Chapter 2

Antenna Fundamentals

2.1 A monopole having $h = 3.25$ cm, $a = 1$ mm, and made from aluminium (assume $\mu = \mu_0$ and $\sigma = 3 \times 10^6$ S/m) is being considered for use in a 433 MHz radio telemetry transmitter. Calculate the impedance, radiation efficiency, directivity, and gain.

At 433 MHz, the free space wavelength $\lambda = 69.3$ cm. Since $h/\lambda = 0.047$, the monopole is electrically-short. Therefore:

$$Z_A \cong 40\pi^2 \left(\frac{h}{\lambda}\right)^2 + R_S \frac{1}{2\pi a}\frac{h}{3} - j\frac{30\Omega}{\pi\,(h/\lambda)}\left[\ln\left(\frac{h}{a}\right) - 1\right] \tag{2.1}$$

The first term is $R_{rad} = 0.869\ \Omega$. The surface resistance needed for the second (R_{loss}) term is

$$R_S \cong \sqrt{\frac{\pi f \mu}{\sigma}} = 84.6\ \mathrm{m}\Omega \tag{2.2}$$

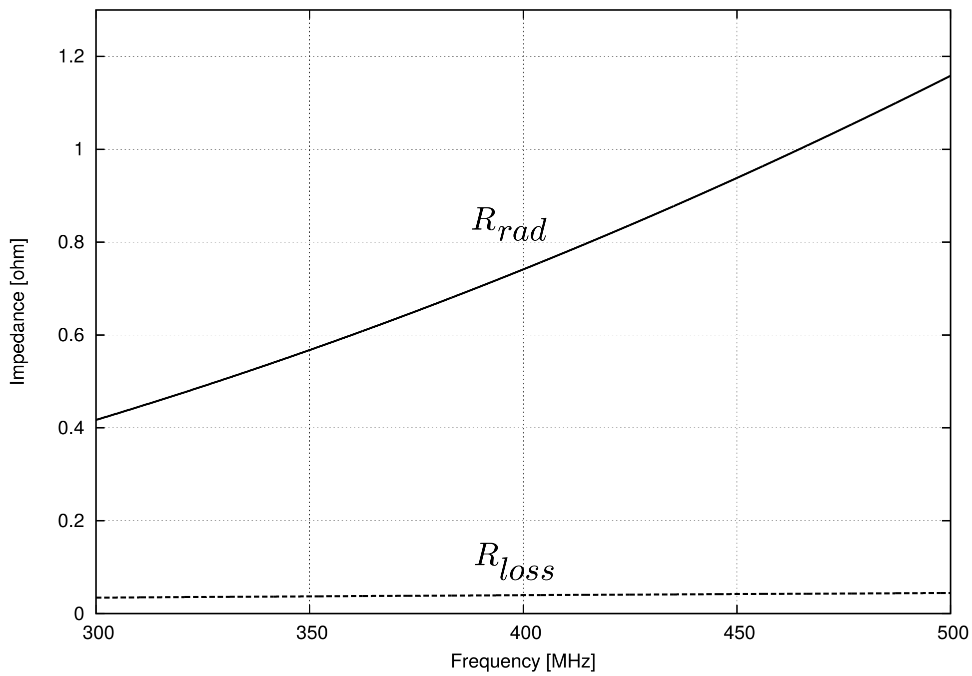
Giving $R_{loss} = 0.146\ \Omega$. The third term is $jX_A = -j505.11\ \Omega$. Adding these terms, we obtain $\boxed{Z_A = 1.01 - j505.1\ \Omega}$.

