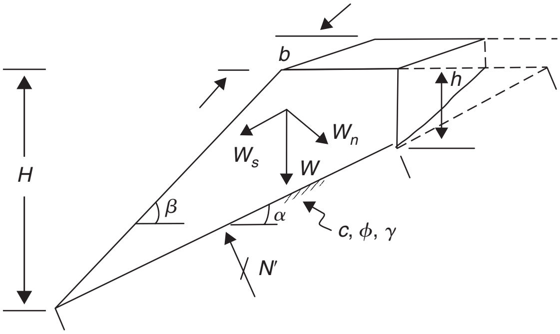
2 Slope Stability

Planar Block Slides

1. Given: Planar block slide
   Find: Algebraic *FS*.

   Solution:

   By definition

   $$FS = \frac{R}{D}$$

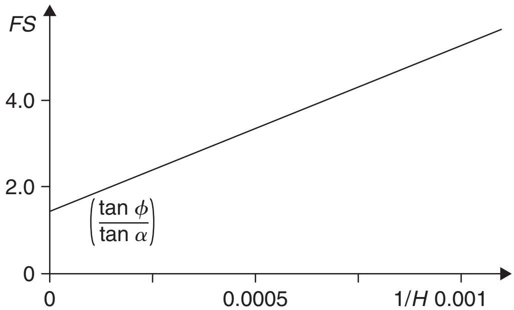
   $R =$ resisting forces: Assume Mohr–Coulomb
   $D =$ driving forces
   $W =$ weight: $W = W_1 - W_2$

   By inspection:

   $$W = \frac{\gamma H^2 b}{2}(\cot\alpha - \cot\beta) - \frac{\gamma h^2 b}{2}\cot\alpha$$

   $R = W_n \tan\phi + cA$

   Note: $\Sigma F_n = 0 \quad \therefore 0 = W_n - N' \ \& \ N' = W_n$



SCHEMATIC PLOT (FS vs H^{-1})
Note: almost linear in $1/H$

Also

$$A = b\left(\frac{H}{\sin\alpha} - \frac{h}{\sin\alpha}\right)$$

And $W_n = W \cos\alpha$

$$\begin{aligned}
\therefore R &= W\cos\alpha\tan\phi + c\mathrm{A} \\
D &= W\sin\alpha \\
FS &= \frac{W\cos\alpha\tan\phi + c\mathrm{A}}{W\sin\alpha} \\
&= \frac{\tan\phi}{\tan\alpha} + \frac{cb\left(\frac{H}{\sin\alpha} - \frac{h}{\sin\alpha}\right)}{(\sin\alpha)\left[\left(\frac{\gamma H^2 b}{2}\right)(\cot\alpha - \cot\beta) - \frac{\gamma h^2 b}{2}\cot\alpha\right]} \\
&= \frac{\tan\phi}{\tan\alpha} + \frac{2c\left(1 - \frac{h}{H}\right)}{(\sin\alpha)^2\left[(\gamma H)\left[(\cot\alpha - \cot\beta) - \frac{h^2}{H^2}\cot\alpha\right]\right]}
\end{aligned}$$

$$\underline{\underline{FS = \frac{\tan\phi}{\tan\alpha} + \frac{2c\left(1 - \frac{h}{H}\right)}{\gamma H(\sin\alpha)^2\left[\left(1 - \left(\frac{h}{H}\right)^2\right)\cot\alpha - \cot\beta\right]}}} \qquad \longleftarrow$$

2. Given: Problem 1 data
   Find: Max β (algebraically).

   Solution:

   Solve FS for β then set $\dfrac{\mathrm{d}f}{\mathrm{d}\beta} = 0$
   Find β^* stationary pts
   Find relative maximum
   Find absolute maximum (end pts) or note by inspection that as β increases the FS decreases so β will be max when FS is min, that is, when $FS = 1$, then

$$[\gamma H(\sin\alpha)^2]\left[\left(1-\left(\frac{h}{H}\right)^2\right)\cot\alpha - \cot\beta\right] = \frac{2c\left(1-\frac{h}{H}\right)}{\left(1.0-\frac{\tan\phi}{\tan\alpha}\right)}$$

$$\underline{\underline{\cot\beta_{\max} = \left(1-\left(\frac{h}{H}\right)^2\right)\cot\alpha - \frac{2c\left(1-\frac{h}{H}\right)}{\gamma H(\sin\alpha)^2\left(1.0-\frac{\tan\phi}{\tan\alpha}\right)}}}$$

Note: $\dfrac{\pi}{2} \geq \beta > \alpha$ (physical constraints) & if no tension crack ($h = 0$)

$$\cot\beta_{\max} = \cot\alpha - \frac{2c}{\gamma H(\sin\alpha)^2\left(1-\frac{\tan\phi}{\tan\alpha}\right)}$$

3. Given: Problem 1, data allow for water (below bottom of tension crack)
Find: FS (with water).

Solution:

(previous) $FS = \dfrac{W\cos\alpha\tan\phi + cA}{W\sin\alpha}$

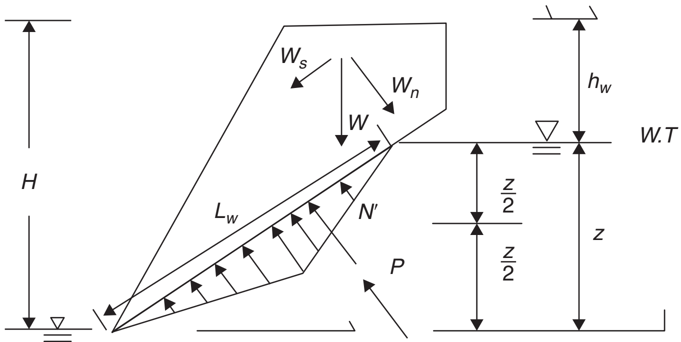
where $W = \dfrac{\gamma H^2 b}{2}(\cot\alpha - \cot\beta) - \dfrac{\gamma h^2 b}{2}\cot\alpha$

and $A = \dfrac{b}{\sin\alpha}(H-h)$

Now

$$\underline{\underline{FS = \frac{N'\tan\phi + cA}{W\sin\alpha}}} \qquad \xleftarrow{(3)}$$

where $\Sigma F_n = 0$ requires $N' = W_n - P$ with P = water force



as before $W_n = W\cos(\alpha)$
but $P = ?$

$$P = \bar{p}A_w, \quad A_w = bL_w$$
$$\bar{p} = \frac{p_{\max}}{2}\text{(rectangle)}$$

$p_{\max} = \gamma_w\, z/2$ (a linear increase of water pressure with depth below water table is assumed)

$$\frac{z}{2} = \left(\frac{1}{2}\right)(H - h_w)$$

where h_w = water table depth below crest

$$L_w = \frac{z}{\sin\alpha}$$
$$P = \left(\frac{\gamma_w}{2}\right)\left(\frac{z}{2}\right)(b)(L_w)$$
$$P = \frac{\gamma_w b z^2}{4\sin\alpha}$$

Hence:

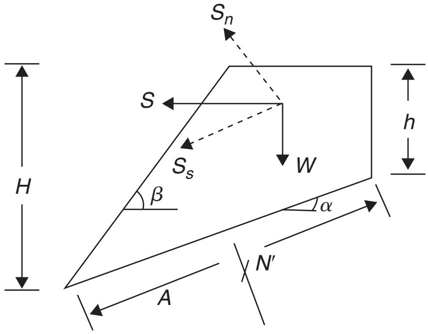
$$\underline{\underline{FS = \frac{(W\cos\alpha - P)\tan\phi + cA}{W\sin\alpha}}} \tag{4}$$

where:

$$\begin{aligned}
P &= \frac{\gamma_w b z^2}{4\sin\alpha}; \quad z = H - h_w \\
A &= \frac{b}{\sin\alpha}(H - h); \quad h = \text{tension crack depth} \\
W &= \frac{\gamma H^2 b}{2}(\cot\alpha - \cot\beta) - \frac{\gamma h^2 b}{2}\cot\alpha
\end{aligned}$$

4. Given: Problem data
   Find: FS for seismic load.

   Solution:



   By definition $S = \dfrac{W}{g} a_s$

   where $a_s =$ seismic acceleration

   i.e. $a_s = a_0 g$

   where $a_0 =$ seismic coefficient

   e.g. $a_0 = 0.15$

$$\therefore S = a_0 W$$

$$\underline{\underline{S_n = a_0 W \sin\alpha}}$$

$$\underline{\underline{S_s = a_0 W \cos\alpha}}$$

$$N' = W_n - S_n$$

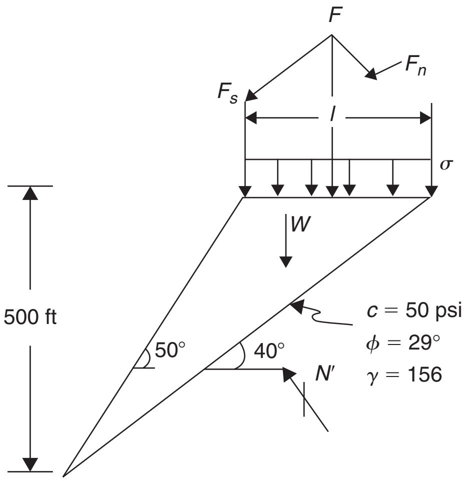
$$R = N' \tan\phi + cA$$

$$D = W_s + S_s$$

$$\underline{\underline{FS = \frac{(W_n - S_n)\tan\phi + cA}{W_s + S_s}}} \qquad (5) \longleftarrow$$

where
$$W_s = W\sin\alpha,$$
$$S_s = a_0 W \cos\alpha$$
$$W_n = W\cos\alpha,$$
$$S_n = a_0 W \sin\alpha$$
$$A = \frac{b}{\sin\alpha}(H - h)$$
$$W = \frac{\gamma b^2 H^2}{2}(\cot\alpha - \cot\beta)$$

5. Given: Data in Fig. 5 and a uniform surcharge σ with $FS = 1.1$ and $b = 25'$
Find: σ.



Solution:

$$FS = \frac{N' \tan\phi + cA + F_n \tan\phi}{W_s + F_s}$$

$F = \sigma b l$
$F_n = F \cos\alpha$
$F_s = F \sin\alpha$
then
$(FS)F_s - F_n \tan\phi$
$\quad = N' \tan\phi + cA - FS(W_s)$
$F[(\sin\alpha)FS - \cos\alpha \tan\phi]$
$$= N' \tan\phi + cA - \left(\frac{FS}{W_s}\right)$$

$$W = \gamma V$$
$$V = \frac{bH^2}{2}(\cot\alpha - \cot\beta)$$
$$= \frac{(25)(500)^2}{2}(\cot 40 - \cot 50)$$
$$V = (3.125)10^6(0.35265)$$
$$V = 1.102(10^6)\ \text{ft}^3$$

$$W = (156)(1.02)(10^6)$$
$$\underline{W = 1.719(10^8)\ \text{lbf}}$$

$$N' = W \cos\alpha$$
$$= 1.719(10^8)\cos 40$$
$$\underline{N' = 1.317(10^8)}$$
$$\underline{N' \tan\phi = 7.3001(10^7)\ \text{lbf}}$$

$$A = \frac{(25)(500)}{\sin 40}$$
$$cA = \frac{(50)(144)(25)(500)}{\sin 40}$$
$$\underline{cA = 1.4002(10^8)\ \text{lbf}}$$

$$FS(W_s) = \left(\frac{1.1}{1.719}\right)10^8(\sin 40)$$
$$\underline{FS(W_s) = 1.2156(10^8)}$$

$$N' \tan\phi + cA - FS(W_s) = 7.3001(10^7) + 1.4002(10^8) - 1.2156(10^8)$$

$$F = \frac{[9.146(10^7)\,\text{lbf}]}{[(\sin 40)(1.1) - \cos 40 \tan 29]}$$

$$= \frac{9.146(10^7)}{0.2824}$$

$$\therefore \underline{F = 3.238(10^8)\,\text{lbf}}$$

$$\sigma = \frac{3.238(10^8)}{(25)(500)(\cot\alpha - \cot\beta)}$$

$$\underline{\underline{\sigma = 7.346(10^4)\,\text{psf}}} \longleftarrow$$

$$\underline{\underline{\sigma = 510\,\text{psi}}}$$

FS without surcharge.

$$FS = \frac{[N' \tan\phi + cA]}{W_s}$$

$$= \frac{1.317(10^8)\tan 29 + 1.4002(10^8)}{1.719(10^8)\sin 40}$$

$$\underline{\underline{FS = 1.93(1.928)}} \longleftarrow$$

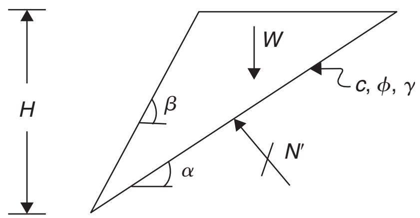
6. Given: Fig. 5 planar block slide and *no* surcharge
   Find: $H_{\max}$.

   Solution:

   $H_{\max}$ occurs when FS is min i.e., when

$$1 = \frac{R}{D} \tag{1}$$

$$\begin{aligned}
R &= N' \tan\phi + cA \\
D &= W_s \\
W_s &= W \sin\alpha \\
W_h &= W\cos\alpha = N' \\
A &= b\frac{H}{\sin\alpha} \\
W &= \frac{\gamma b H^2}{2}(\cot\alpha - \cot\beta)
\end{aligned}$$



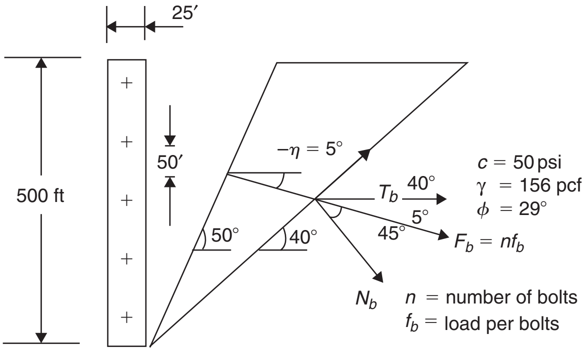
Solving (1)

$$\begin{aligned}
1 &= \frac{\tan\phi}{\tan\alpha} + \frac{Rb\left(\frac{H}{\sin\alpha}\right)}{\frac{\gamma b H^2}{2}(\cot\alpha - \cot\beta)\sin\alpha} \\
H_{\max} &= \left[\frac{2c}{\gamma(\sin\alpha)^2(\cot\alpha - \cot\beta)}\right]\left[\frac{1}{1 - \frac{\tan\phi}{\tan\alpha}}\right] \\
&= \frac{(2)(50)(144)}{(156)(\sin 40)^2(\cot 40 - \cot 50)\left(1 - \frac{\tan 29}{\tan 40}\right)}
\end{aligned}$$

$$\underline{\underline{H_{\max} = 1{,}867\ \text{ft}}} \quad \longleftarrow$$

7. Given: Sketch data and cable bolts
   Bolt spacing vertical = 50 ft   Bolt angle $\eta = -5°$
   Bolt spacing horizontal = 25 ft   Bolt tension = 60% ultimate
   Bolts: 12 strand type 270
   Find: FS, ΔF.

