Chapter 2

2.1 The electric current j and the direction and magnitude of electron and ion drift ($\mathbf{v}_e$ and $\mathbf{v}_i$) relative to an ionosphere electric field $\mathbf{E}$ parallel to the x-axis (while $\mathbf{B}_E$ is along the z-axis) are plotted in the Figure 2.25 for three different heights. The angle a between electrons and ions relative to $\mathbf{E}$ is given, for a charged particle k, by $a_k = \tan^{-1} (\Omega_k/\nu_{kn})$ (Ω and ν are gyrofrequency and collision frequency, respectively). For a height of about 180 km, $\nu_{kn} << \Omega_k$; electrons and ions move in the same direction, with a velocity $\mathbf{E} \times \mathbf{B}/B^2$. This drift produces no net current. Discuss the importance of collisions for two cases. In case a, assume that the elctron-neutral collision frequency greatly exceeds the electron gyrofrequency (as, for example, near 70 km). In case b, assume that the electron gyrofrequency greatly exceeds the electron-neutral collision frequency (as, for example, near 180 km).

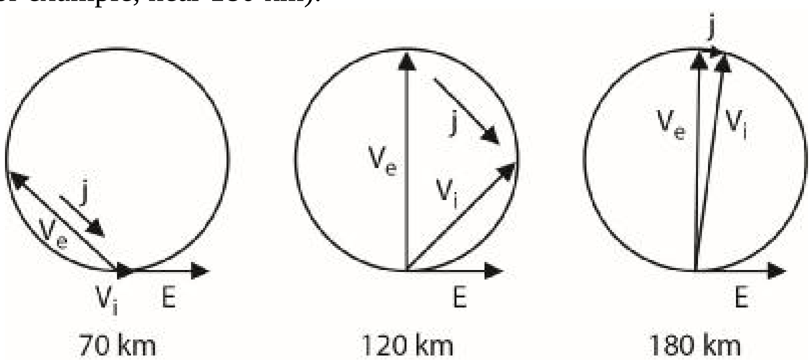


Fig. 2.25. Electric current j and the direction and magnitude of electron and ion drift ($\mathbf{v}_e$ and $\mathbf{v}_i$) relative to an ionospheric electric field $\mathbf{E}$ parallel to the x-axis (while $\mathbf{B}_E$ is along the z-axis) for three different heights.

Answer:

When the electron-neutral collision frequency greatly exceeds the electron gyrofrequency, as at 70 km in Fig. 2.25, the electron moves at an angle to the electric field given by the inverse tangent of the ratio of the gyrofrequency to the electron collision frequency, the ions are stationary, and a current flows roughly along the electric field. When the electron gyrofrequency far exceeds the electron-neutral collision frequency as at 180 km, then both the electron and the ion drift perpendicular to the electric field, and there is no current.

2.2 Find the expression for h_m, the height of the production peak for the Chapman production function in terms of neutral atmosphere quantities only. [Hint: Use equation (2.57).] Does the production peak height double if some event such as a dust storm increases the neutral gas density so that n_0 occurs at twice the original reference height h_0 (with no change to the neutral scale height)?

Answer:

Using equation 2.57, we find that $h_m = h_0 + H_0 \ln [\sigma H_n n_0 \sec \chi]$, where h_0 is the reference altitude, H_0 is the neutral scale height, n_0 is the density at the reference altitude, and χ is the solar zenith angle. Inspection of this equation shows that if the dust storm raises the reference altitude to a new h_0, the production peak moves up by the same distance as the reference height moved, if the neutral scale height H_n remains the same and if n_0 is the same at the new h_0. Otherwise, the height of the maximum density does not simply follow the rise in h_0.