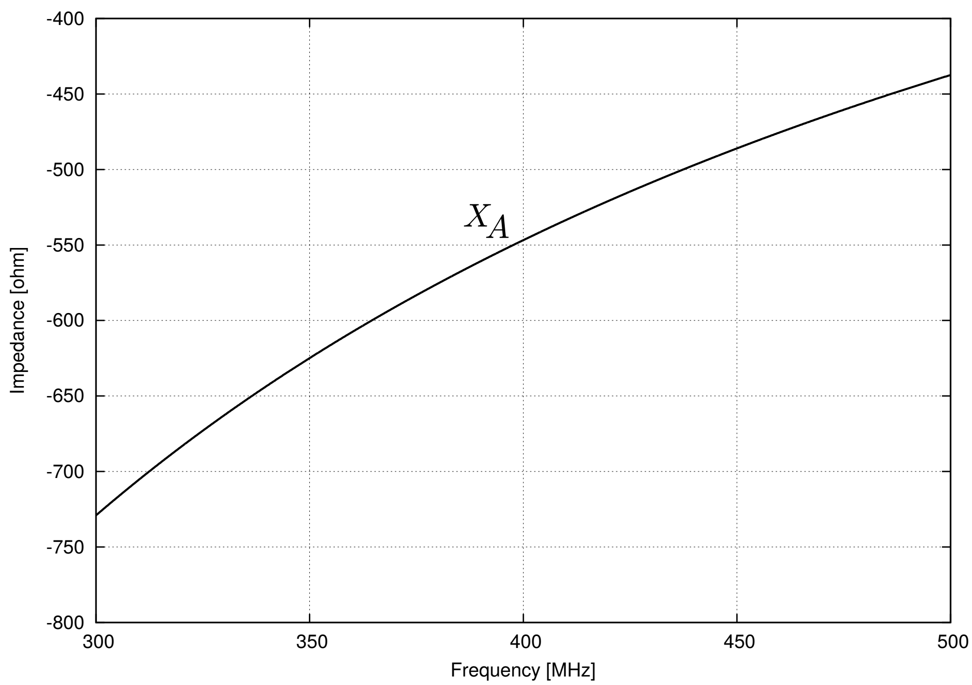
The radiation efficiency is

$$\epsilon_{rad} = \frac{R_{rad}}{R_{rad} + R_{loss}} = 0.856 = \boxed{85.6\%} \tag{2.3}$$

Since this is an electrically-short monopole, the directivity is $\boxed{4.77\ \text{dBi}}$ and does not change significantly with h or a. The gain is the product of the radiation efficiency and the directivity: $4.77\ \text{dBi} + 10\log_{10} 0.856 = \boxed{4.09\ \text{dBi}}$

2.2 Repeat the analysis shown in Figure 2.5 for the antenna described in the above problem. Consider the range 300–500 MHz.





2.3 Derive Equation 2.32 (vector effective length of an electrically-short dipole).

For a $\hat{\mathbf{z}}$ wire antenna:

$$\mathbf{l}_e = \hat{\mathbf{z}} \frac{1}{I_T^{(t)}} \int_{z=-L/2}^{+L/2} I^{(t)}(z') dz' \tag{2.4}$$

For an electrically-short dipole:

$$I_T^{(t)}(z') = I(z') \cong I_T \left(1 - \frac{2}{L} |z'| \right) \tag{2.5}$$

substituting:

$$\mathbf{l}_e = \hat{\mathbf{z}} \int_{z=-L/2}^{+L/2} \left(1 - \frac{2}{L} |z'| \right) dz' = \hat{\mathbf{z}} \int_{z=-L/2}^{+L/2} dz' - \hat{\mathbf{z}} \frac{2}{L} \int_{z=-L/2}^{+L/2} |z'| \, dz' \tag{2.6}$$

$$= \hat{\mathbf{z}} L - \hat{\mathbf{z}} \frac{2}{L} \int_{z=-L/2}^{0} (-z') \, dz' - \hat{\mathbf{z}} \frac{2}{L} \int_{z=0}^{+L/2} (+z') \, dz' \tag{2.7}$$

$$= \hat{\mathbf{z}} L - \hat{\mathbf{z}} \frac{1}{L} \left(-\frac{L}{2} \right)^2 - \hat{\mathbf{z}} \frac{1}{L} \left(+\frac{L}{2} \right)^2 \tag{2.8}$$

$$= \hat{\mathbf{z}} L - \hat{\mathbf{z}} \frac{L}{4} - \hat{\mathbf{z}} \frac{L}{4} \tag{2.9}$$