   Solution:

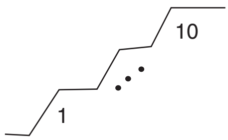


f_b: using Table A1.1 270 k ultimate strength = 495,600 lbf for 12 strands.

$$\underline{f_b = 4.956(10^5)\,\text{lbf}}$$

$$n = \frac{H}{(\text{vert. space})} = \frac{500}{50}$$

$$\underline{n = 10 \text{ holes (benches)}}$$



$$\underline{F_b = (10)(4.956)10^5\,\text{lbf}}$$

$$N_b = F_b \cos 45$$

$$\underline{\underline{N_b = (10)^6(4.956)\frac{1}{\sqrt{2}}}}$$

$$T_b = F_b \sin 45$$

$$\underline{\underline{T_b = (10^6)(4.956)\left(\frac{1}{\sqrt{2}}\right)}}$$

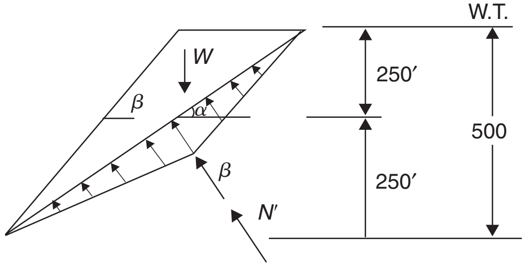
$$FS = \frac{N' \tan\phi + cA + (N_b \tan\phi + T_b)(0.6)}{W \sin\alpha} \quad (60\% \text{ mobilized})$$

$$
\begin{aligned}
FS &= FS^{\circ} + \frac{(N_b \tan\phi + T_b)(0.6)}{W \sin\alpha} \\
&= 1.928 + \frac{(0.6)\left[(10^6)\left(\frac{1}{\sqrt{2}}\right)(4.956)\tan 29 + 10^6\left(\frac{1}{\sqrt{2}}\right)\right]}{(\sin 40)\, 1.719(10^8)} \\
&= 1.928 + \frac{5.447(10^6)}{1.105(10^8)} 0.6 \\
&= 1.928 + (0.049)(0.6)
\end{aligned}
$$

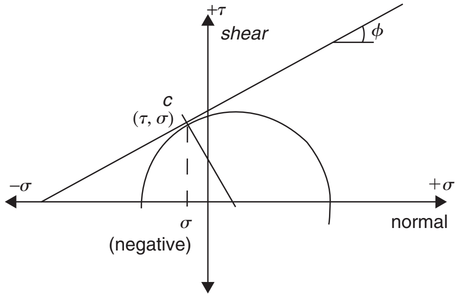
$$\underline{\underline{FS = 1.958, \quad \Delta F = 0.30}}$$

8. Given: Planar block slide data and water table at crest
   Find: *FS*.

   Solution:



$c = 50$ psi $\gamma = 156$ pcf
$\phi = 29°$ $\alpha = 40°$
$\beta = 50°$



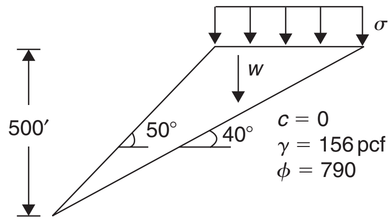
Note: (normal load is negative)

$$FS = \frac{R}{D}$$
$$R = N' \tan\phi + cA$$
$$D = W_s$$
$$N' = W_n - p$$
$$W = \frac{\gamma b H^2}{2}(\cot\alpha - \cot\beta)$$

$$\left.\begin{aligned} \underline{W_n = 1.317(10^8)\,\text{lbf}} \\ \underline{W_s = 1.105(10^8)\,\text{lbf}} \\ \underline{cA = 1.400(10^8)\,\text{lbf}} \end{aligned}\right\} \left(\begin{matrix}\text{prior}\\ \text{calculations}\end{matrix}\right)$$

$$P = \bar{p} A_w$$
$$A_w = \frac{bH}{\sin\alpha}$$
$$\bar{p} = \frac{p_{\max}}{2}$$
$$p_{\max} = \frac{\gamma_w z}{2}$$
$$\frac{z}{2} = 250'$$
$$P = (67.4)\left(\frac{250}{2}\right)(25)\left(\frac{500}{\sin 40}\right)$$
$$P = 1.5168(10^8)$$
$$R = N' \tan\phi + cA$$
$$= (1.317 - 1.517)10^8 \tan 29 + 1.400(10^8)$$
$$R = -1.109(10^7) + 1.4(10^8)$$
$$D = 1.105(10^8)$$
$$FS = \frac{1.289(10^8)}{1.105(10^8)}$$
$$\underline{\underline{FS = 1.167}}$$

9. Given: Planar block slide in sketch with cohesion destroyed
Find: acceleration a



Solution:

Assume $\sigma = 0$ (no surcharge)

then $FS = \dfrac{R}{D}$

But also for the slide mass center

$$F = ma$$
$$m = \frac{W}{g}$$
$$F = D - R$$
$$\therefore D(1 - FS) = \left(\frac{W}{g}\right) a$$
$$\therefore a > 0 \text{ if } FS < 1$$

$$\begin{aligned} FS &= \frac{N' \tan\phi}{W_s} \\ &= \frac{W \cos\alpha \tan\phi}{W \sin\alpha} \\ &= \frac{\tan\phi}{\tan\alpha} \\ &= \frac{\tan 29}{\tan 40} \\ FS &= 0.66 \end{aligned}$$

acceleration – yes downhill = parallel to slide surface

$FS = 0.66$

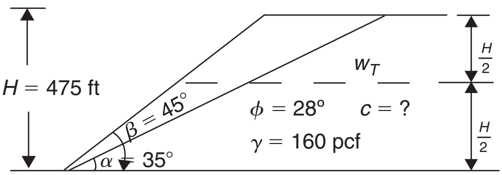
then $D(1 - 0.66) = \dfrac{W}{g} a$

$$D = W \sin\alpha$$
$$\begin{aligned} \therefore a &= g \sin\alpha (1 - 0.66) \\ &= 32.2(\sin 40)(0.34) \end{aligned}$$
$$\underline{\underline{a = 7.04 \text{ ft/s}^2}}$$

$\xleftarrow[\text{(parallel to failure surface)}]{\text{tangential direction}}$

10. Given: Planar block slide

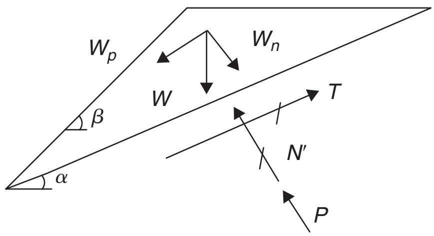
Find: α for $FS = 1.5$.



Solution:

(Free body)

$$\Sigma_n F = 0 \qquad W_n - N' - P = 0$$
$$N' = W_n - P$$
$$W_s = W \sin\alpha \quad W_n = W\cos\alpha$$
$$\begin{aligned} W &= \frac{\gamma b H^2}{2}(\cot\alpha - \cot\beta) \\ &= (150)(1)\left(\frac{475}{2}\right)^2 (\cot 35 - \cot 45) \\ &= 7.728 \end{aligned}$$
$$W = 7.252(10^6)\ \text{lbf}$$
$$W_n = 7.252(10^6)\cos 35$$
$$\underline{W_n = 5.941(10^6)\ \text{lbf}}$$
$$W_s = 7.252(10^6)\sin 35$$
$$\underline{W_s = 4.156(10^6)\ \text{lbf}}$$



Water force must check for water head if $\tan\beta > 2\tan\alpha$, then $h_w = \frac{z}{2}$ else

$$h_w = \frac{z}{2}\left(-1 + \frac{\tan\beta}{\tan\alpha}\right)$$

check

$\tan 45 > 2\tan 35$

$(1) > 2(0.700)$

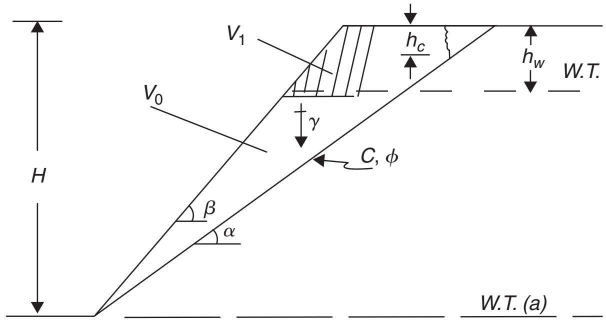
no.

$$\therefore h_w = \left(\frac{475}{2}\right)\left(\frac{1}{2}\right)\left(-1 + \frac{\tan 45}{\tan 35}\right)$$
$$\underline{\underline{h_w = 50.84\ \text{ft}}}$$

$$p_{\max} = (62.4)(50.84)$$
$$\bar{p} = \frac{p_{\max}}{2} = \frac{(6.24)(50.84)}{2}$$
$$\underline{\bar{p} = 1.586(10^3)\ \text{psf}}$$
$$\begin{aligned} P &= \bar{p}A \\ &= 1.586(10^3)(1)\left(\frac{H_w}{\sin\alpha}\right) \\ &= 1.586(10^3)(1)\left(\frac{\frac{475}{2}}{\sin 35}\right) \end{aligned}$$
$$\underline{\underline{P = 6.567(10^5)\ \text{lbf}}}$$

$$N' = 5.941(10^6) - 0.6567(10^6)$$
$$\underline{\underline{N' = 5.284(10^6)}}$$
$$FS = \frac{N' \tan\phi + cbL}{W_s}$$
$$cbL = (1.5)(4.156)(10^6) - \tan 28(5.284)10^6$$
$$= 3.424(10^6)$$
$$c = \frac{3.424(10^6)}{\frac{(1)(475)}{\sin 35}}$$
$$\underline{\underline{c = 4.135(10^3)\text{ psf}}}$$

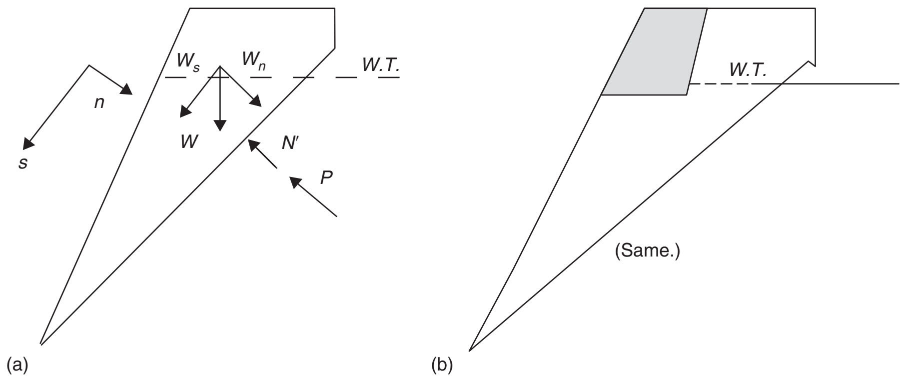
11. Given: Planar block slide in sketch:



Find:
(a) Formula for *FS* with relieving bench V_1
(b) Formula for *FS* with toe berm

Solution:

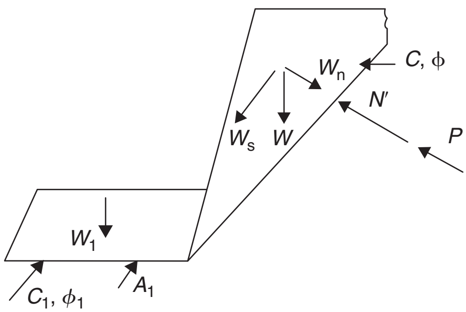
By definition: $FS = \dfrac{R}{D}$
$R =$ resisting forces
$D =$ driving forces



Assume M–C criterion

$$FS_a = \frac{N' \tan\phi + C}{W \sin\alpha} \qquad FS_a = \frac{N' \tan\phi + C}{W \sin\alpha}$$

$$\therefore FS_a = \frac{\tan\phi}{\tan\alpha} + \frac{C}{\gamma V_0 \sin\alpha} \qquad FS_b = \frac{\tan\phi}{\tan\alpha} + \frac{C}{\gamma (V_0 - V_1)\sin\alpha}$$



$C = cA$,
$A =$ area of failure surface
$N' = (W\cos\alpha - P)$
$W = \gamma V_0$

$$\therefore FS_a = \frac{(W\cos\alpha - P)\tan\phi + C}{W\sin\alpha}$$

But $P = 0$ when W.T. is below to toe. The first terms are the same, but the second term in FS_b is greater because $V_0 - V_1$ is less then V_0 is FS.

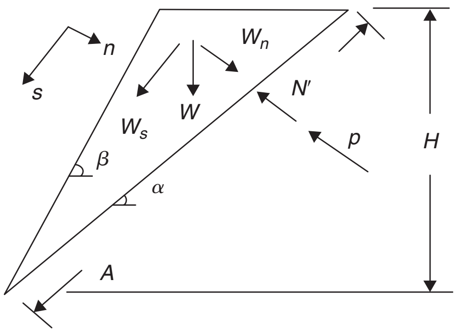
$$FS = \frac{(N'\tan\phi + C) + W_1\tan\phi + C_1}{W_s} \quad \text{(a)}$$

$$FS(with\ berm) = FS(without\ berm) + \frac{W_1\tan\phi_1 + c_1A_1}{W_s} > FS(without\ berm) \quad \text{(b)}$$

The added resistance $W_1\tan\phi_1 + C_1$, comes without added driving force and thus increases the FS.

12. Given: sketch, data
No tension crack, no benches, $\alpha = 29°$, $\gamma = 156$ pcf, $\beta = 50°$, persistence $= 0.87$
$c_r = 64{,}800$ psf $\quad c_j = 1{,}620$ psf
$\phi_r = 32°$ $\quad \phi_j = 25°$

Find:
(a) $H_{\max}$ when WT at crest
(b) $H_{\max}$ when WT at toe



Solution:

$$FS = \frac{R}{D}$$
$$D = W \sin\alpha$$
$$R = N' \tan\phi + C$$
$$N' = (W_n - p)$$
$$C = cA$$
$$W_n = W\cos\alpha$$

(b) When depressurized $P = 0$

$$W = \frac{\gamma H^2}{2}(\cot\alpha - \cot\beta)$$
$$A = \frac{H}{\sin\alpha}$$
$$FS = \frac{W\cos\alpha\tan\phi}{W\sin\alpha} + \frac{\frac{cH}{\sin\alpha}}{W\sin\alpha}$$
$$FS = \frac{\tan\phi}{\tan\alpha} + \frac{\frac{cH}{\sin\alpha}}{\frac{\gamma H^2}{2}(\cot\alpha - \cot\beta)\sin\alpha}$$
$$\therefore \left(FS - \frac{\tan\phi}{\tan\alpha}\right)\left(\frac{\gamma}{2}\right)\left(\frac{\cot\alpha - \cot\beta}{\frac{c}{\sin^2\alpha}}\right) = \frac{1}{H}$$

By inspection $\frac{1}{H}$ is minimum when FS is minimum

$\therefore H_{\max}$ occurs of $FS = 1$.

need rock mass c, ϕ

$$c = (1-p)c_r + pc_j$$
$$= (1 - 0.87)64{,}800 + 0.87(1{,}620)$$
$$= 8{,}424 + 1{,}409$$
$$\underline{\underline{c = 9{,}833 \text{ psf}}}$$

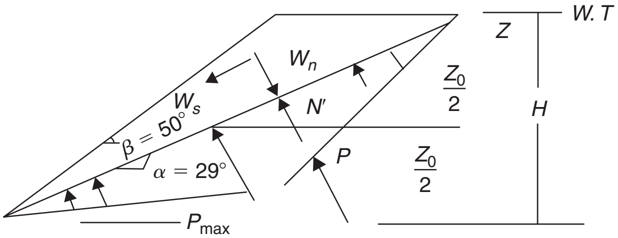
$$\tan\phi = (1-p)\tan\phi_r + p\tan\phi_j$$
$$= (1 - 0.87)\tan 32 + 0.27\tan 25$$
$$\tan\phi = 0.08123 + 0.4057$$
$$= 0.4829$$
$$\underline{\underline{\phi = 25.96 \approx 26°}}$$

$$\frac{1}{H} = \left(1 - \frac{0.4829}{0.5543}\right)\left(\frac{156}{2}\right)\left(\frac{0.96495}{\frac{9833}{(0.4848)^2}}\right)$$

$$= (0.12157)(78)(0.2307)(10^{-4})$$

$$\frac{1}{H} = 2.188(10^{-4})$$

$$\underline{\underline{H = 4.571\,\text{ft}}} \qquad \longleftarrow H_{\max}\text{ dry}$$



Water force:

if $\tan\beta > 2\tan\alpha$ then use $\dfrac{z_0}{2}$

$\tan(50) > 2\tan 29$

$1.1918 > 2(0.5543) = 1.1086$

o.k.

$$P = 2\left(\gamma_w \frac{z_0}{2}\right)\left(\frac{1}{2}\right)L_w$$

$$= \gamma_w \frac{H}{2}\frac{\frac{H}{2}}{\sin\alpha}$$

$$P = \frac{\gamma_w H^2}{4\sin\alpha}$$

$$FS = \frac{(W\cos\alpha - p)\tan\phi + \frac{cH}{\sin\alpha}}{W\sin\alpha}$$

$$= \frac{\tan\phi}{\tan\alpha} + \frac{\frac{c}{\sin\alpha}}{\left(\frac{\gamma}{2}\right)H(\cot\alpha - \cot\beta)\sin\alpha}$$

$$- \frac{\frac{\gamma_w H^2}{4\sin\alpha}\tan\phi}{\left(\frac{\gamma}{2H^2}\right)(\cot\alpha - \cot\beta)\sin\alpha}$$

$$K = \frac{\gamma_w \tan\phi}{\left(\frac{\gamma}{2}\right)(\sin^2\alpha)(\cot\alpha - \cot\beta)}$$

$$= \frac{\left(\frac{1}{4}\right)(62.4)\tan 26}{\left(\frac{156}{2}\right)(\sin^2 29)(\cot 29 - \cot 50)}$$

$$K = 0.4301$$

$$\underline{\underline{\left[FS - \frac{\tan\phi}{\tan\alpha} + K\right]\left[\left(\frac{\gamma}{2}\right)\frac{\cot\alpha - \cot\beta}{\frac{c}{\sin^2\alpha}}\right] = \frac{1}{H}}} \qquad \longleftarrow \text{formula}$$

$$\underline{\underline{[0.5502]\left[\left(\frac{156}{2}\right)\frac{\cot 29-\cot 50}{(9{,}833)\left(\frac{1}{\sin^2 29}\right)}\right]=\frac{1}{H}}}$$

$$9.899(10^{-4})=\frac{1}{H}$$

$$\underline{\underline{H=1{,}010\,\text{ft}}} \qquad \xleftarrow{H_{\max}\ \text{wet}}$$

13. Given: Planar block slide

$\gamma = 158$ pcf

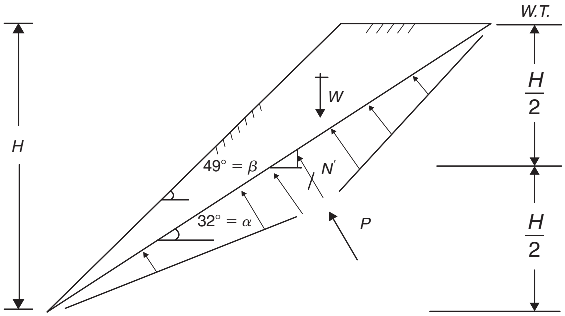
$\left.\begin{array}{ll} \phi_n = 38^\circ & \phi_j = 27^\circ \\ c_r = 1{,}000 \text{ psi} & c_j = 15 \text{ psi} \end{array}\right\}$ Mohr–Coulomb

$A_j/A = 0.93$

bench height = 55 ft

b = breadth

Find: H_{max}.