2.3 Figure 2.18 shows some altitude profiles of energy deposition for protons in an exponential oxygen atmosphere (air). If you had a monoenergetic 100-keV proton flux of 10^5 cm^{-2} ·s^{-1} entering the atmosphere from the magnetosphere or a solar source, what would the approximate peak production rate be (ions cm^{-3}· s^{-1})? How does this compare with the peak subsolar ion production rate from solar EUV photons (Fig. 2.16)? If instead of protons, 10 keV electrons precipitated, how would the altitude and peak production change? Describe how you would calculate the peak altitude if these were alpha articles (He^{2+}). (Note: dE/dx is independent of the incident ion mass, but depends on the square of its charge.) [Hint: Remember the range of the particles is determined in *x* with units of g cm^{-2}, which must be translated to altitude.]

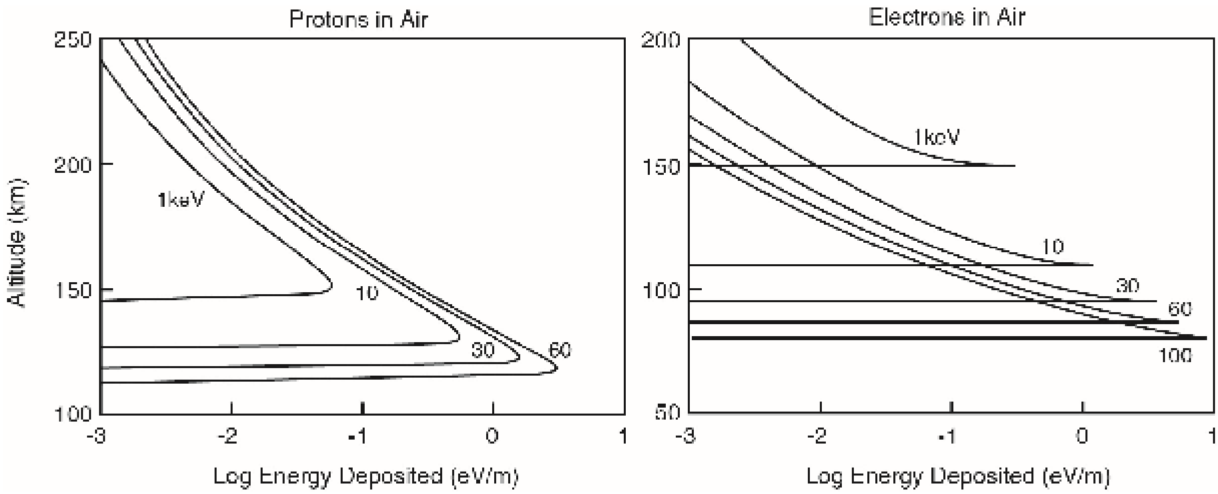


Figure 2.18. Energy-deposition profiles for protons (left) and electrons (right) of various energies incident on the Earth's atmosphere, as calculated from the method described in the text.

Answer:

In Figure 2.18, the peak production rate for 10 keV protons is suggested to be ~ 1 eV m^{-1}. An incident proton flux at this energy of 10^5 cm^{-2} s^{-1} times this gives about 10^3 eV cm^{-2} s^{-1}. For air, ~35 eV is required to make an ion pair. Thus 10^5 cm^{-2}s^{-1} x 1 eV m^{-1} x 10^{-2} m/cm x (35 eV/ion pair)$^{-1}$ = ~30 ion pairs cm^{-3} s^{-1} are produced at the deposition peak height of ~130 km. Comparing this number with the peak subsolar production rate due to solar EUV at about the same altitude in Figure 2.16 which is ~3x10^3 cm^{-3} s^{-1}, we conclude that, in general, in Earth's atmosphere, particle precipitation as a source of ionosphere is mainly important at night.

For 10 eV electrons, the production peak in Figure 2.18 is slightly lower at ~115 km, but the deposition rate is roughly the same, near ~1 eV m^{-1}.

