_{fg}}{T_{sat} v_{fg}}$$

$$\Rightarrow \quad \Delta T \approx \Delta P \, \frac{T_{sat} v_g}{h_{fg}} \qquad (5)$$

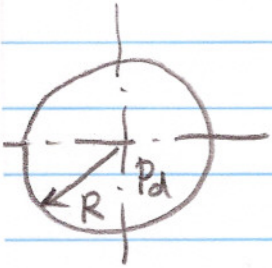
(4) & (5) $\Rightarrow$

$$T_L - T_{sat}\Big|_{P_\infty} = \frac{2\sigma T_{sat} v_g}{R \, h_{fg}} \qquad (6)$$

This expression represents the liquid superheat needed for the bubble with radius R to be at equilibrium with its surrounding liquid.

The attached Table 1 shows some numbers.

Now, for a droplet



$$P_\infty = P_v$$
$$T_\infty = T_d = T_v$$

$$\ln \frac{P_v}{P_{sat}|_{T_v}} = + \frac{2\sigma}{\rho_L \frac{R_u}{M} T_v R} \qquad (7)$$

$$\Rightarrow P_v \approx P_{sat}|_{T_v} \left[1 + \frac{2\sigma}{\rho_L \frac{R_u}{M} T_v R} \right] \approx P_{sat}|_{T_v} + \frac{2\sigma \rho_v}{\rho_L R} \qquad (8)$$

$$P_v - P_{sat}|_{T_v} = \frac{2\sigma \rho_v}{\rho_L R}$$

$$\Rightarrow T_{sat}|_{P_v} - T_v \approx \frac{2\sigma T_v}{\rho_L R h_{fg}} \qquad (9)$$

Thus, for a microdroplet with radius R to be stable in pure vapor, the vapor must be supercooled by the above amount.

The attached Table 2 shows some numerical results.