Solution:

H_{max} occurs when $FS = 1$

$$FS = \frac{R}{D}$$

$R = W_n' \tan\phi + cLb$

$D = W_s$

$\therefore W_s = W_n \tan\phi + cLb$

$W \sin\alpha = (W \cos\alpha - P)\tan\phi + cLb$

$$W = \frac{\gamma H^2}{2}(\cot\alpha - \cot\beta)$$

$$= (158)\left(\frac{1}{2}\right) H^2(\cot 32 - \cot 49)$$

$$\underline{W = 57.75\, H^2}$$

$$57.75\, H^2 \sin 32 = 57.75\, H^2 \cos 32 \tan\phi - P\tan\phi + \frac{cH(1)}{\sin 32}$$

$$\underline{\underline{30.60 = 48.98 \tan\phi + \frac{c}{H}1.887 - \frac{P\tan\phi}{H^2}}}$$

need, ϕ, c for rock mass

$$c = (1 - 0.93)c_r + 0.93c_j$$

$$= 0.07(1{,}000) + 0.93(10.0)$$

$$\underline{\underline{c = 79.3 \text{ psi } (11{,}719 \text{ psf})}}$$

$$\tan\phi = 0.07 \tan 38 + 0.93 \tan 27$$

$$\underline{\underline{\tan\phi = 0.5286}} \quad \phi = 27.9^\circ$$

return

$$30.60 = 48.98(0.5286) + \frac{(11{,}419)(1.887)}{H} - \frac{P}{H^2}(0.5286)$$

$$\text{need: } P = \bar{p}Lb$$

$$= \left(\frac{\gamma_w H}{4}\right)\left(\frac{H}{\sin\alpha}\right)(1)$$

$$\frac{P}{H^2} = \frac{(62.4)}{4}\frac{H^2}{\sin\alpha}$$

$$30.60 = 25.89 + \frac{1}{H}(22{,}812) - \frac{62.4}{4\sin 32}(0.5286)$$

$$\frac{1}{H} = \frac{30.60 - 25.89 + 15.56}{21{,}549}$$

$$\underline{\underline{H = 1{,}065\,\text{ft}}}$$

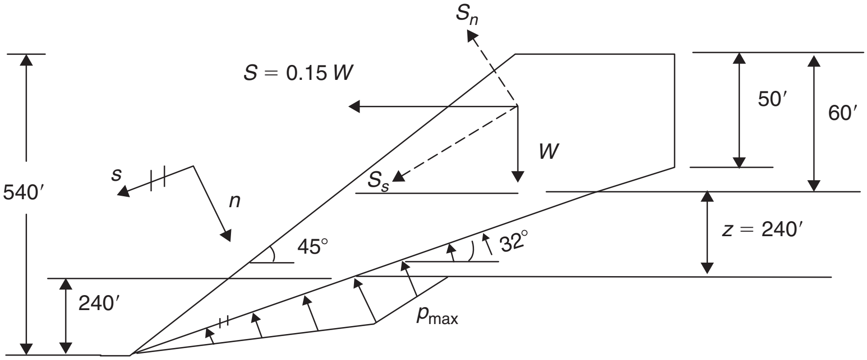
14. Given: Planar block slide data

$FS(\min) = 1.05$

Bench height $= 60'$

Persistance $= 79\% \left(\dfrac{A_j}{A}\right)$

$Wt = 1.351(10^7)$ lbf per ft of thickness.



$\phi_r = 33°$ $c_r = 2{,}870$ psi $\phi_j = 28°$ $c_j = 10.0$ psi $\gamma = 158$ pcf

Find: If $FS = 1.05$ possible.

Solution:

Assume water distribution as $p = \gamma_w z$ where z is 1/2 distance (vertical) to toe

$$z = \frac{(540 - 60)}{2} \qquad p_{\max} = \frac{(62.4)(240)}{144}$$

$$\underline{z = 240\text{ ft},} \qquad \underline{p_{\max} = 104\text{ psi}}$$

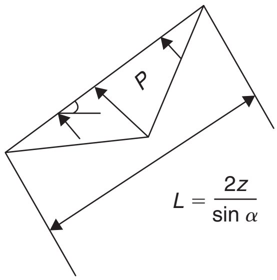
$$\overline{P} = \frac{p_{\max}}{2}$$

$$\overline{P} = 52\text{ psi}$$

$$P = \overline{p}A$$

$$= \overline{p}(1)\left(\frac{480}{\sin(32)}\right)(144)$$

$$\underline{P = 6.783(10^6)\text{ lbf}}$$



$$W_n = W\cos\alpha$$

$$= 1.351(10^7)\cos 32$$

$$\underline{W_n = 1.146(10^7)\text{ lbf}}$$

$$W_s = 1.351(10^7)\sin 32$$

$$\underline{W_s = 7.159(10^6)\text{ lbf}}\text{ (no seismic force)}$$

$$
\begin{aligned}
W_n' &= W_n - P \\
&= 1.146(10^7) - 6.783(10^6) \\
\underline{W_n'} &\underline{= 4.6745(10^8)\ \text{lsf}}\ \text{(no seismic force)}
\end{aligned}
$$

$$W_n' = W_n - P - S_s$$

$$
\begin{aligned}
S_n &= S\sin\alpha & S_s &= S\cos\alpha \\
&= 0.15\ W\sin 32 & &= 0.15\ W\cos\alpha \\
&= 0.15\ W_s & &= 0.15\ W_n \\
&= 0.15\,(7.159)(10^6) & &= 0.15\,(1.146)10^7 \\
\underline{S_n} &\underline{= 1.074(10^6)\ \text{lbf}} & \underline{S_s} &\underline{= 1.719\,(10^6)\ \text{lbf}}
\end{aligned}
$$

$$
\begin{aligned}
W_n' &= 4.6745 - 1{,}074(10^6) \\
\underline{W_n'} &\underline{= 3{,}600(10^6)\ \text{lbf} \qquad \text{with seismic force}}
\end{aligned}
$$

need: c, ϕ

$$
\begin{aligned}
c &= 0.79\,c_j + 0.21\,c_r \\
&= (0.79)(10) + (0.21)(2870) \\
&= 7.9 + 603 \\
\underline{c} &\underline{= 610.9\ \text{psi}}
\end{aligned}
$$

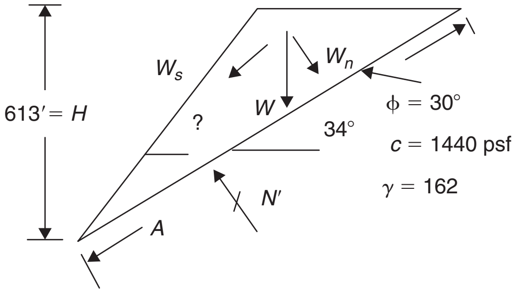
$$
\begin{aligned}
\tan\phi &= (0.79)\tan 28 + (0.21)\tan 33 \\
\tan\phi &= 0.556 \\
\underline{\phi} &\underline{= 29.1^\circ}
\end{aligned}
$$

$$
\begin{aligned}
FS &= \frac{W_n'\tan\phi + cA}{W_s + S_s}, \\
&= \frac{3.60(10^6)\tan 29.1 + (611)(144)(92.5)(1)}{7.159(10^6) + (1.719)(10^6)} \\
&= \frac{2.004(10^6) + 8.139(10^7)}{8.878(10^6)} \\
&= \frac{81.72(10^6)}{8.878(10^6)} \\
\underline{\underline{FS}} &\underline{\underline{= 9.39}} \qquad \text{Yes.}
\end{aligned}
$$

Note:
$$L = \frac{540 - 50}{\sin 32}$$

Note: The actual water pressure is less because of the slope and failure surface angles, so *FS* is even higher.

15. Given: Planar block slide Mohr–Coulomb failure
Find: $\beta_{\max}$ (dry).



Solution:

$$F_S = \frac{R}{D} = \frac{N' \tan\phi + cA}{W_s}$$

$$N' = W_n$$

$$W_n = W\cos\alpha$$

$$A = \frac{H}{\sin\alpha} \qquad \text{(1 ft thick)}$$

$$W_s = W\sin\alpha$$

$$\therefore FS = \frac{\tan\phi}{\tan\alpha} + \frac{cH}{(\sin\alpha)^2 W}$$

But $W = \frac{\gamma H^2}{2}(1)(\cot\alpha - \cot\beta)$

So FS is min when β is max.

$$FS_{\min} = 1.0$$

$$\therefore 1 = \frac{\tan\phi}{\tan\alpha} + \frac{cH}{\sin^2(\alpha)W}$$

$$1 = \frac{\tan 30}{\tan 34} + \frac{(1{,}440\ \text{psf})(613\ \text{ft})(1\ \text{ft})}{\sin^2(34)\ W}$$

$$1 = 0.856 + \frac{2.823(10^6)}{W}$$

$$W = \frac{2.823(10^6)}{0.144}$$

$$\underline{\underline{W = 19.598(10^6)\ \text{lbf}}}$$

$$W = \frac{\gamma H^2}{2}(\cot\alpha - \cot\beta)$$

$$\cot\alpha - \cot\beta = \frac{(2)(19.598)(10^6)}{(162)(613)^2}$$

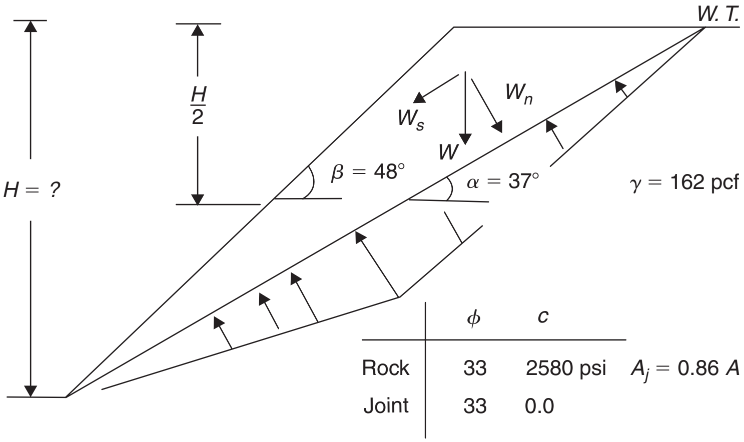
$$\cot\alpha - \cot\beta = 0.64387$$

$$\cot\beta = \cot 34 - 0.64357$$

$$\cot\beta = 0.83869$$

$$\underline{\underline{\beta_{\max} = 50^\circ}} \qquad \xleftarrow{\beta_{\max}}$$

16. Given: Slope, sketch, no seismic load, etc.



Find: H at $FS = 1.15$.

Solution:

$$
\begin{aligned}
FS &= \frac{F_R}{F_D} \\
F_D &= W_s \\
&= W \sin\alpha \\
F_D &= \frac{\gamma H^2}{2}(b)(\cot\alpha - \cot\beta)\sin(\alpha_j) \\
&= H^2\frac{(162)(1)}{2}(\cot 37 - \cot 48)\sin(37) \\
F_D &= H^2(81)(0.4266)\sin(37) \\
&\underline{\underline{F_D = 20.80\,H^2}} \\
F_R &= W_n' \tan\phi + cL \\
W_n' &= W_n - P \\
W_n &= W\cos\alpha \\
&= H^2(81)(0.4266)\cos(37) \\
&\underline{\underline{W_n = 27.60\,H^2}}
\end{aligned}
$$

$$
\begin{aligned}
P &= \bar{p}A \qquad \bar{p} = \frac{p_{\max}}{2},\; A = Lb \\
&= \left(\gamma_w \frac{H}{2}\right)\left(\frac{1}{2}\right)(1)\left(\frac{H}{\sin\alpha}\right) \\
&= \left(\frac{62.4}{4}\right)H^2\frac{1}{\sin 37} \\
&\underline{\underline{P = 25.92\,H^2}}
\end{aligned}
$$

$$cL = \left(c_r \frac{A_n}{A} + c_j \frac{A_j}{A}\right)\left(\frac{H}{\sin\alpha}\right)$$

$$= (0.14)(2{,}580)(144)\left(\frac{H}{\sin 37}\right)$$

$$\underline{\underline{cL = 8.643(10^4)H}}$$

$$\therefore \frac{F_R}{F_D} = \frac{(27.60\,H^2 - 25.92\,H^2)\tan(33) + 8.643(10^4)H}{20.80\,H^2}$$

$$\frac{F_R}{F_D} = 0.0525 + \frac{4.155(10^3)}{H}$$

But $F_S = \dfrac{F_R}{F_D}$

$$1.15 = 0.0525 + \frac{4.155(10^3)}{H}$$

$$\frac{1}{H} = 2.641(10^{-4})$$

$$\underline{\underline{H = 3{,}786\text{ ft}}} \qquad \longleftarrow \text{depth at } FS = 1.15$$

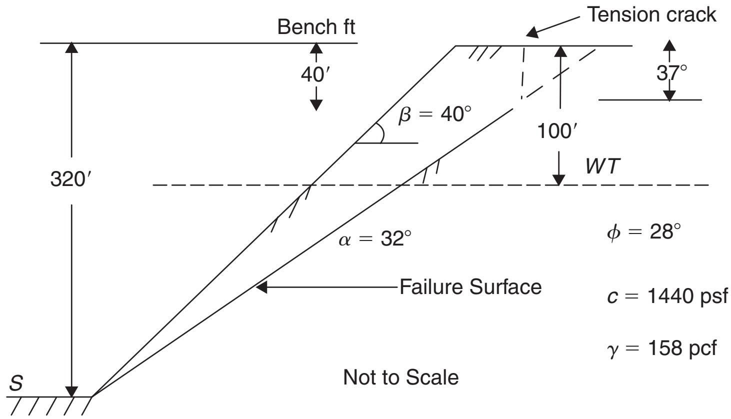
17. Given: Sketch of the potential slope failure shown in the sketch,

Find:

(a) Factor-of safety of a cable bolted slope when the water table is drawn down 100 ft
Bench height = 40′ (vertical bolt spacing)
Horizontal bolt spacing = 20′
Bolt angle = 5° down
Bolt loading = 700 kips/hole

(b) Factor of safety of the same slope but without bolts when the water table is drawn below the toe

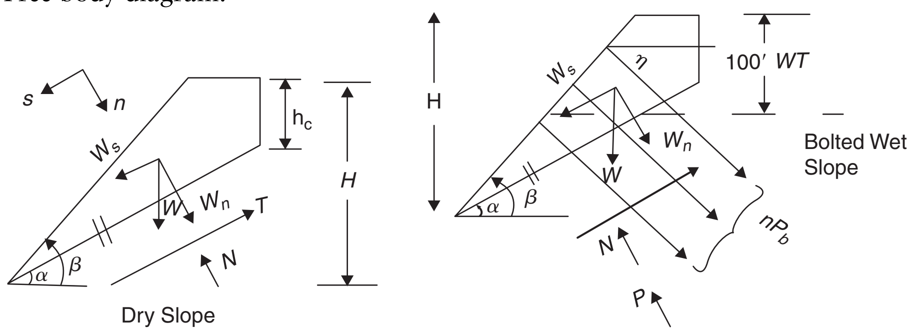
(c) Reasons for preferring one over the other.



Sketch for problem with given data.

Solution:

Free body diagram.