For alpha particles, the energy loss rate dE/dx for protons is nominally increased by 4 times so that the range in x (g cm^{-2}) is reduced by this amount compared to the protons. To translate this change in range to a change in the peak altitude, one can use the integral that describes x in terms of altitude (Eq 2.64), together with an exponential atmospheric density profile approximation, to evaluate the x equivalent of the peak deposition altitude for protons. Then divide this by 4 and equate it to the same expression for the new peak altitude desired for alphas. The solution of the equation

$$\exp(-(h_{amax}-h_o)/H) = (\exp(-(h_{pmax}-h_o)/H))/4$$

for h_{amax}, where h_{amax} is the desired peak altitude for alphas and h_{pmax} is the proton peak deposition altitude, should give the desired approximation.

2.4 Compare the ranges of protons and electrons in air (see Figure 2.17). At a particular energy, which are most penetrating? Using Figure 2.14, how do the energies of photons that penetrate to the same altitudes compare?

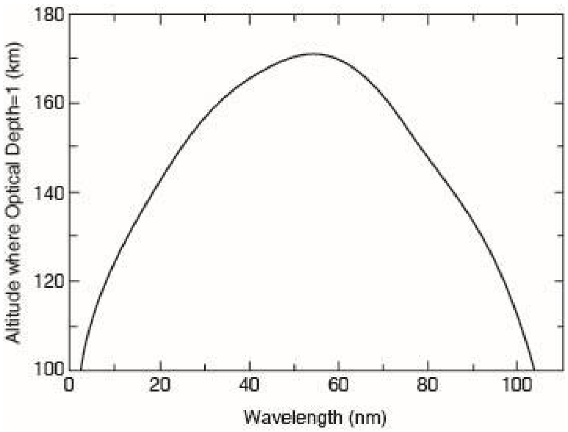


Figure 2.14. Photon wavelength versus the altitude in the Earth's atmosphere where the optical depth is equal to 1 (effectively, the depth of penetration of a photon). (Adapted from Rishbeth and Garriott, 1969.)

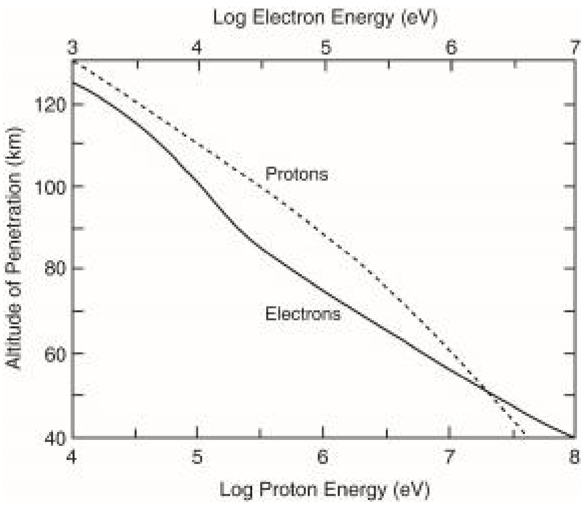


Figure 2.17. Penetration altitudes for electrons and protons in the Earth's atmosphere versus their incident energies.

Answer:

Using Figure 2.17, it is clear that at an electron energy of 7eV, the electrons penetrate to 40 km altitude, but at this same energy, protons reach only about 60 km. A 7eV photon would have a wavelength of about 60 nm. This photon would be absorbed at about 170 km altitude. Note that photons decrease in energy with increasing wavelength so the decreasing altitude from left to right in Fig. 2.14 corresponds to decreasing photon energy, while in Fig. 2.17, the energy increases to the right.

2.5 Using the assumption that B is along z, derive the expression for the Pedersen and Hall conductivities from the electron- and ion-momentum equations.

Answer:

Extending the momentum equations (2.88) and (2.89) to include the magnetic field force, we find for ions and electrons that

$$(\quad + \quad \times \quad) =$$
$$-\ (\quad + \quad \times \quad) =$$

respectively, assuming single-charged positive ions (=), and ignoring pressure gradients and inertia. The momentum equations are written in the frame of the neutral gas, and we have included only collisions with the neutrals.