So we see that

$$\mathbf{l}_e = \hat{\mathbf{z}} \frac{L}{2} \tag{2.10}$$

2.4 Derive the result that $A_e = 0.12\lambda^2$ for the electrically-short dipole, starting with the expressions for A_e and R_{rad} for this antenna (Equations 2.39 and 2.22, respectively).

Effective aperture may be expressed in terms of effective length as follows:

$$A_e = \frac{\eta\, l_e^2}{4R_{rad}} \tag{2.11}$$

and the radiation resistance for an electrically-short dipole is

$$R_{rad} \cong \frac{\eta\,\pi}{6}\left(\frac{L}{\lambda}\right)^2 \tag{2.12}$$

Substituting:

$$A_e \cong \frac{\eta\, l_e^2}{4}\frac{6}{\eta\,\pi}\left(\frac{\lambda}{L}\right)^2 = \frac{3}{2\pi}l_e^2\left(\frac{\lambda}{L}\right)^2 \tag{2.13}$$

since $l_e \cong L/2$ for an electrically short dipole:

$$A_e \cong \frac{3}{2\pi}\left(\frac{L}{2}\right)^2\left(\frac{\lambda}{L}\right)^2 = \frac{3}{8\pi}\lambda^2 \cong 0.12\lambda^2 \tag{2.14}$$

2.5 Equation 2.44 ($D = 4\pi A_e/\lambda^2$) is presented without derivation. But it can be checked. Confirm that this equation is consistent with the independently-derived expressions for the directivity and effective aperture of an electrically-short dipole.

For an electrically-short dipole, $A_e \cong 0.12\lambda^2$. Substituting:

$$D = 4\pi \frac{A_e}{\lambda^2} \cong 4\pi \frac{0.12\lambda^2}{\lambda^2} = 4\pi \cdot 0.12 \cong 1.5 \tag{2.15}$$

This is in fact the directivity of an electrically-short dipole, which suggests the original equation is correct.

2.6 Using the same method employed for the electrically-short dipole, show that the radiation resistance of a half-wave dipole is about 73 Ω.

The general strategy is (1) Find **E** by integrating over the current distribution, (2) Find S_{rad}, (3) Integrate S_{rad} to get P_{rad}, (4) Relate P_{rad} to I_T to get R_{rad}. Here we go: For a linear (wire) antenna aligned along the z-axis:

$$\mathbf{E}\left(\mathbf{r}\right) \cong \ \hat{\theta}\ j\frac{\eta}{2}\left(\sin\theta\right)\frac{e^{-j\beta r}}{r}\left[\frac{1}{\lambda}\int_{-L/2}^{+L/2} I(z')e^{+j\beta z'\cos\theta}dz'\right] \tag{2.16}$$

Let us deal with the integral first. An appropriate expression for $I(z)$ for a half-wave dipole is:

$$I(z) = I_T\cos 2\pi\frac{z}{\lambda} \tag{2.17}$$

so the integral becomes

$$\int_{-L/2}^{+L/2} I_T\left(\cos 2\pi\frac{z'}{\lambda}\right)\ e^{+j\beta z'\cos\theta}\ dz' = I_T\int_{-L/2}^{+L/2}\left(\cos\beta z'\right)\ e^{+j\beta z'\cos\theta}\ dz' \tag{2.18}$$

Let $u' \equiv \beta z'$; then $du' = \beta dz'$ and the integral becomes

$$\frac{I_T}{\beta}\int_{-\pi/2}^{+\pi/2}\left(\cos u'\right)\ e^{+ju'\cos\theta}\ du' \tag{2.19}$$