(a) Factor of safety of a bolted slope with water table

$$\eta = \text{bolting angle} = -5°$$

$$n = \text{number of holes per row}$$

$$= \frac{320'}{40'} = 8$$

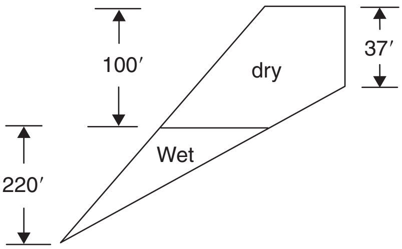
$$\Sigma F_n = 0$$

$$N = W\cos(\alpha) - P_s + nP_b\sin(\alpha-\eta)$$
$$T = N\tan(\phi) + c\left[\frac{H-h_c}{\sin(\alpha)}\right]b$$
$$R = T + nP_b\cos(\alpha-\eta)$$
$$D = W\sin(\alpha)$$

$$FS = \frac{R}{D}$$
$$= \frac{[W\cos(\alpha) - P_s + nP_b\sin(\alpha-\eta)]\tan(\phi) + c\left[\frac{H-h_c}{\sin(\alpha)}\right]b + nP_b\cos(\alpha-\eta)}{W\sin(\alpha)}$$
$$FS = \frac{\tan(\phi)}{\tan(\alpha)} - \frac{P_s\tan(\phi)}{W\sin(\alpha)} + \frac{c(H-h_c)b}{W\sin^2(\alpha)} + \frac{nP_b[\sin(\alpha-\eta)\tan\phi + \cos(\alpha-\eta)]}{W\sin\alpha}$$
$$= \frac{\tan(\phi)}{\tan(\alpha)} - \frac{P_s\tan(\phi)}{W\sin(\alpha)} + \frac{c(H-h_c)b}{W\sin^2(\alpha)} + \frac{nP_b\cos(\alpha-\eta-\phi)}{W\sin\alpha\cos\phi}$$

using—$\sin(A)\sin(B) + \cos(A)\cos(B) = \cos(A-B)$

$\underline{FS = FS_{\text{wet}}\text{ unbolted} + \Delta FS\text{ bolt}}$



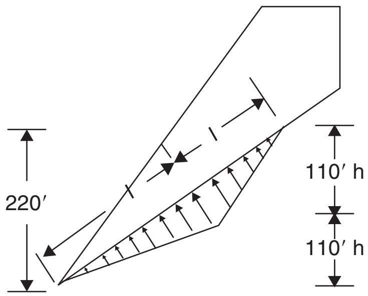
Assume $e = 18\%$

$$G = \frac{\gamma_{\text{dry}}}{\gamma_w}(1+e)$$
$$G = \frac{158}{62.4}(1.18) = 2.99 \rightarrow \text{grain SG gravity}$$

$$\gamma_{\text{wet}} = \left(\frac{2.99+0.18}{1.18}\right)62.4 = \left(\frac{G+Se}{1+e}\right)\gamma_w$$
$$\underline{= 168\text{ pcf}}$$
$$W_{\text{wet}} = \gamma_{\text{dry}}V_{\text{dry}} + (\gamma_{\text{wet}} - \gamma_{\text{dry}})V_{\text{wet}}$$
$$V_{\text{wet}} = \frac{(H_w)^2}{2}[\cot(\alpha) - \cot(\beta)]b$$
$$= \frac{(220)^2}{2}[\cot(32) - \cot(40)]1$$
$$\underline{= 9{,}888\text{ ft}^3}$$

$$V_{\text{dry}} = \frac{H^2}{2}[\cot(\alpha) - \cot(\beta)]b - \left[\frac{1}{2}h_c^2\cot(30)\right]$$
$$= \frac{320^2}{2}[\cot(32) - \cot(40)]1 - \left[\frac{37^2}{2}\cot(32)\right]$$
$$\underline{= 1{,}9824\text{ ft}^3}$$
$$W_{\text{wet}} = (158)1{,}9824(20) + (168-158)9{,}888(20)$$
$$\underline{= 64{,}621{,}440\text{ lbs}}$$

$$P_s = 2\int_0^l Pb\,dl$$
$$= 2\int_0^l b\gamma_w l \sin(\alpha)dl$$
$$= 2\frac{\gamma_w h\, l\, b}{2} = \gamma_w hlb$$
$$= (62.4)(110)\left[\frac{110}{\sin(32)}\right](20) = \underline{28{,}496{,}420\,\text{lbf}}$$



$$FS = \frac{\tan(28)}{\tan(32)} - \frac{(28{,}496{,}420)\tan(28)}{(64{,}621{,}440)\sin(32)} + \frac{1{,}440(320-37)(20)}{(64{,}621{,}440)\sin^2(32)}$$
$$+\frac{(8)(700\times 10^3)\cos(32+5-28)}{(64{,}621{,}440)\sin(32)\cos(28)}$$
$$= 0.851 - 0.442 + 0.449 + 0.183$$
$$\underline{\underline{FS = 1.041}} \quad \leftarrow FS \text{ wet, bolted slope}$$

(b) Factor of safety of a dry slope.

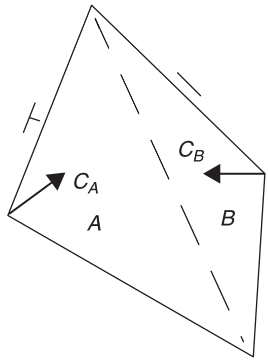
$$W = \gamma_{\text{dry}}\, V_{\text{dry}}$$
$$= (158)19{,}824(20) = \underline{6{,}26{,}43{,}840\,\text{lbs}}$$

$$\Sigma F_n = 0$$
$$N = W\cos(\alpha)$$
$$T = N\tan\phi + c(H-h_c)\frac{b}{\sin(\alpha)}$$
$$R = T$$
$$= [W\cos(\alpha)]\tan(\phi) + c(H-h_c)\frac{b}{\sin(\alpha)}$$
$$D = W\sin(\alpha)$$

$$FS_{\text{dry}} = \frac{R}{D} = \frac{W\cos(\alpha)+\tan(\phi)+c(H-h_c)\frac{b}{\sin(\alpha)}}{W\sin\alpha}$$
$$= \frac{\tan(\phi)}{\tan(\alpha)} + \frac{c(H-h_c)b}{W\sin^2(\alpha)}$$
$$= 0.851 + \frac{1440\,(320-37)\,(20)}{6{,}26{,}43{,}840\sin^2(32)}$$
$$= 0.851 + 0.463$$
$$\underline{\underline{FS = 1.314}} \quad \leftarrow FS \text{ dry, unbolted}$$

Wedge Failures

18. Given: Wedge data



A: $\alpha = 0^\circ \quad \delta = 60^\circ$
B: $\alpha = 90^\circ \quad \delta = 60^\circ$

Find: normals C_A, C_B.

Solution:

	x	y	z
C	$\sin\delta\sin\alpha$	$\sin\delta\cos\alpha$	$\cos\delta$
A:	$\left(\frac{\sqrt{3}}{2}\right)(0)$	$\left(\frac{\sqrt{3}}{2}\right)(1)$	$\left(\frac{1}{2}\right)$
B:	$\left(\frac{\sqrt{3}}{2}\right)(1)$	$\left(\frac{\sqrt{3}}{2}\right)(0)$	$\left(\frac{1}{2}\right)$

$$\therefore \quad \left.\begin{array}{l} \underline{\underline{\vec{C}_A = \left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)}} \\ \underline{\underline{\vec{C}_B = \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)}} \end{array}\right\} \text{direction cosines}$$

$$\left.\begin{array}{l} \underline{\underline{\vec{C}_A\text{: } (90^\circ, 30^\circ, 60^\circ)}} \\ \underline{\underline{\vec{C}_B\text{: } (30^\circ, 90^\circ, 60^\circ)}} \end{array}\right\} \text{direction angles}$$

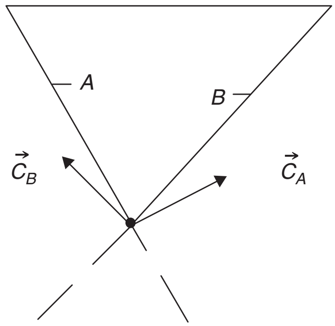
19. Given: The wedge from problem 18,

$c_A = \left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$c_B = \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$

Find: Dip direction and dip of the line of intersection $\vec{s}$.

Solution:



$\vec{s} = \vec{c}_A \times \vec{c}_B$	form "determinant"		
	i	j	h
c_A:	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
c_B:	$\frac{\sqrt{3}}{2}$	0	$\frac{1}{2}$

$\vec{s} = \left(\frac{\sqrt{3}}{4}\ \frac{\sqrt{3}}{4} - \frac{3}{4}\right)$ dir. numbers

$|s| = \left[\left(\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 + \left(-\frac{3}{4}\right)^2\right]^{1/2}$

$|s| = \frac{\sqrt{15}}{\sqrt{16}} = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{5}}{1}\right)$

direction cosines:

	S_x	S_y	S_z
$\vec{S}$	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{\sqrt{3}}{\sqrt{5}}$

$\alpha_s = \underline{\underline{45^\circ}}$

$\delta_s = \underline{\underline{50.8^\circ}}$ $\xleftarrow{\alpha_s, \delta}$

$$\tan\alpha_s = \frac{S_x}{S_y}$$

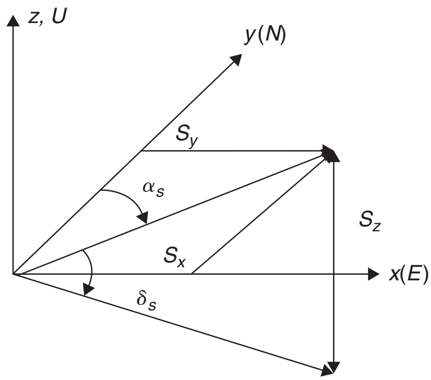
$$\tan\alpha_s = \frac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{5}}}$$

$$\tan\alpha_s = 1$$

$$\therefore \underline{\underline{\alpha_s = 45^\circ}}$$

$$\tan\delta_s = \frac{-S_3}{[S_x^2 + S_y^2]^{1/2}}$$

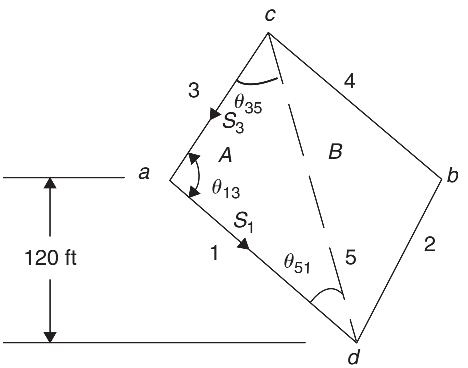
$$\tan\delta_s = \frac{+\frac{\sqrt{3}}{\sqrt{5}}}{\sqrt{\frac{2}{5}}} = +\frac{\sqrt{3}}{\sqrt{2}} \quad \therefore \delta_s = \underline{\underline{50.8^\circ}}$$



20. Given: Table 2.7 data and $\overline{ad} = 120$ ft
Find: Joint plane areas A_A, B_A (no tension crack).

Solution:

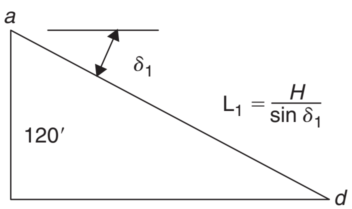
Vectors S_1, S_3 along lines 1 and 2 can be found from intersections of (1) Joint plane A and Face F, and (2) Joint Plane A and Upland U, since normals to F and U can be found from their dip directions and dips. The normal to A is known from problem 18.

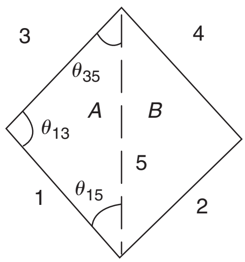


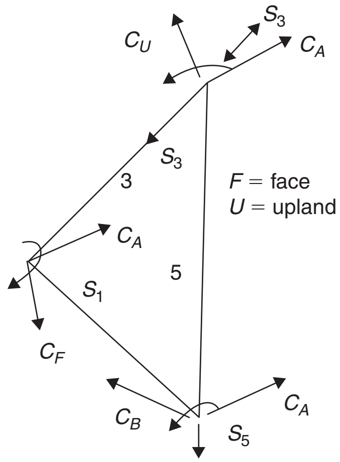
- The angle θ_{13} can be found from $S_1 \cdot S_3 = |S_1||S_3| \cos\theta_{13}$ $|S_1| = 1$ $|S_3| = 1$ by normalization
- Similarly angles θ_{35} & θ_{51} can be found
- The distance L_1 can be found from the dip of line 1 (S_1) and $H = 120'$.
- The distance L_3 can be found.

From the sine law

$$\frac{L_3}{\sin\theta_{15}} = \frac{L_1}{\sin\theta_{35}} = \frac{L_5}{\sin\theta_{13}}$$







Need vectors S_1, S_3, S_5 that are formed by planes of intersection e.g. $S_5 = \vec{C}_A \times \vec{C}_B$ (as in problems 18 & 19) Need direction cosines of C_A, C_F, C_U, C_B

	C: $\sin\delta\sin\alpha$	$\sin\delta\cos\alpha$	$\cos\delta$	
(problem 18)	$C_A = 0$	0.8660	0.5000	$\lvert C_A\rvert = 1$
(Problem 18)	$C_B = 0.8660$	0	0.5000	$\lvert C_B\rvert = 1$
	C_F sin 85 sin 45	sin 85 cos 45	cos 85	
	$C_F = 0.7044$	0.7044	0.08716	$\lvert C_F\rvert = 1$
	$C_U = 0.06163$	0.06163	0.9962	$\lvert C_U\rvert = 1$

CROSS PRODUCTS

$C_A \times C_B =$	0.4330 (0.4472)	0.4330 (0.4472)	−0.7500 (−0.7746)	$\lvert S_5\rvert = 0.9682$ normalized
$C_A \times C_F =$	−0.2767 (−0.3657)	0.3522 (0.4654)	−0.6100 (−0.8060)	$\lvert S_1\rvert = 0.7568$ normalized
$C_A \times C_Z =$	0.8321 (0.9973)	0.0308 (0.03692)	−0.0534 (−0.06400)	$\lvert S_3\rvert = 0.8344$ normalized

dir. cos

	x	y	z
S_5	0.4472	0.4472	−0.7746
S_1	−0.3657	0.4654	−0.8060
S_3	0.9973	0.03692	−0.06400

DOT PRODUCTS

$$S_{13} = |S_1||S_3|\cos\theta_{13} = S_{1x}S_{3x} + S_{1y}S_{3y} + S_{1z}S_{3z}$$
$$S_{13} = -0.2959 \quad \therefore \theta_{13} = 107^\circ \quad \text{(or } 73^\circ \text{ alt. sol.)}$$
$$S_{15} = 0.6689 \quad \therefore \underline{\theta_{15} = 48.0^\circ} \quad \text{[Look of direction of } S_3\text{]}$$
$$S_{35} = 0.5102 \quad \therefore \underline{\theta_{35} = 59.3^\circ}$$

Note: $\theta_{15} + \theta_{35} + \theta_{13} = 180^\circ \therefore \underline{\theta_{13} = 73^\circ}$
Sine law for length

$$\frac{L_1}{\sin\theta_{35}} = \frac{L_3}{\sin\theta_{15}}$$
$$L_1 = \frac{H}{\sin\delta_1}$$

need dip of L_1, δ_1

$$\tan\delta_1 = \frac{-S_{1z}}{[S_{1x}^2 + S_{1y}^2]}$$
$$= \frac{-(-0.8060)}{[0.3657^2 + 0.4634^2]^{1/2}}$$
$$\tan\delta_1 = 1.3617$$

$$\therefore \delta_1 = 53.70^\circ$$
$$L_1 = \frac{120}{\sin 53.7}$$
$$\underline{\underline{L_1 = 148.9\text{ ft}}}$$
$$L_3 = (148.9)\frac{(\sin 48)}{\sin(59.3)}$$
$$\underline{\underline{L_3 = 128.7\text{ ft}}}$$
$$A_A = \tfrac{1}{2}L_1L_3|S_1||S_3|\sin\theta_{13}$$
$$= \left(\tfrac{1}{2}\right)(148.9)(128.7)\sin 73$$
$$\underline{\underline{A_A = 9{,}161\text{ sq ft}}} \qquad \xleftarrow{A_A}$$

by symmetry of this problem

$$\underline{\underline{B_A = 9{,}161\text{ sq ft}}} \qquad \xleftarrow{B_A}$$

Computer check using WEDGE (course download) $\underline{\underline{A_A = B_A = 9{,}161\text{ sq ft o.k.}}}$

20. (Alternative)
Given: Data in Table where the vertical distance between $\underline{a}$ and $\underline{d}$ is 120 ft,
Find: Areas of A and B (without a tension crack)

Solution:

This is a lengthy calculation best done with the aid of a computer program:
Using WEDGE from course downloads

$$A(\text{area}) = B(\text{area}) = \underline{\underline{9{,}160\ \text{ft}^2}} \quad \longleftarrow$$

Using SWEDGE from ROCSCIENCE (same results)
Given: Data in Table & vertical distance $\overline{ad} = 120$ ft, $\gamma = 158$ pcf water below toe slope tension crack offset = 90 ft
Find: A, B areas and FS_{dry} and volume.

Solution:

Using WEDGE course download.

$\underline{\underline{A = B = 8{,}072\ \text{ft}^2}}$

$\underline{\underline{FS_{\text{dry}} = 1.28}}$

$\underline{\underline{\text{Volume} = 11{,}683\ \text{yds}^3.}}$

Same results using SWEDGE from ROCSCIENCE.