We shall now assume that the magnetic field is in the *z*-direction, and further the ion and electron gyro-frequencies are defined as $\Omega \; = \; /$ and $\Omega \; = \; /$. Then

$$+ \quad = (\quad /\Omega\ )$$
$$- \quad = (\quad /\Omega\ )$$
$$+ \quad = -(\quad /\Omega\ )$$
$$- \quad = -(\quad /\Omega\ )$$

Taking the ions as an example, from the first equation

$$= - \quad + (\quad /\Omega\ )$$

On substituting this is in the second equation.

$$- \quad = \frac{\quad}{\Omega} - \quad + \frac{\quad}{\Omega}$$

or

$$= \frac{1}{(1 + \quad /\Omega\ )}\ \frac{\quad}{\Omega} \quad +$$

Consequently

$$= - \quad + \frac{\quad}{\Omega}\frac{1}{(1 + \quad /\Omega\ )}\ \frac{\quad}{\Omega} \quad +$$
$$= \frac{1}{(1 + \quad /\Omega\ )}\ - \quad + \frac{\quad}{\Omega}$$

Similarly, for the electrons,

$$= \frac{1}{(1 + \quad /\Omega\ )}\ -\frac{\quad}{\Omega} \quad +$$

and

$$= \frac{1}{(1 + \quad /\Omega\ )}\ - \quad - \frac{\quad}{\Omega}$$

To determine the current density, we assume the ions and electrons have the same number density , and

$$= \quad (\quad - \quad)$$

Therefore

$$= \frac{}{}\left[\frac{/\Omega}{1+\ \ /\Omega}+\frac{/\Omega}{1+\ \ /\Omega}\right]$$

$$+\left[\frac{1}{1+\ \ /\Omega}-\frac{1}{1+\ \ /\Omega}\right]$$

and

$$= \frac{}{}\left[\frac{/\Omega}{1+\ \ /\Omega}+\frac{/\Omega}{1+\ \ /\Omega}\right]$$

$$-\left[\frac{1}{1+\ \ /\Omega}-\frac{1}{1+\ \ /\Omega}\right]$$

These should be compared with equations (2.91) (2.92), and (2.93). From the expressions given above, on rearranging terms, we find

$$= \frac{1}{}\ \frac{}{+\ \Omega} + \frac{1}{}\ \frac{}{+\ \Omega}$$

and

$$= \frac{}{}\ \frac{}{} - \frac{}{}\ \frac{}{}\ .$$

These are the Pedersen and Hall conductivities, respectively.

Comment:

It should be noted that, as defined here, both conductivities are positive quantities for the Earth's ionosphere, where $/\Omega \ll /\Omega$.

We also note that the matrix representation in equation (2.91) has a sign error. The formalism as presented in Chapter 2 follows that of Hargreaves [1979, 1992]. Other texts (e.g., Prölss, G. W., *Physics of the Earth's Space Environment*, Springer-Verlag, Berlin, 2004) are consistent with the following representation:

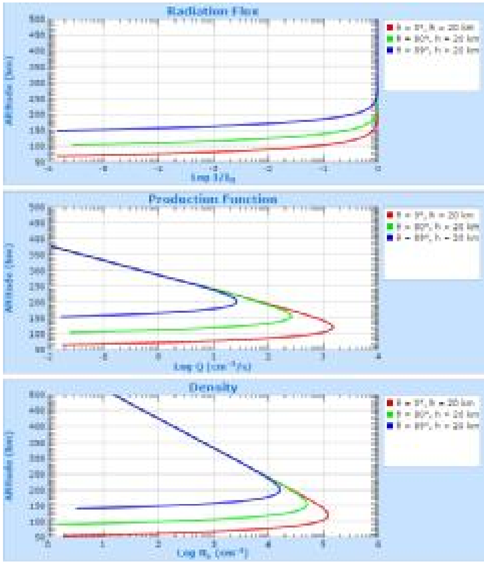
$$= \begin{pmatrix} & - & 0 \\ & & 0 \\ 0 & 0 & \end{pmatrix}$$

Notice the sign change for the off-diagonal terms in the conductivity matrix. This will be corrected in future editions.