This can be solved using the integral identity:

$$\int e^{bx}\left(\cos ax\right)dx = \frac{1}{a^2+b^2}\ e^{bx}\left(a\sin ax + b\cos ax\right) \tag{2.20}$$

using $x = u'$, $a = 1$, and $b = j\cos\theta$, the integral becomes

$$\frac{I_T}{\beta}\left[\frac{1}{1-\cos^2\theta}\ e^{ju'\cos\theta}\left(\sin u' + j\left(\cos\theta\right)\cos u'\right)\right]_{-\pi/2}^{+\pi/2} \tag{2.21}$$

$$= \frac{I_T}{\beta}\frac{1}{\sin^2\theta}\left[e^{+j\frac{\pi}{2}\cos\theta} + e^{-j\frac{\pi}{2}\cos\theta}\right] \tag{2.22}$$

$$= \frac{I_T}{\beta}\ \frac{1}{\sin^2\theta}\ 2\cos\left(\frac{\pi}{2}\cos\theta\right) \tag{2.23}$$

Substituting this result back into the original expression for **E**, we obtain:

$$\mathbf{E}\left(\mathbf{r}\right) \cong \ \hat{\theta}\ j\eta\ \frac{e^{-j\beta r}}{r}\frac{I_T}{2\pi}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \tag{2.24}$$

Now step (2):

$$S_{rad} = \frac{\left|\mathbf{E}\right|^2}{2\eta} \cong \frac{\eta}{8\pi^2}\ \frac{1}{r^2}\left|I_T\right|^2\ \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \tag{2.25}$$

Now step (3):

$$P_{rad} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} S_{rad}\ r^2 \sin\theta' d\theta' d\phi' \tag{2.26}$$

$$\cong \frac{\eta}{8\pi^2} |I_T|^2 \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta'\right)}{\sin\theta'} d\theta' d\phi' \tag{2.27}$$

$$= \frac{\eta}{4\pi} |I_T|^2 \int_{\theta=0}^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta'\right)}{\sin\theta'} d\theta' \tag{2.28}$$

The remaining integral is relatively hard to evaluate, but is quite simple to compute numerically. It is $\cong 1.219$; thus:

$$P_{rad} \cong \frac{\eta}{4\pi} |I_T|^2 \cdot 1.219 \tag{2.29}$$

Now step (4): Applying circuit theory to the equivalent circuit for the antenna in transmission:

$$P_{rad} = \frac{1}{2} |I_T|^2 R_{rad} \tag{2.30}$$

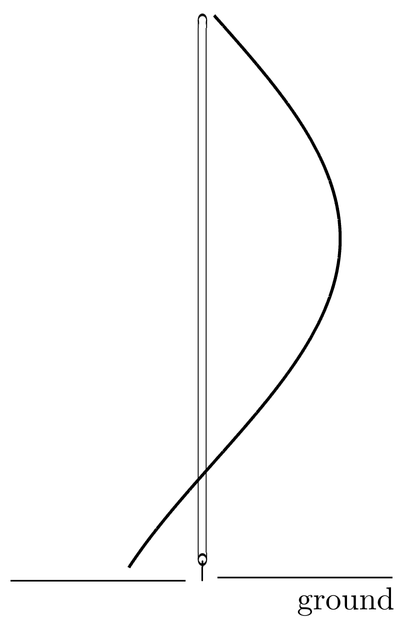
Solving for R_{rad}:

$$R_{rad} = \frac{2P_{rad}}{|I_T|^2} \cong \frac{\eta}{2\pi} \cdot 1.219 = 73.1\ \Omega \tag{2.31}$$

as expected.

2.7 A popular alternative to the quarter-wave monopole is the 5/8-λ monopole. As the name implies, $h = 5\lambda/8$. Sketch the current distribution for this antenna in same manner shown in Figure 2.3. What advantages and disadvantages does this antenna have relative to the quarter-wave monopole? Consider size and gain.