21. (no tension crack)

WEDGE	DIP DIRECTION DEGREES	DIP ANGLE DEGREES
PLANE A	0.0	60.0
PLANE B	90.0	60.0
LINE OF INTERSECTION	45.0	50.8
SLOPE FACE	45.0	85.0
UPLAND	45.0	5.0
TENSION CRACK	45.0	75.0
EXTERNAL LOAD	0.0	0.0

WEDGE HEIGHT (LEFT SIDE)	120.0 FT.
LENGTH PLANE A TRACE (UPLAND)	128.8 FT.
TENSION CRACK OFFSET (PLANE A TRACE)	128.8 FT.
EXTERNAL LOAD MAGNITUDE	0.0 KIPS
ROCK UNIT WEIGHT	158.0 PCF.

	FRICTION ANGLE DEGREES	COHESION PSF.
PLANE A	32.0	1080.0
PLANE B	37.0	1640.0

	AREA SQ. FT.	WATER FORCE KIPS
PLANE A	9160.1	0.0
PLANE B	9160.1	0.0
TENSION CRACK	0.0	0.0

WEDGE VOLUME = 3,27,142.6 CU. FT. = 12116.4 CU. YD.
WEDGE WEIGHT = 51,688.5 KIPS

FACTOR OF SAFETY = 1.33

Checks Table 18.

WEDGE	DIP DIRECTION DEGREES	DIP ANGLE DEGREES
PLANE A	0.0	60.0
PLANE B	90.0	60.0
LINE OF INTERSECTION	45.0	50.8
SLOPE FACE	45.0	85.0
UPLAND	45.0	5.0
TENSION CRACK	45.0	75.0
EXTERNAL LOAD	0.0	0.0

WEDGE HEIGHT (LEFT SIDE)	120.0 FT.
LENGTH PLANE A TRACE (UPLAND)	128.8 FT.

TENSION CRACK OFFSET (PLANE A TRACE)	90.0 FT.
EXTERNAL LOAD MAGNITUDE	0.0 KIPS
ROCK UNIT WEIGHT	158.0 PCF.

	FRICTION ANGLE DEGREES	COHESION PSF.
PLANE A	32.0	1080.0
PLANE B	37.0	1640.0

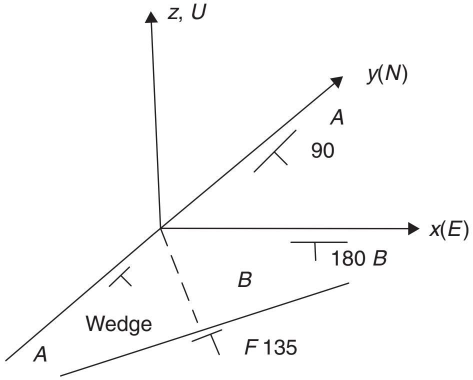
	AREA SQ. FT.	WATER FORCE KIPS
PLANE A	8071.9	0.0
PLANE B	8071.9	0.0
TENSION CRACK	1312.0	0.0

WEDGE VOLUME = 315429.3 CU. FT. = 11682.6 CU. YD.
WEDGE WEIGHT = 49837.8 KIPS

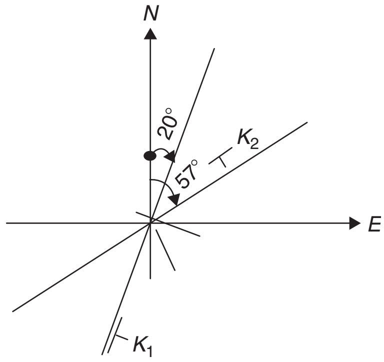
FACTOR OF SAFETY = 1.28 *F.S.*

22. Given: Wedge data

Wedge forms as shown in sketch. Line of intersection has dip direction of 135° and is in the same direction as the face dip direction. When the face dips ±90 to line of intersection, then kinematic failure is impossible. Thus, the range of concern is for face dip directions. (45°, 225°).



23. Given: Wedge data



K_1 joints $\alpha = 110°$ $\delta = 38°$
K_2 joints $\alpha = 147°$ $\delta = 42°$
n_1: $(0.5785, -0.2106, 0.7880)$
n_2: $(\sin\delta\sin\alpha, \sin\delta\cos\alpha, \cos\delta)$
$\sin(42)\sin(147)\sin(42)\cos(147)\cos(42)$

n_2: $(0.3644, -0.5662, 0.7431)$
$\vec{n}_1 \times \vec{n}_2 = \vec{S}(0.2857 \quad -0.1427 \quad -0.2479)$
$|s| = [(0.285)^2 + (-0.1427)^2 + (-0.2479)^2]^{1/2}$
$|s| = 0.4043$
dir. cos : $\vec{s} = (0.7067, -0.3530, -0.6132)$

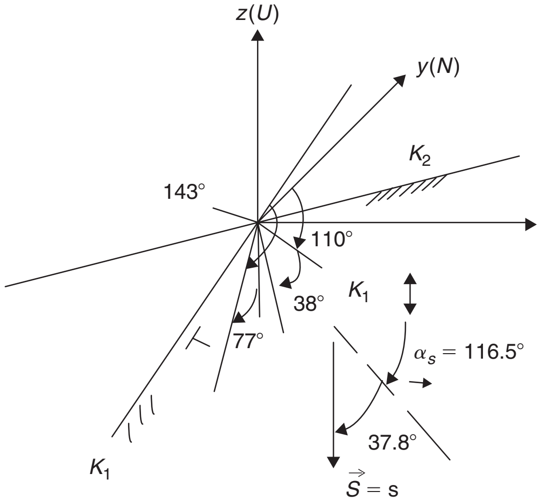
$$\tan\alpha_s = \frac{s_x}{s_y} = \frac{0.7067}{-0.3530} \qquad \cos\delta' = -0.6132, \quad \delta' = 127.8°, \quad \delta = \delta' - 90°$$

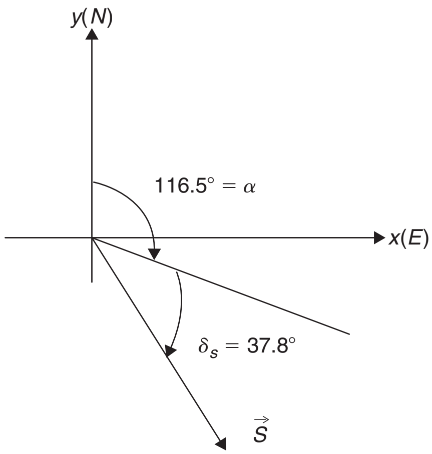
$$\tan\alpha_s = -0.2002 \qquad \underline{\underline{\delta_s = 37.8°}}$$

δ

$\alpha_s = -63.5°$ or $116.5°$
$s_x > 0$ $s_y < 0$ $\therefore$ in 4th quadrant
$\underline{\underline{\alpha_s = 116.5°}}$

α_s





24. Given: Wedge data

Find:

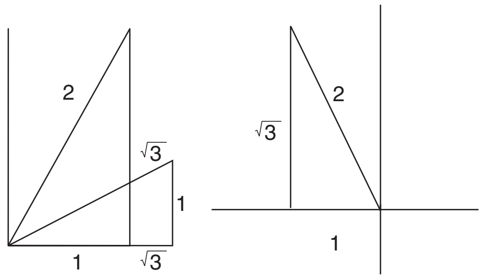
(a) dip direction and dip of lines of intersection of A & B
(b) Range of azimuths that are safe
(c) Length of line formed by A & F (Face)

Solution:

(a) Need normals to A & B

Notes:

	$C_x = \sin\delta\sin\alpha$	$C_y = \cos\delta\cos\alpha$	$C_z = \cos\delta$
	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
B:	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)$	$\frac{1}{2}$
S:	$\left[\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{4}\frac{1}{2}\right]$	$-\left(\frac{\sqrt{3}}{4}\frac{1}{2}-\frac{3}{4}\frac{1}{2}\right)$	$\left[\frac{\sqrt{3}}{4}\left(-\frac{\sqrt{3}}{4}\right)-\frac{3}{4}\cdot\frac{3}{4}\right]$
S:	$\frac{3+\sqrt{3}}{8}$	$\frac{3-\sqrt{3}}{8}$	$-\left(\frac{9+3}{16}\right)$
S:	0.9915	0.1585	−0.7500



$$\sin\alpha = \frac{0.5915}{(0.5915^2 + 0.1585^2)^{1/2}} = \frac{S_x}{(S_x^2 + S_y^2)^{1/2}}$$

$$\sin\alpha = 0.9659$$

$$\underline{\underline{\alpha = 75^\circ}} \qquad \xleftarrow{\alpha_{AB}}$$

$$\sin\delta = \frac{-S_z}{(S_x^2 + S_y^2 + S_z^2)^{1/2}}$$

$$= \frac{0.75}{(0.5915^2 + 0.1585^2 + 0.75^2)^{1/2}}$$

$$\sin\delta = 0.7746$$

$$\underline{\underline{\delta = 50.8^\circ}} \qquad \xleftarrow{\delta_{AB}}$$

(b) The face dip direction is 75° if a line of intersection does not penetrate the face, then sliding cannot occur. Thus 75° ± 90° defines range of dip direction that are unsafe (−15°, +165°)
∴ safe (165°, 345°) ← safe α_s'

(c)

	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
F:	sin 85 sin 75	sin 85 cos 75	cos 85
F:	0.9623	0.2578	0.0872
S_{AF}:	0.0654 − 0.1289	−(0.03776 − 0.48115)	0.1116 − 0.7217
S_{AF}:	−0.0635	0.44339	−0.6101

$$\sin\alpha = \frac{-0.0635}{[(-0.0635)^2 + (0.44339)^2]^{1/2}}$$

$$\sin\alpha = -0.1418$$

$$\underline{\underline{\alpha = -8.2^\circ}} \quad \text{quadrant!}$$

$$\sin\delta = \frac{-(-0.6101)}{(0.0635^2 + 0.4433^2 + 0.6101^2)^{1/2}}$$

$$\sin\delta = 0.8062$$

$$\underline{\underline{\delta = 53.7^\circ}}$$

$$L = \frac{H}{\sin\delta}$$

$$= \frac{85}{0.8062}$$

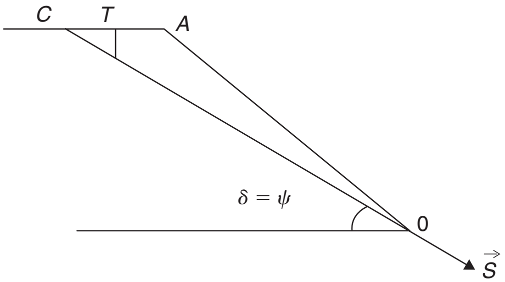
$$\underline{\underline{L = 105.5\ \text{ft}}}$$

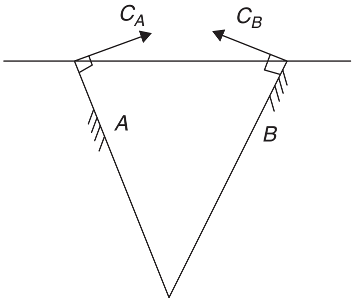
25. Given: Wedge slide data and required $FS = 1.10$

Find: If can obtain $FS = 1.10$.

Solution:

Try quick screening for sliding down line of intersection assuming zero cohesion and lowest $\phi(28°)$





Direction cosines (from T.2)

	x	y	z
$\vec{C}_A$	0.7501	0.4321	0.5000
$\vec{C}_B$	−0.4337	−0.7501	0.5000

$$\begin{aligned}\vec{S} &= \vec{C}_A \times \vec{C}_B = (S_x, S_y, S_z)\\ &= (0.4321)(0.5000) - (-0.7501)(0.5000)\\ &\quad -(0.7501)(0.5000) + (-0.4333)(0.5000)\\ &\quad +(0.7701)(-0.7501) - (-0.4337)(0.4321)\end{aligned}$$

$$\vec{S} = (0.5911, -0.5911, +0.3753)$$

$$\tan\alpha = \frac{0.5911}{(-0.5911)}$$

$$\alpha = 135° \quad \text{dip direction}$$

$$\tan\delta = \frac{S_z}{\sqrt{S_x^2 + S_y^2}}$$

$$= \frac{0.3753}{[(0.5911)^2 + (0.5911)^2]^{1/2}}$$

$$\tan\alpha = 0.44895$$

$$\underline{\underline{\delta = 24.2°}}$$

$$FS(\text{dry}, C = 0,\ \phi = \phi_{\min}) = \frac{\tan\phi}{\tan\delta}$$

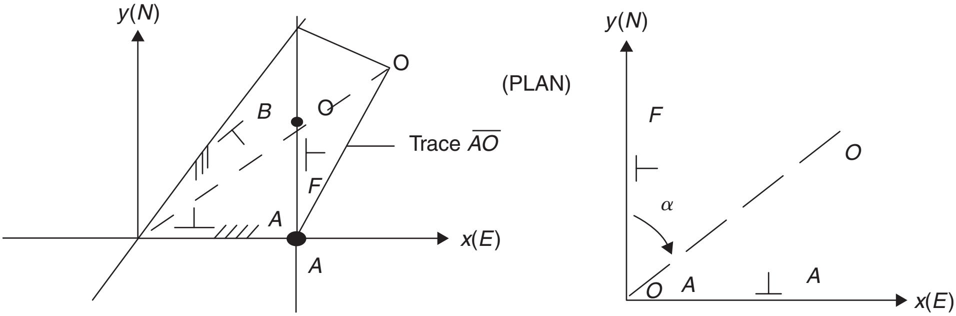
$$= \frac{\tan 28}{\tan 24.2}$$

$$\underline{\underline{FS = 1.18}} \quad \xleftarrow[(>1.10)]{\textit{yes}}$$

(added cohesion and added friction will increase this FS)

26. Given: Wedge data in table, sketch
Find: dip, dip direction, length of $\overline{AO}$

Solution:



Note:

	x	y	z
C_a	$\sin\alpha_a \sin\delta_a$	$\cos\alpha_a \sin\delta_a$	$\cos\delta_a$
C_b	$\sin\alpha_f \sin\delta_f$	$\cos\alpha_f \sin\delta_f$	$\cos\delta_f$

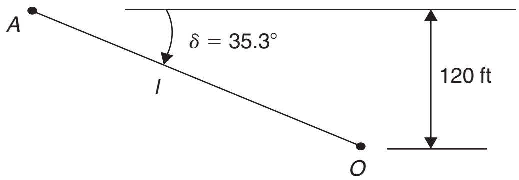
$\alpha_a = 0°$ $\quad \delta_a = 45°$
$\alpha_f = 90°$ $\quad \delta_f = 45°$

S	S_x	S_y	S_z
C_a	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
C_b	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$
S_s	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

$$\tan\alpha_{AO} = \frac{S_x}{S_y} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1\text{: } \underline{\underline{\alpha_{AO} = 45°}} \qquad \xleftarrow{\alpha = 45°}$$

$$\tan\delta_{AO} = \frac{S_z}{\sqrt{S_x^2 + S_y^2}} = \frac{-\frac{1}{2}}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}} = -\frac{1}{\sqrt{2}} : \underline{\underline{\delta_{AO} = -35.3°}} \qquad \xleftarrow{\delta = 35.3°}$$

length



$l \sin 35.5 = 120'$
$\underline{\underline{l = 208 \text{ ft}}}$ $\qquad \xleftarrow{\text{length}}$

27. Given: Wedge data
Find:

(a) Dip direction and dip of lines of intersection of A & B
(b) Range of azimuths that are safe
(c) Length of line formed by A & F (face)

Solution:

(a) Need normals to A & B
Notes:

$C_x = \sin\delta \sin\alpha$ $\quad C_y = \sin\delta\cos\alpha$ $\quad C_z = \cos\delta$

	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
B:	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)$	$\frac{1}{2}$
S:	$\left[\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{4}\frac{1}{2}\right]$	$-\left(\frac{\sqrt{3}}{4}\frac{1}{2}-\frac{3}{4}\frac{1}{2}\right)$	$\left[\frac{\sqrt{3}}{4}\left(-\frac{\sqrt{3}}{4}\right)-\frac{3}{4}\cdot\frac{3}{4}\right]$
S:	$\frac{3+\sqrt{3}}{8}$	$\frac{3-\sqrt{3}}{8}$	$-\left(\frac{9+3}{16}\right)$
S:	0.5915	0.1585	−0.7500

$$\sin\alpha = \frac{0.5915}{(0.5915^2+0.1585^2)^{1/2}} = \frac{S_x}{(S_x^2+S_y^2)^{1/2}}$$

$$\sin\alpha = 0.9659$$

$$\underline{\underline{\alpha = 75^\circ}} \qquad \xleftarrow{\alpha_{AB}}$$

$$\sin\delta = \frac{-S_z}{(S_x^2+S_y^2+S_z^2)^{1/2}}$$

$$= \frac{0.75}{(0.5915^2+0.1585^2+0.75^2)^{1/2}}$$

$$\sin\delta = 0.7746$$

$$\underline{\underline{\delta = 50.8^\circ}} \qquad \xleftarrow{\delta_{AB}}$$

(b) The face dip direction is 75° if a line of intersection does not penetrate the face, then sliding cannot occur. Thus 75° ± 90° defines range of dip direction that are unsafe (−15°, +165°)