2.6 Use the Space Physics Exercises, select the Ionosphere tab. Select the blue Altitude Profile tab.

a) Turn on the overlay option and use the default values for scale height and the plotting range. Using three different colors plot the ionospheric profiles for zenith angles of the sun of 0^{o}, 80^{o} and 89^{o}. Print out your results. Describe how the changing height of the sun affects the ionospheric density profile.

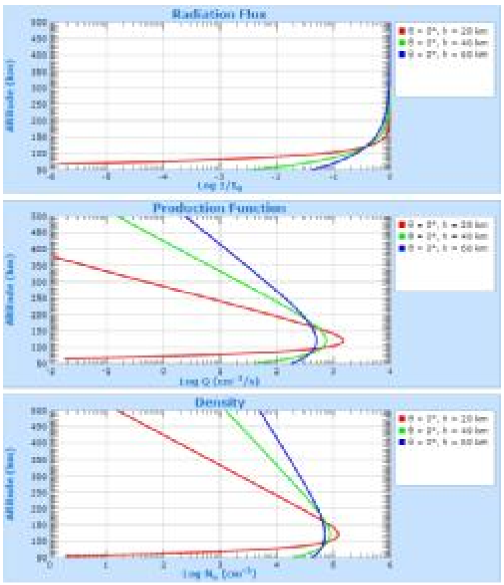
Answer:



An increase in solar zenith angle causes the solar radiation flux to be absorbed at a higher altitude. The production function and the density profile remain the same at highest altitudes, but the altitude of the peak production and density move upward, and the values of the peak production and peak density decrease.

b) Turn on the overlay option after erasing the old plots and reset the solar zenith angle to 0°. Set the scale height to 20, 40 and 60 km using different colors. Print out your results. How does the increased scale height affect the absorption of radiation and the density of electrons with altitude?

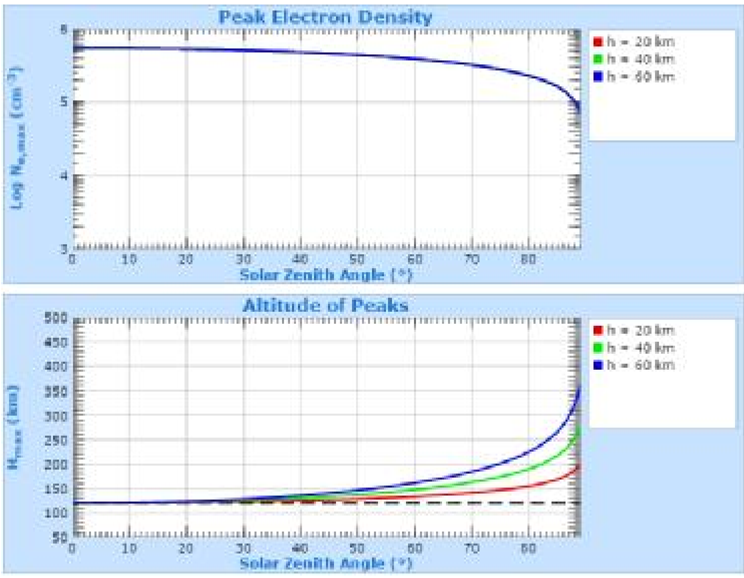
Answer:



An increase in the scale height increases the absorption of radiation at high altitudes, but maintains the altitude at which the intensity has decreased to e^{-1} of its incoming strength. The production function increases at high altitude and decreases at its peak value, as does the density.

c) Click on the Solar Zenith Angle module. Turn on the overlay option. For scale heights of 20, 40 and 60 km plot the peak electron density and altitude of the maximum altitude versus solar zenith angle. Print out your results. How does the peak density change? How does the altitude of the maximum density change?

Answer:



The peak electron density is not dependent on the scale height of the atmosphere. The altitude of the peaks all increase with increasing scale height at all solar zenith angles.

Comment:

The space physics exercises may not produce the optimum size plot for a student's report. Remind the students that they can resize the screen or change from portrait to landscape to improve the image. For example, ALT F opens the print preview in Firefox, allowing resizing of image.