This monopole can be viewed as one-half of a $5\lambda/4$ dipole, shown in Fig. 2.9. Here's a sketch:



Advantage: This monopole has higher gain than the quarter-wave monopole (8.16 dB vs. 5.15 dB). Disadvantage: This monopole is much larger (0.625λ vs. 0.25λ). Because it is much larger relative to a wavelength, it is normally used only at higher frequencies, i.e., UHF and above, where wavelengths are smaller.

2.8 Calculate the directivity of a z-aligned half-wave dipole at $\theta = 45°$. Give your answer in dBi. Repeat for the electrically-short dipole, and both corresponding monopole versions. Compare.

Half-wave dipole:

$$D \cong 1.64 \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} = 1.64 \frac{\cos\left(\frac{\pi}{2}\cos 45°\right)}{\sin 45°} = 1.0298 = \boxed{0.13 \text{ dBi}} \tag{2.32}$$

Electrically-short dipole:

$$D \cong 1.5\sin^2\theta = 1.5\sin^2 45° = 0.75 = \boxed{-1.25 \text{ dBi}} \tag{2.33}$$

Quarter-wave monopole: 0.13 dBi (from above result for half-wave dipole) +3 dB = $\boxed{3.13 \text{ dBi}}$.

Electrically-short monopole: -1.25 dBi (from above result for quarter-wave monopole) +3 dB = $\boxed{1.75 \text{ dBi}}$.

2.9 A particular handheld receiver design requires that the GPS patch from Example 2.4 be reduced in area by 20%. You may modify the size of the patch and the dielectric constant, but not the height of the dielectric slab. What are the required dimensions and dielectric constant for the new design?

In the example, the patch was 20 mm $\times$ 21 mm, and the dielectric has relative permittivity $\epsilon_r = 5.4$. A 20% reduction in area means each dimension must be reduced by $\sqrt{0.8}$, therefore the new dimensions are $\boxed{\text{17.9 mm and 18.8 mm}}$, respectively.

The wavelength in the medium is $\lambda_0/\sqrt{\epsilon_r}$ where λ_0 is the wavelength in free space. ϵ_r must therefore be increased by $1/0.8$ in order for the electrical dimensions of the patch – that is, the patch dimensions in wavelengths – to remain unchanged. Therefore $\boxed{\epsilon_r = 6.75}$.

2.10 A reflector intended for satellite communications at 4 GHz is circular with a diameter of 3 m. The aperture efficiency is 75% and the feed has a radiation efficiency of 95%. What is the gain of this antenna in dBi?

Here:

$$G = \epsilon_{rad}\epsilon_a \frac{4\pi}{\lambda^2} A_{phys} \tag{2.34}$$

From the problem statement, $f = 4$ GHz, $\epsilon_a = 0.75$, $\epsilon_{rad} = 0.95$, and $D = 3$ m. The wavelength $\lambda = c/f = 7.5$ cm. The physical aperture is the projected area of the dish:

$$A_{phys} = \pi \left(\frac{D}{2}\right)^2 = 7.069 \text{ m}^2 \tag{2.35}$$

Therefore $G = 1.125 \times 10^4 = \boxed{40.5 \text{ dBi}}$.

2.11 A certain patch antenna has a gain of 4.2 dBi. Seven of these are arranged into a vertical stack array. What is the directivity of this array in dBi?

The array factor AF is $\leq N$, where N is the number of elements in the array. The upper bound is achieved by selecting the appropriate spacing, and lower values are associated with non-optimum spacings. Assuming AF is to be maximized, AF $= N = 7$ here. By the principle of pattern multiplication, the directivity is greater than AF by the directivity of the element. Thus we have

$$D \leq 10 \log_{10} 7 + 4.2 \text{ dBi} \cong \boxed{12.6 \text{ dBi}}. \tag{2.36}$$