$\therefore$ safe $\underline{\underline{(165^\circ, 345^\circ)}}$ $\qquad \xleftarrow{\text{Safe } \alpha_s'}$

(c)

	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
F:	sin 85 sin 75	sin 85 cos 75	cos 85
F:	0.9623	0.2578	0.0872
S_{AF}:	0.0654 − 0.1289	−(0.03776 − 0.48115)	0.1116 − 0.7217
S_{AF}:	−0.0635	0.44339	−0.6101

$$\sin\alpha = \frac{-0.0635}{[(-0.0635)^2 + (0.4433)^2]^{1/2}}$$

$$\sin\alpha = -0.1418$$

$$\underline{\underline{\alpha = -8.2^\circ}} \quad \text{quadrant!}$$

$$\sin\delta = \frac{-(-0.6101)}{(0.0635^2 + 0.4433^2 + 0.6101^2)^{1/2}}$$

$$\sin\delta = 0.8062$$

$$\underline{\underline{\delta = 53.7^\circ}}$$

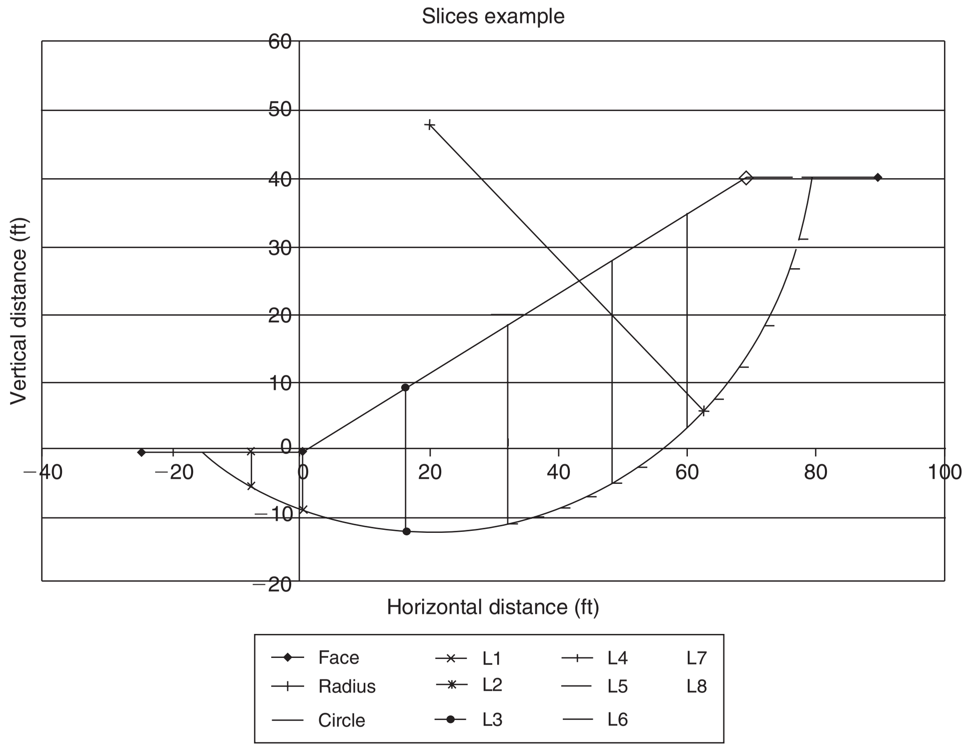
$$L = \frac{H}{\sin\delta}$$

$$= \frac{25.9}{0.8062}$$

$$\underline{\underline{L = 32.13\,\text{m}}}$$

28.

Slice	coordinates x(ft)	h(ft)	h(ave)ft	vol(ft^3)	wt(lbf)	δy	δx	chord slope angle = deg	rads	$W_n =$	$W_s =$	water P(slice)	$\phi =$ deg $N'\tan(\phi)$	$L =$ ft	C = psf $cLb =$ lbf
	Slices Problem														
	30 deg slope, dips below toe														
	$R = 60$ ft			$b = 1$ ft	spwt = 100 pcf										
	−16	0.00													
1			2.53	20.26	2,026	−5.07	8.00	−32.34	−0.56451	1,712	−1,084	1,497	70	9.47	6,818
	−8	5.07										0			
2			6.82	54.54	5,454	−3.50	8.00	−23.64	−0.41268	4,996	−2,187	3,715	416	8.73	6,288
	0	8.57										0			
3			14.84	237.38	23,738	−3.30	16.00	−11.65	−0.20328	23,249	−4,792	15,124	2,640	16.34	11,762
	16	21.10										0			
4			25.18	402.94	40,294	1.08	16.00	3.86	0.06732	40,202	2,711	25,200	4,874	16.04	11,546
	32	29.26										0			
5			31.02	496.33	49,633	5.72	16.00	19.68	0.34344	46,735	16,713	32,892	4,498	16.99	12,234
	48	32.78										0			
6			32.07	384.85	38,485	8.34	12.00	34.81	0.60762	31,596	21,972	29,250	762	14.62	10,524
	60	31.36										0			
7			28.79	267.21	26,721	10.49	9.28	48.51	0.84674	17,701	20,017	25,171	−2,427	14.01	10,086
	69.28	26.23										0			
8			18.04	144.35	14,435	16.37	8.00	63.95	1.11614	6,339	12,968	20,510	−4,605	18.22	13,116
	77.28	9.86										0			
9			4.93	10.80	1,080	9.90	2.19	77.53	1.35316	233	1,054	3,120	−938	10.14	7,303
	79.47	0.00													
									Totals	1,72,764	75,435 dry =		56,134		
				Total =	2,01,865						−8,064 wet =		13,261		89,677
									$FS =$	1.471 wet			WET	DRY	
										2.040 dry		MR =	6.6601E+06	9.2325E+06	
										dry	wet	MD =	4.5261E+06	4.5261E+06	
							RocSci:		Simplified Bishop =	2.318	1.58				
							“Slide”		Janbu =	2.000	1.425				



Slices example
60
50
40
30
20
10
0
−10
−20
−40
−20
0
20
40
60
80
100
Vertical distance (ft)
Horizontal distance (ft)
Face
Radius
Circle
L1
L2
L3
L4
L5
L6
L7
L8

29.

SLICES														
A part W.T. 1														
	SCALE	$1'' = 40'$	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_S	W_N	P	$W_{N'}$	TANϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	25,958.4	−11,695.8	0.726543	0	0		0
2	1.211			220,960	57	185,312.6	120,343.4	161,304	−40,960.6	0.649408	0	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110											
TOT-KJ	0.885		110	155,760										
3	3.407			6,41,108.8	38	394,706	505,200.6	349,752	155,448.6	0	0	2,300	59	135,700
B-YOU	1.051		110											
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG-MED	0.609		125	121,800										
4	7.654			1,435,600	13	322,939.7	1398,806	652,579.2	746,226.5	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD-LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		952,697.9		Total friction =		0		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	0.503937							

$FS = 0.504$

SLICES														
B part W.T.2														
	SCALE =	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_s	W_N	P	$W_{N'}$	TAN ϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	0	14,262.58	0.726543	10,362.37	0		0
2	1.211			220,060	57	185,312.6	120,343.4	0	120,343.4	0.649408	78,151.94	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110											
TOT-KJ	0.885		110	155,760										
3	3.407			641,108.8	38	394,706	505,200.6	1,03,084.8	402,115.8	0	0	2,300	59	135,700
B-YOU	1.051		110											
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG–MED	0.609		125	121,800										
4	7.654			1,435,600	13	322,939.7	1,398,806	4,71,744	92,7061.7	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD-LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		952,697.9		Total friction =		88,514.31		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	0.596846							

$FS = 0.597$

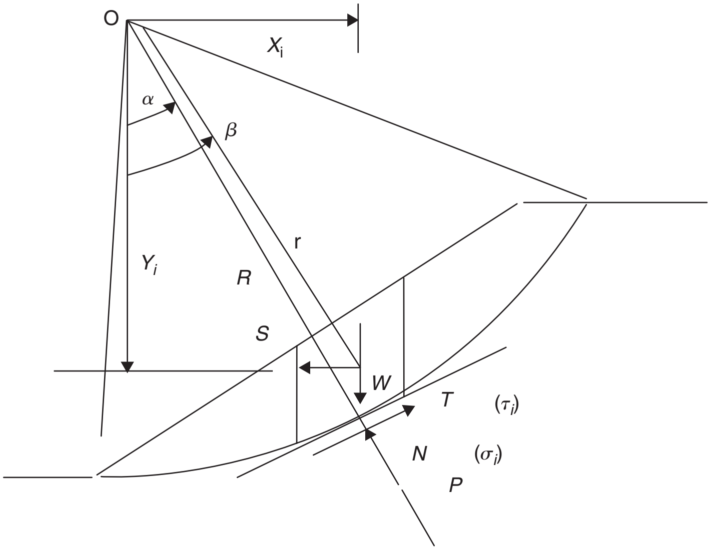
SLICES														
C part W.T. 2 with 140c60 ft relieving bench (crest)														
	SCALE =	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_s	W_N	P	$W_{N'}$	TAN ϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	0	14,262.58	0.726543	10,362.37	0		0
2	1.211			207,936	57	174,389.8	113,250.1	0	113,250.1	0.649408	73,545.45	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110	13,024										
TOT-KJ	0.885		110	155,760										
3	3.407			456,132.8	38	280,823.4	359,437.6	103,084.8	256,352.8	0	0	2,300	59	135,700
B-YOU	1.051		110	184,976										
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG-MED	0.609		125	121,800										
4	7.654			907,600	13	2041,65.6	884,338.3	471,744	412,594.3	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD-LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		709,118.3		Total friction =		83907.82		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	0.795365							

$FS = 0.795$

D part W.T. 1 with 140c60 ft relieving bench (crest)														
	SCALE =	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_s	W_N	P	$W_{N'}$	TAN ϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	25,958.4	−11, 695.8	0.726543	0	0		0
2	1.211			207,936	57	174,389.8	113,250.1	161,304	−48, 053.9	0.649408	0	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110	13,024										
TOT-KJ	0.885		110	155,760										
3	3.407			456,132.8	38	280,823.4	359,437.6	349,752	9,685.551	0	0	2,300	59	135,700
B-YOU	1.051		110	184,976										
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG-MED	0.609		125	121,800										
4	7.654			907,600	13	204,165.6	884,338.3	652,579.2	231,759.1	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		709,118.3		Total friction =		0		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	1.05	$L =$	377.8203	ft				

$L = 378$ ft

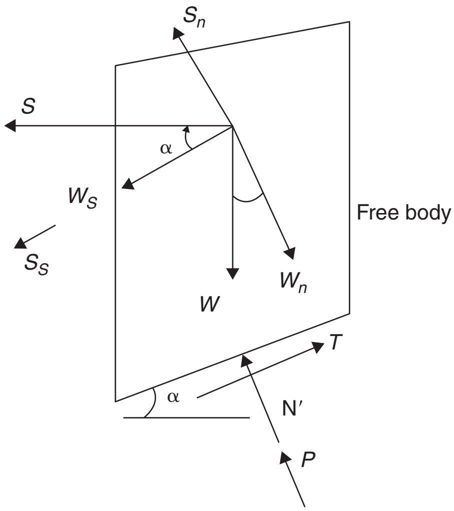
30. Given: Slices analysis with *S* loads



Show: $FS = \dfrac{R\Sigma_1^n(N_i' \tan\phi + C_i)}{R\Sigma_1^n(W_i \sin\alpha_i + S_i \frac{r_i}{R}\cos\alpha_i)}$

when $N_i' = W_i \cos\alpha_i - P_i - S_i \sin\alpha_i$.

Solution:



Overall equilibrium

$\Sigma_1^n(W_i x_i + S_i y_i) = \Sigma_1^n \tau_i A_i R$

$\Sigma[W_i R \sin(\alpha_i) + S_i R_i \cos(\beta_i)]$

$= \Sigma\tau_i \max \dfrac{A_i R}{fs_i}$

- assume $fs_i = fs_j = fs = FS$
- and MC strength so

$\tau_i \max = \sigma_i' \tan\phi_i + c_i$

$\therefore\ \Sigma(W_i R \sin\alpha_i + S_i R_i \cos\beta_i)$

$= \dfrac{\Sigma(\sigma_i' + \tan\phi_i + c_i)A_i R}{FS}$

$\therefore FS = \dfrac{\Sigma_i^n(N_i' \tan\phi_i + C_i)R}{\Sigma_i^n\left[W_i \sin\alpha_i + S_i\left(\frac{r_i}{R}\right)\cos\beta_i\right]}$

when $N_i' = \sigma_i' A_i$ & $C_i = c_i \cdot A_i$

- Normal force equilibrium
- Neglect side forces

$\Sigma_n F = 0$

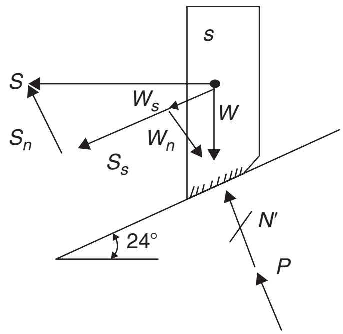
$0 = W_n - N' - P - S_n$

$\underline{\underline{N_i' = W_i \cos\alpha_i - P_i - S_i \sin\alpha_i}}$ (each slice *i*)

31. Given: Figure and table data
$R = 300\,\text{ft} \quad H = 120\,\text{ft} \quad \gamma = 95\,\text{psf} \quad C = 367\,\text{psf} \quad \phi = 16^\circ \quad \beta = 29^\circ$

Find:

(1) Seismic force on slice 7, $a_s = 0.15$
(2) Water force on bottom of 7
(3) FS of 7
(4) FS slip surface, dry & no seismic force.



Solution:

(1)
$$S = ma = \frac{W}{g}0.15\,g$$
$$S = 0.15\,W$$
$$S = 0.15(10.29)(10^5)$$
$$\underline{\underline{S = 1.544(10^5)\,\text{lbf}}} \qquad \longleftarrow (1)$$

(2)
$$P = \bar{p}A \quad \bar{p} = rH$$
$$= (62.4)(82)(75)(1) \qquad \longleftarrow (2)$$
$$\underline{\underline{P = 3.838(10^5)\,\text{lbf}}}$$

(3)
$$FS(7) = \frac{R(7)}{D(7)}$$
$$D = W_s + S_s$$
$$= (10.29)(10^5)\sin 24 + 1.544(10^5)\cos 34$$
$$\underline{D = 5.596(10^5)\,\text{lbf}}$$
$$R = N'\tan\phi + cL$$
$$N' = W_n - P - S_s$$
$$= 10.29(10^5)\cos 24 - 3.838(10^5) - 1.544(10^5)\sin 24$$
$$\underline{N' = 4.934(10^5)\,\text{lbf}}$$
$$FS(7) = \frac{4.934(10^5)\tan 16 + (367)(75)(1)}{5.596(10^5)}$$
$$\underline{\underline{FS(7) = 0.302}} \qquad \longleftarrow (3)$$

Note: (Slopes 1–5 are neg.)

(4)
$$FS = \frac{\Sigma M_R}{\Sigma M_D} \quad \text{(dry, no seismic force)}$$
$$= \frac{\sum_1^{10} W_r \tan\phi + \sum_1^{10} C + \sum_1^{5} W_s}{\sum_6^{10} W_s}$$
$$= \frac{52.67(10^5)\tan 16 + (367)(540) + 5.97(10^5)}{12.93(10^5)}$$
$$\underline{\underline{FS = 1.78}} \qquad \longleftarrow (4)$$

32. This amounts to part 4 of 31.

$$\underline{\underline{FS = 1.78}}$$

$$FS = \frac{\Sigma M_R}{\Sigma M_D}$$

$$FS = \frac{\sum_1^{10} R(W\cos\alpha\tan\phi + cL) + \sum_1^5 R\ W\sin\alpha}{\Sigma_6^{10} R\ W\sin\alpha}$$

$\therefore$ as in 31(4).

33. Given: Circular are failure data in Fig. 2.33 and Table 2.14.
Find: $FS(2)$, $FS(7)$, show total FS.

Solution:

$FS = \Sigma M_R / \Sigma M_D$, $R =$ radius of slips circle
Slice 2

$$FS = \frac{R[(W_n' \tan\phi) + C]}{RW_s}$$

$$P = \gamma_w WL(1)$$
$$= (62.4)(1)(52)(30)$$
$$\underline{P = 9.734(10^4)\,\text{lbf}}$$

$$C = cL(1)$$
$$= (367)(30)(1)$$
$$\underline{C = 1.101(10^4)\,\text{lbf}}$$

normal equilibrium:

$$W_n' = W\cos\alpha - P$$
$$= 27.78(10^4)(\cos 28) - 9.734(10^4)$$
$$= 24.49(10^4) - 9.734(10^4)$$
$$\underline{W_n' = 14.76(10^4)\,\text{lbf}}$$

$$\underline{W_s = W\sin\alpha \quad \text{But is resisting!}}$$
$$\therefore FS = \frac{[(W_n' \tan\phi + C)R + RW_s]}{0}$$
$$\underline{\underline{FS \to \infty \text{ Slice 2}}}$$

$\leftarrow$ Slice 2

Slice 7

$$P = (62.4)(1)(75)(82)$$
$$\underline{P = 3.838(10^5)\,\text{lbf}}$$
$$W_n' = 10.29(10^5)\cos(24) - 3.838(10^5)$$
$$\underline{W_n' = 5.563(10^5)\,\text{lbf}}$$

$$C = (367)(75)(1)$$
$$\underline{C = 2.753(10^4)\,\text{lbf}}$$

$$W_s = 10.29(10^5)\sin 24$$
$$W_s = 4.185(10^5)\,\text{lbf}$$

$$M_R = R(W_n' \tan\phi + C)$$
$$= R(5.563(10^5)\tan 16 + 0.275(10^5))$$
$$\underline{M_R = R(1.870)10^5 \text{ lbf}}$$

$$M_D = RW_n$$
$$\underline{M_D = R(4.185)(10^5)}$$

$$FS = \frac{R(1.870)(10^5)}{R(4.185)(10^5)}$$
$$\underline{FS = 0.447}$$

Slice 7

Algebraically

$$FS = \frac{\Sigma M_R}{\Sigma M_D}$$
$$FS = \frac{\sum_{i=1}^{10} R(W_n' \tan\phi + C)_i + \sum_{i=1}^{5} R(W_s)_i}{\sum_{i=6}^{10} R(W_s)_i}$$

34. Given: Circular failure in sketch
Find: Expression for safety factor

Solution:

By definition

$$FS = \frac{M_R}{M_D} = \left(\frac{\text{Moments resistance}}{\text{Moments driving}}\right)$$
$$M_R = \sum_{\text{slices}} (\text{forces})(\text{lever arm})_R$$
$$M_D = \sum_{\text{slices}} (\text{forces})(\text{lever arm})_D$$

lever arm = R
since circular failure

$$M_R = R\sum_{i=1}^{5} (N \tan\phi + C)_i + \sum_{i=1}^{2} RT_i$$
$$M_D = R\sum_{i=1}^{5} T$$

T = Tangential component of weight
N = Normal component of weight
α = inclination of slice bottom from horizontal
$N = W\cos(\alpha)$
$T = W\sin(\alpha)$

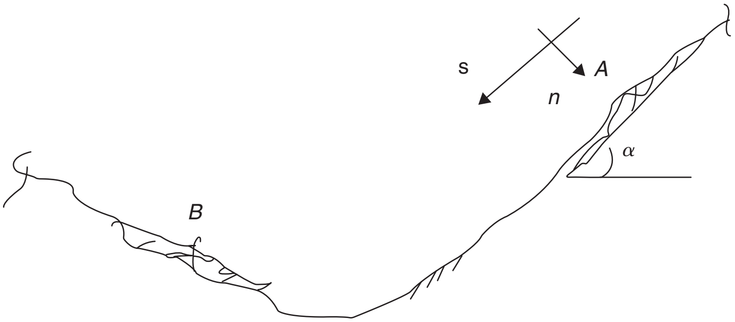
Note: No water table shown, assume dry

$$FS = \frac{\sum_{i=1}^{5} R(N\tan\phi + C)_i + \sum_{i=1}^{2} RT_i}{\sum_{i=3}^{5} RT_i}$$

where $cA = C$ and A = area of slice base

Dynamics, Toppling

35. Given: Slope situation



Find: a, v, s, of mass center.

Solution:

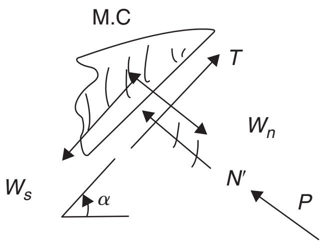
Mass center obeys
F $= m\ddot{s}$ where $s =$ down hill distance
$\dot{s} =$ velocity
$\ddot{s} =$ acc. of mass center

- Choose origin of coordinates n, s at mass center starting point A

At time $t = 0$, $s = \dot{s} = \ddot{s} = 0$

- Downhill forces $= D$, Resisting up hill forces $= F$

So $F = D - R$
$D =$ downhill component of weight
Free body of slide mass
$D = W \sin\alpha$
$N = W \cos\alpha$
$P = 0$ dry assumption



- Frictional resistance

$F_f = N' \tan\phi$

$$\therefore W \sin\alpha - W \cos\alpha \tan\phi = \frac{W}{g}\ddot{s}$$

$$W \sin\alpha - W \cos\alpha \tan\phi = \frac{W}{g}\ddot{s}$$

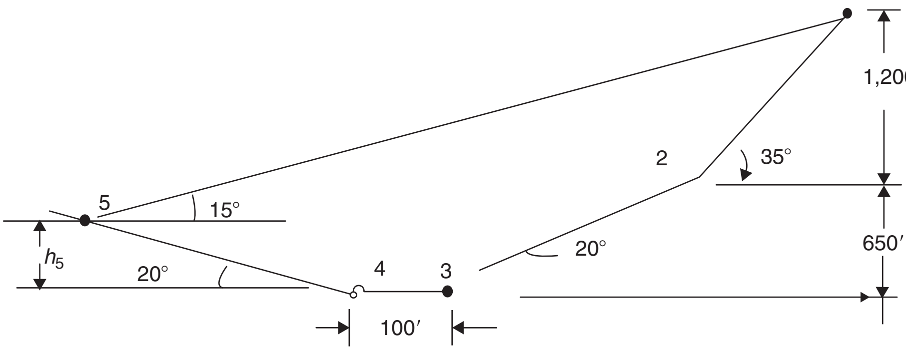
$$\left.\begin{array}{ll} \underline{\ddot{s} = g\left[\dfrac{\sin(\alpha-\phi)}{\cos\phi}\right]} & (\alpha \geq \phi) \\ \underline{(\text{If } \phi > \alpha \text{ then no sliding occurs and } \ddot{s} = 0)} & (\alpha \leq \phi) \end{array}\right\} \longleftarrow$$

At constant slope

$$\dot{s} = \ddot{s}t + \dot{s}(0)$$

$$\left.\begin{aligned} \therefore \underline{\dot{s} = g\left[\frac{\sin(\alpha - \phi)}{\cos\phi}\right]t} \quad & (\alpha \geq 0) \\ \therefore \underline{s = g\left[\frac{\sin(\alpha - \phi)}{\cos\phi}\right]\frac{t^2}{2} \quad (\alpha \geq 0)} \end{aligned}\right\}$$

36. Given: Profile $\phi = 15°$



Find: h_5
Solution:

(1) Draw ϕ-line
(2) From notes on dynamics of sliding, $0 = Wh - Wd \tan\phi$

$$\begin{aligned}
\tan\phi &= \frac{h}{d} \\
h &= 1{,}200 + 650 - h_5 \\
d &= d_1 + d_2 + 100 + d_5 \\
\tan 35 &= \frac{1{,}200}{d_1} \\
\tan 70 &= \frac{650}{d_2} \\
\tan 70 &= \frac{h_5}{d_5} \\
\therefore d &= \frac{1{,}200}{\tan 35} + \frac{65°}{\tan 20} + 100 + \frac{h_5}{\tan 20}
\end{aligned}$$

$$\tan 15° = \frac{(1{,}200 + 650 - h_5)}{\left(\frac{1{,}200}{\tan 35} + \frac{65°}{\tan 20} + 100 + \frac{h_5}{\tan 20}\right)}$$

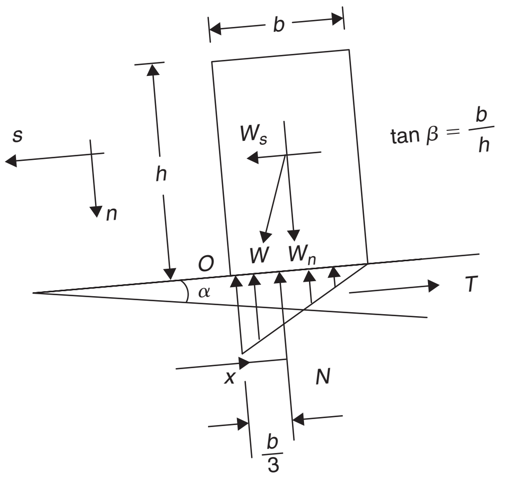
$$(0.260)\left(1{,}713.0 + 1{,}786 + 100 + \frac{h_5}{\tan 20}\right) = 1{,}850 - h_5$$

$$\begin{aligned}
964 + \frac{0.268\,h_5}{\tan 20} &= 1{,}850 - h_5 \\
1{,}738\,h_5 &= 886 \\
\underline{\underline{h_5 = 510}}
\end{aligned}$$

$\xleftarrow{h_5}$

37. Given: Topping rock block

Show: $\tan(\alpha) < \frac{1}{3}\tan\beta$ for stability against toppling.



Solution:

Equilibrium requires

$$\begin{aligned}
\Sigma_n F &= W \qquad W\cos\alpha = N \\
\Sigma_s F &= 0 \qquad W\sin\alpha = T \\
\Sigma_o M &= 0 \qquad 0 = W_s\frac{h}{2} + Nx - N\frac{b}{2}
\end{aligned}$$

where $x =$ distance from 0 to N

$$\begin{aligned}
\therefore\ W\sin\alpha\frac{h}{2} + W\cos\alpha\left(x - \frac{b}{2}\right) &= 0 \\
\tan\alpha\frac{h}{2} &= \frac{b}{2} - x \\
x &= \frac{b}{2} - \frac{h}{2}\tan\alpha
\end{aligned}$$

but with a triangular stress distribution

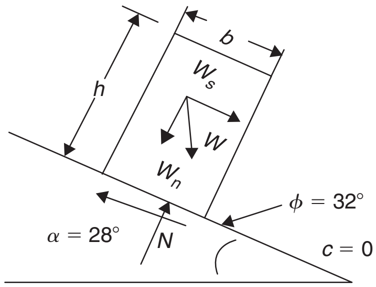
$$\begin{aligned}
& x \geq \frac{b}{3} \\
& \therefore\ \frac{b}{2} - \frac{h}{2}\tan\alpha \geq \frac{b}{3} \\
& \frac{b}{6} \geq \frac{h}{2}\tan\alpha,\ \frac{b}{h} = \tan\beta \\
& \therefore\ \underline{\underline{\tan\alpha \leq \frac{1}{3}\tan\beta}}
\end{aligned}$$

(for stability w.r.t. toppling)

38. Given: Rock block on slope

Find: Dimension necessary to prevent toppling.

To prevent toppling, must be in equilibrium



$\Sigma F = 0$: $N = W_n = W\cos\alpha$

$T = W_s = W\sin\alpha$

$$\Sigma M = 0 \quad Nx - T\frac{h}{2} = 0$$

$$\therefore x = \left(\frac{h}{2}\right)\left(\frac{T}{N}\right)$$

$$x = \left(\frac{h}{2}\right)(\tan\alpha) \;\; \& \;\; x < \frac{b}{2}$$

$$\therefore \left(\frac{h}{2}\right)\tan\alpha \le \frac{b}{2}$$

$$\frac{b}{h} \ge \tan\alpha$$

$$b \ge h\tan 28$$

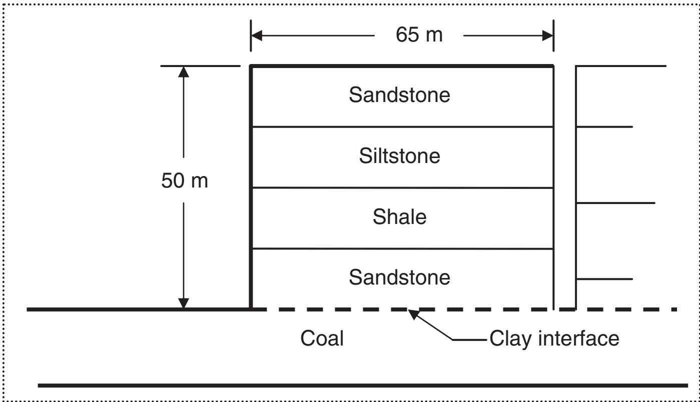
$$\underline{\underline{b \ge h(0.5317)}}$$

square base $\therefore$ b' into page $= b$.

e.g. if $h = 10'$ $b = b' \ge 5.317$ ft

Added Problems

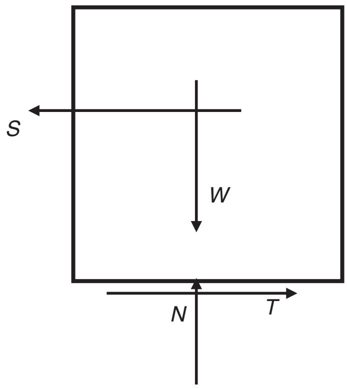
39. Consider a "highwall" in flat sedimentary strata as shown in the sketch. Three sets of joints are present. One set, J1, is composed of bedding plane joints. The other two sets, J2 and J3, are at right angles to the bedding and to each other as is often the case. The vertical joints J2 and J3 allow vertical separation of a large block of ground. For each meter of block into the page, estimate the acceleration (from blasting) that will just move the block forward as it slips along the clay interface. Note: the cohesion of the clay interface is 100 kPa; the friction angle is zero.



Given: large rock block, material properties of the interface
Find: acceleration a necessary to move the block.

Solution:

Draw a free body diagram with inertial force.

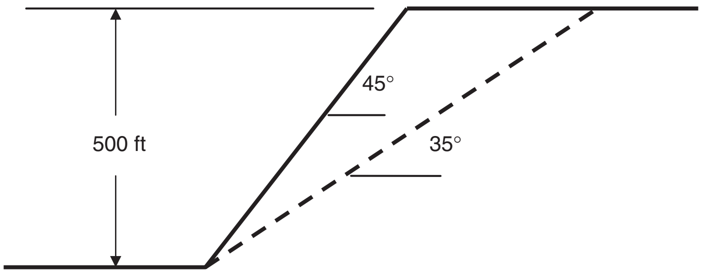


Thus, W = weight, N = normal reaction, S = inertial force, T = tangential reaction. Now consider a safety factor with respect to sliding. Thus,

$$\begin{aligned} FS &= R/D \\ &= (N\tan(\phi) + cA)/S \\ &= (0 + cA)/S \\ &= (0.1)(65)(1)/(W/g)a \\ &= (0.1)(65)(1)/(22.6)(65)(50)(a/g) \\ FS &= 0.0885/(a/g) \end{aligned}$$

where a reasonable specific weight of 22.6 kN/m^3 is assumed. The ratio (a/g) is a seismic coefficient. At the instant of slip, $FS = 1$, so $(a/g) = 0.0885$.
Thus, $a = (0.0885)(9.807)$, so $\underline{\underline{a = 0.868\ \text{m/s}^2}}$

40. Suppose the cohesion of the failure surface in the sketch is diminished with each blast by fracturing of the interface, so the persistence p, defined as the ratio of facture area to total area, decreases by steps with each blast. Further suppose blasting occurs twice a day and the slope moves downhill 0.1 inch per blast. Is there a day when the slope continues to accelerate after a blast? Note: cohesion and angle of friction of the failure surface are (900 psi, 40°). The fractured portion of the failure surface has cohesion and friction angle (0 psi, 30°). Specific weight is 150 pcf.



Given: slope data
Find: days to failure.

Solution:

By inspection, the failure surface inclination is less than the friction angle before any blast damage has occurred (40 versus 35 degrees), so the slope is safe at the outset. However, the friction angle of damaged, fractured failure surface is less than the failure surface slope (30 versus 35 degrees) and is also cohesionless, so failure will occur when blast damage extends the length of the slope. Blasting occurs twice per day, so the fracturing is 0.2 inch per day. The slope length is 500/sin(35) = 872 ft or 10,461 inches. Days to failure are no more than 10,461/0.2 = 52,300 or about 14.3 yrs.

When the factor of safety is unity, then the next blast will accelerate the slope. In this situation

$$FS = R/D = (W\cos(35)\tan(\phi) + cA)/W\sin(35)$$

The weight of the slide mass remains constant as do the area and inclination of the potential failure surface. However, the cohesion and properties of the failure surface change with persistence. These properties may be estimated with the help of the Terzaghi jointed rock mass model. Thus, from equations 2.4 in the text

$$\begin{aligned} \tan(\phi) &= (1-p)\tan(\phi_r) + (p)\tan(\phi_j) \\ c &= (1-p)c_r + (p)(c_j) \end{aligned}$$

Thus,

$$\begin{aligned} \tan(\phi) &= (1-p)\tan(40) + (p)\tan(30) = 0.8391 - 0.2617p \\ c &= (1-p)(900) + (p)(0) = 900 - 900p \text{ (psi)} \end{aligned}$$

After substitution into the factor of safety equation and setting $FS = 1$,

$$W\sin(35) = W\cos(35)(0.8391 - 0.2617p) + (900 - 900p)A$$

that may be solved for persistence p after computing W and A.

$$\begin{aligned} W &= (\gamma H^2/2)(\cot(\alpha) - \cot(\beta) \\ &= (150)(500)(500/2)(\cot(35) - \cot(45)) \\ W &= 8.028(10^6) \text{ lbf} \\ A &= H/\sin(\alpha) = 500/\sin(35) = 871.7 \text{ ft} \end{aligned}$$

After substitution

$$\begin{aligned} (8.028)(10^6)(0.5736) &= (8.028)(10^6)(0.8192)(0.8391 - 0.2617p) \\ &\quad +(900 - 900p)(871.7)(144) \end{aligned}$$

and solving: $p = 0.993$ indicating the entire failure surface will sustain blast damage before slope failure.

Check so far:

$$\begin{aligned} \tan(\phi) &= (1 - 0.993)\tan(40) + (0.993)\tan(30) = 0.5792 \\ c &= (1 - 0.993)(900) + (0.993)(0) = 6.3 \text{ psi} \end{aligned}$$

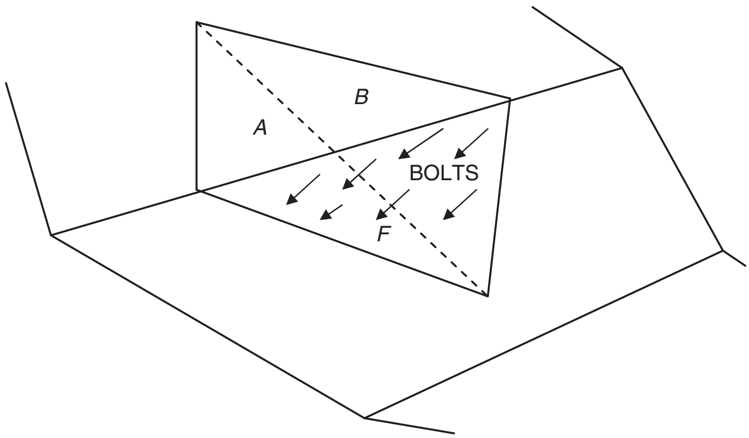
$$\begin{aligned} 8.0280(10^6)\sin(35) &= 8.028(10^6)\cos(35)\tan(30) + 6.3(144)(500/\sin(35)) \\ 4.605(10^6) &= 3.797(10^6) + 7.908(10^5) \\ &= 4.588(10^6) \end{aligned}$$

which is a reasonably close check, although not exact.

The time to failure is therefore as before, **14.3 yrs**.

Comment: Slopes sometimes appear to fail for no apparent reason. Day after day, operations are much the same, so the question that arises is what has changed? Why the failure? One answer is blasting causes slope acceleration but with passing of the transient, safety returns. However, the some loss of cohesion occurs with each blast. Eventually, the slope may fail as cohesion is destroyed entirely.

41. Bolting reinforcement of a potential wedge failure is anticipated. Bolts will be installed perpendicular to the face of the wedge as indicated in the sketch. All bolts will be tensioned the same amount and installed on a square pattern S × S m. Derive a formula for the wedge factor of safety with respect to sliding down the line of intersection of joints *A* and *B* that includes the effect of bolting. Assume there are m_A and m_B bolts intersecting planes *A* and *B*.



Given: Wedge failure data (dip and dip direction, cohesion and friction angles, specific weight, upland, face and foreland dip and dip directions, water table) and bolting data (spacing, number, tension)
Find: Bolted wedge safety factor formula.

Solution:

The *FS* expression without bolting is equation 2.23 in the text. Thus,

$$FS_w = R/D = (R_A + R_B)/D$$

Bolting will improve the resistance by mobilizing more frictional resistance via normal force (N) components and directly by uphill components (T) of bolt tension in much the same way as in planar block slides. Thus,

$$FS_w = R/D = (R_A + N_A \tan(\phi_A) + T_A + R_B + N_B \tan(\phi_B) + T_B)/D$$

An actual computation requires procedures for computing the bolting force components. Because all bolts have the same direction and tension, the total forces in this formula are simply the number of bolts times the force per bolt, that is,

$$F_A\ (bolt) = m_A\, f(bolt), \quad F_B\ (bolt) = m_B\, f(bolt)$$

These forces are directed normal to the face, that is, parallel to the face normal, n_F. With the normal to the joint faces, n_A and n_B. These normal directions may be obtained from Table 2.1 in the text.

Projection of the bolt forces onto the normal directions of the joint planes is achieved with the vector dot product $\vec{n}_F \cdot \vec{n}_A = \cos(\theta_{AF})$, and similarly, $\vec{n}_F \cdot \vec{n}_B = \cos(\theta_{BF})$. Thus,

$$N_A = F_A \cos(\theta_{AF}) \quad \text{and} \quad N_B = F_B \cos(\theta_{BF})$$

The uphill components of the bolt forces should point up the line of intersection. If a unit vector $\vec{s}$ down the line of intersection has been computed in the process of determining the dip and dip direction of the line of intersection, then the vector dot product may again be used, such that

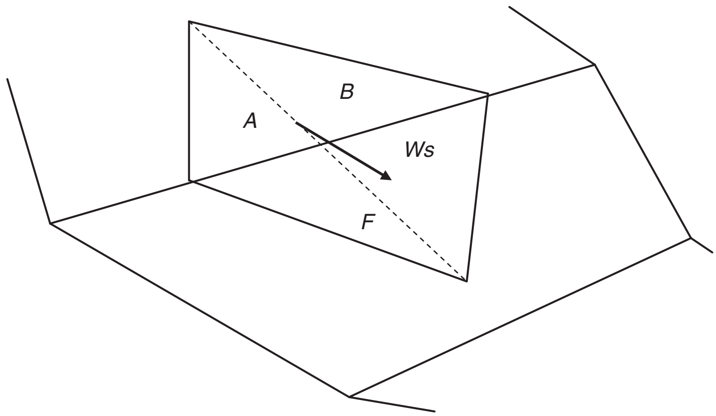
$$\vec{s} \cdot \vec{n}_F = \cos(\theta_{sF}).$$

The uphill bolt forces are then

$$T_A = F_A \cos(\theta_{sF}) \quad \text{and} \quad T_B = F_B \cos(\theta_{sF})$$

which completes the analysis.

42. Consider a potential wedge failure with seismic load *Ws* illustrated in the associated sketch. The seismic force is estimated as wedge weight *W* times a seismic coefficient a_s appropriate for the region. The seismic force acts horizontally through the center of the wedge and has an azimuth equal to the dip direction of the line of intersection. Modify a wedge safety factor formula to include such a seismic force.



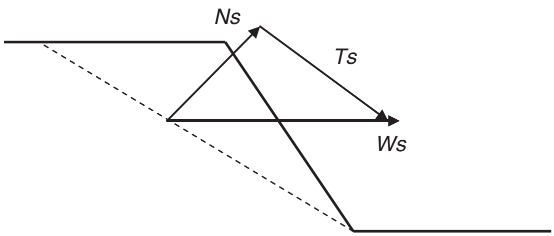
Given: Given: wedge failure data (dip and dip direction, cohesion and friction angles, specific weight, upland, face and foreland dip and dip directions, water table) and seismic coefficient.
Find: Factor of safety formula.

Solution:

The seismic force (*Ws*) will decrease resistance by decreasing frictional resistance via normal force (*N*) components, but also will add to the driving force directly by downhill components in much the same way as in planar block slides. Thus,

$$FS_w = \frac{[R_A + (N_A - N_{sA})\tan(\phi_A) + R_B + (N_B - N_{sB})\tan(\phi_B)]}{D + T_{sA} + T_{sB}}$$

To be useful, details for calculating the seismic force components are needed. With reference to the sketch that shows the line of intersection in true dip, the normal and tangential components are seen to be similar to weight components, although the normal component of seismic force acts in the opposite direction of weight. Therefore, the formulas for normal weight components may be used to compute the normal components of seismic force.



Thus, from equations 2.31 in the text

$$N_{sA} = \frac{(Ns)\sin(\delta_b)}{\sin(\delta_a + \delta_b)}, \quad N_{sB} = \frac{(Ns)\sin(\delta_a)}{\sin(\delta_a + \delta_b)}, \quad Ws = Wa_s, \quad Ns = (Ws)\cos(\delta_s)$$

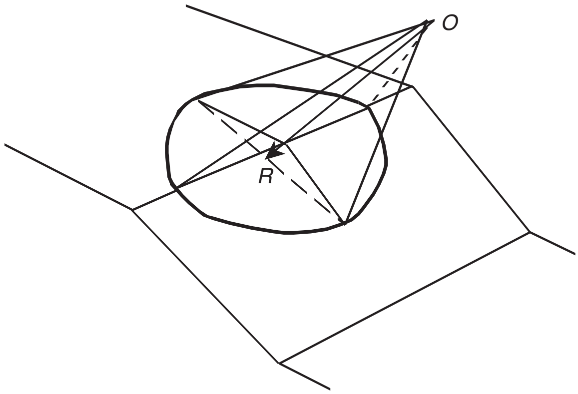
where Ws follows from the definition and Ns is simply the normal component of Ws.

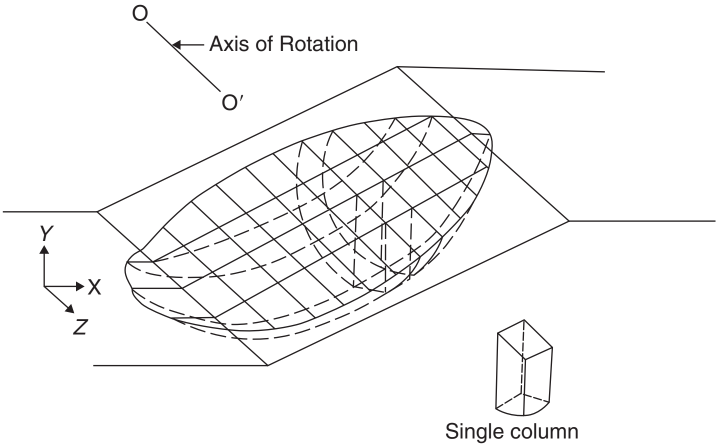
The angle between the horizontal and the downhill direction is simple the inclination of the line of intersection δ_s. The downhill seismic forces are then

$$T_{sA} = Ws\cos(\delta_s) \quad \text{and} \quad T_{sB} = Ws\cos(\delta_s)$$

which completes the analysis.

43. Consider a rotational slide in the form of a spherical segment as shown in the sketches. Outline a "method of columns" analogous to the method of slices that accounts for three-dimensional effects not present in the two-dimensional analysis using the method of slices. (After Chen, 1981)

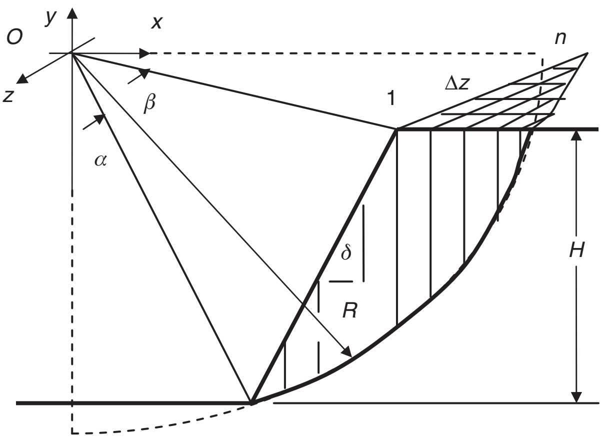




Given: Spherical slid mass with center of rotation at O with radius R in a slope of height H with angle δ from the horizontal.
Find: Method of columns similar to the method of slices.

Solution:

First fix a coordinate system with origin at the center of rotation, then partition the slide mass into slabs. Begin with a conventional slices partition of a central slab; proceed to the outer boundary of the spherical segment as shown in the sketch using a distance Δz for a total of n slabs. Let the angles from the toe of the central slab to the crest be α and β (sphere radius $= R$).



The mid-planes of a slab "i" have coordinates $-z(i) = (2i-1)(\Delta z/2)$ from $i=1$ to $i=n$ slabs. The radius $r(i)$ of the ith slab is $r(i) = R^2 - z(i)^2$. Each slab may now be treated in the same manner in the two-dimensional method of slices. Recall, moment equilibrium requires $\int_A RT\mathrm{d}A = \int_V r\gamma \mathrm{d}V$ that is approximated by $\sum R\sin(\alpha_j)W_j = \sum RT_j(strength)/fs_j$. The last is just equation 2.43 in the text. Assuming, as in the method of slices, that the global FS is equal to the local slice fs, one has for the central slab with radius R

$$FS = \frac{\sum RT_j(strength)}{\sum R\sin(\alpha_j)W_j}$$

The same analysis applies to the other slabs with radius $r(i)$. In consideration of all slabs (i) requiring summation over (i):

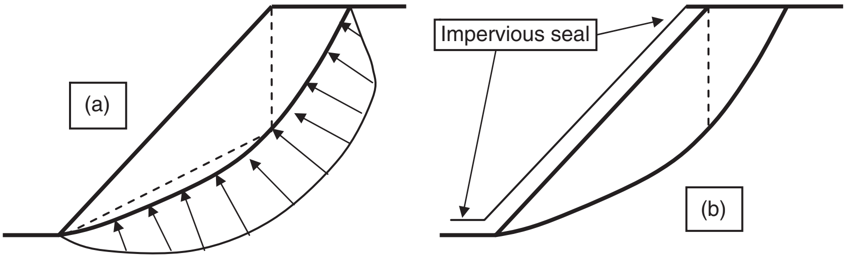
$$FS = \frac{\sum\left[\sum r(i)T_j(strength)\right]_i}{\sum\left[\sum r(i)\sin(\alpha_j)W_j\right]_i}$$

Geometric details could be computed systematically in a spread sheet for a site-specific spherical segment slip surface. Obviously, only the outline is given here. Two references that lead to more details about the method of columns are:

Chen, R. H. (1981) "Three-dimensional slope stability analysis." Joint Highway Research Project, Engineering Experiment Station, Purdue University, Report JHRP-81-17.

Hovland, H. J. (1977) "Three-dimensional slope stability analysis method." Journal of the Geotechnical Engineering Division, ASCE, Vol. 103, No. 9, pp. 971–986.

44. Consider a rotational failure as shown in the sketch with the water table at the crest. The water pressure on the circular slip surface is given by $p = \gamma_w z$, that is, by the product of water specific weight times water depth. The pressure distribution will show an increase from zero at the crest to a maximum and then a decrease to zero at the toe of the slope (a). An impervious seal is applied to the slope, but no drain holes are provided (b). Derive a formula that shows the effect of the seal on the slope safety factor



In this cases (a) and (b), the water pressure on the slip surface will increase steadily with depth below the water table to a point directly below the crest. At points further down the slip surface the water pressure in (a) depends on the vertical distance between the slip surface and the slope face. In case (b), the water pressure below the crest will be greater than in case (a) from the difference in water heads (distance below water tables). This difference will affect all slices below the slope face and is equal to the distance from the crest elevation to the slope face. Thus, the pressure difference between the two cases is $\Delta p = \gamma_w \Delta z$ and according to equation 2.45 in the text the *decrease* in slope safety factor is

$$\Delta FS = \frac{\sum_j (W_n - \Delta P)\tan(\phi)}{\sum_j W_s}$$

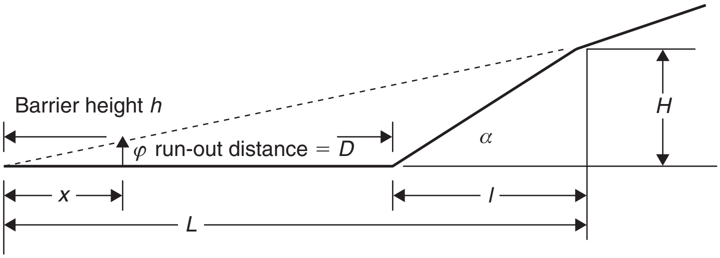
where $\Delta P = A\Delta p$ (A = area of slice base) and summation is over only the slices below the crest to the left of the dotted lines. The decrease in safety factor is the effect of the seal without drains.

45. An unstable slope poses a threat to a housing development below. The height to the unstable mass is $H = 185$ ft and the slope angle $\alpha = 30$ degrees. The effective angle of sliding friction is ϕ. (a) Estimate the "run-out" distance the mass center moves from the slope toe before coming to rest. (b) Estimate a barrier height required to stop the slide mass before traveling the entire run-out distance.

Given: Unstable slope
Find: (a) run-out distance, (b) barrier height.

Solution:

First sketch the situation.



From the sketch:

$$D = L - l$$
$$\underline{\underline{D = [H/\cot(\varphi)] - [H\cot(\alpha)]}}$$

The height of a barrier necessary to stop the mass center short of the full run-out distance is given by

$$\tan(\varphi) = h/x$$
$$\therefore \quad \underline{\underline{h = x\cot(\varphi)}}$$