SOLUTIONS TO PROBLEMS – CHAPTER 2

Note to instructors:
Problems 2.1-2.7 and their solutions are unchanged from Problems 4.1-4.7 of the second edition, apart from the references to Figure numbers and eqn numbers.

Solution to problem 2.1

(a) The Solar Constant, G_0^*, is the irradiance perpendicular to the solar beam just outside our Earth's atmosphere (see RER sec.2.2.2).

Assume that the Earth maintains a constant distance *L* from the Sun (i.e. neglect the slightly elliptical orbit of the Earth).

The power emitted by the Sun (as a black body) from its surface area is:

$P_S = (\sigma T_S^4)(4\pi R_S^2)$

= power passing through the surface of a pseudo sphere of radius equal to the Earth's distance, L

$= (G_0^*)(4\pi L^2)$

Hence

$$G_0^* = = \frac{\sigma T_S{}^4 (4\pi) R_S{}^2}{4\pi L^2} = \frac{\sigma T_S{}^4 (4\pi)(\tfrac{1}{2} D_S\ )^2}{4\pi L^2}$$

$$= (5.67\times10^{-8}\,\mathrm{Wm^{-1}K^{-4}}) \times (5780\mathrm{K})^4 \times \left(\frac{\tfrac{1}{2}\times 1.392\times10^9\,\mathrm{m}}{1.498\times10^{11}\,\mathrm{m}} \right)^2$$

$$= 1365\ \mathrm{Wm^{-2}}$$

Comment: this answer equals the observed solar constant of 1366 W m^{-2} within 0.15% (i.e. within rounding errors) ; indeed working this sum backwards from satellite measured data is one way to determine T_S .

(b) The area intercepting solar radiation at the Earth is $\pi R_E{}^2$, but the area of the Earth emitting infrared radiation to maintain thermal equilibrium is almost exactly a spherical surface of area $4\pi R_E{}^2$. The depth of the Earth's atmosphere is negligible compared with its radius for this calculation. We initially assume the Earth is a black body of uniform absolute temperature T. and zero reflectance. Hence at equilibrium:

$$(\sigma T^4)(4\pi R_E{}^2) = G_0^* \pi R_E{}^2$$

Hence this approximate analysis gives the radiative temperature of the Earth as

$$T = \left(\frac{G_0^*}{4\sigma} \right)^{\frac{1}{4}} = 278\mathrm{K} = 5^\circ\mathrm{C}$$

However there is significant reflection of incoming solar radiation from white cloud and the Earth's non-black surfaces. Therefore, reflection should be included, as in the related analysis of RER sec.2.9.1. Then:

$$(\sigma T^4)(4\pi R_E{}^2) = G_0^* \pi R_E{}^2 (1 - \rho_0)$$

where ρ_0 is the effective short-wave reflectance of the Earth from Space. Approximately $\rho_0 \approx 0.3$, hence

$$T = \left(\frac{G_0^*(1-\rho_0)}{4\sigma}\right)^{\frac{1}{4}} = \left[\frac{(1367\text{Wm}^{-2})\times 0.7}{4(5.67\times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}}\right]^{1/4} = \left[42.0\times 10^8\right]^{1/4} K = 2.55\times 10^2 = 255\text{K} = -18^{\circ}\text{C}$$

These calculations demonstrate the fundamental physical processes by giving reasonable answers to an accuracy of about +/- 5%. The same principles, but with more detail and, are used for computer-simulations, e.g. of weather.

Solution to Problem 2.2

Sketch diagrams should be drawn for the southern hemisphere, to replicate Figs 2.8 and 2.9. As compared with the diagrams for the northern hemisphere, latitude ϕ is now negative and solar device orientation γ shifts by 180°. Working through the equations in Sect 4.5.2, angle of incidence θ from Equ 4.8 is seen to be unchanged.

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Solution to Problem 2.3

This problem is a numerical example of eqns (2.8)-(2.11) of RER section 2.5.2. So, as in RER Example 2.1, the first step is to use the data to identify (determine) the various 'input' angles required to calculate θ_z and θ. Here, we have:

latitude ϕ : given for Suva, capital of Fiji, in the Southern Hemisphere $\phi = -18^{\circ}$
declination δ: calculated from RER eqn (4.5) thus:
day number n = 139 for 20 May (counted on a calendar)
Hence, from RER (2.5):

$$\delta = \delta_0 \sin[360\times(284+139)/365)]$$

$$= 19.7\,\text{deg with } \delta_0 = 23.5\text{deg}$$

Hour angle ω: at 9 a.m., using RER eqn (2.4):

$$\omega = (15^{\circ}\text{h}^{-1})(-3\text{h}) = -45^{\circ}$$

Neglecting ω_{eq} and assuming $\psi = \psi_{zone}$ (which is in fact true for Suva).

(a) With the above angles as input, RER eqn (2.11) gives the angle θ_z between the beam radiation and the vertical:

$$\begin{aligned}\cos\theta_z &= \sin\phi\sin\delta + \cos\phi\cos\omega\cos\delta \\ &= \sin(-18^{o})\sin 19.7^{o} + \cos(-18^{o})\cos 45^{o}\cos 19.7^{o} \\ &= (-\sin 18^{o})\sin 19.7^{o} + (\cos 18^{o})\cos 45^{o}\cos 19.7^{o} \\ &= 0.528, \quad \text{and so} \\ \theta_z &= 58^{o}\end{aligned}$$

If the diffuse irradiance is insignificant compared with the beam irradiance, then the variation of G^* is that of the beam component, which is given by RER eqn (2.2) :

$$G_{bh} = G_b^* \cos\theta_z$$

Hence the irradiance in the beam direction is

$$G^* \approx G_h / \cos\theta_z$$
$$= 1.0\,\mathrm{MJ\,h^{-1}} / 0.528 = 1.9\mathrm{MJ\,h^{-1}}$$

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(b) Although the angle θ_c can be calculated from the full formula [RER eqn (2.8)], since we have just calculated θ_z , it is less work to use RER eqn (2.9) instead. To do this we need three additional 'input' angles:

Slope β: given as $\beta = +30$ degrees.
Azimuth γ: +180 degrees for a surface facing due north (see RER, p.93).
Solar azimuth γ_s:
This is closely related to the hour angle, but measured in a different way. As on RER p.94, (γ_s -γ) is the angle between the beam and the surface (both projected onto the horizontal plane). Since at 9 a.m. the sun is 45^o East of N, then
(γ_s -γ) = 45^o.

Putting these values into RER eqn (2.9) yields:

$$\cos\theta_c = \cos\theta_z \cos\beta + \sin\theta_z \sin\beta \cos(\gamma_s - \gamma)$$
$$= (0.528)(0.866) + (0.84)(0.5)(0.707) = 0.754$$
$$\rightarrow \theta_c = 41^o$$

Assuming, as in (a) that total irradiance varies with angle in the same way as beam irradiance (i.e. according to RER eqn (2.2)) we find that the irradiance on the collector is

$$G_c \approx G_{bc} = G_b^* \cos\theta_c$$
$$= (1.9\mathrm{MJ\,h^{-1}})(0.754) = 1.4\mathrm{MJ\,h^{-1}}$$

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(c) Now we are to assume that half the radiation on a horizontal surface is diffuse, so

$$G_{dh} = G_{bh} = 0.5 G_{th} = 0.5 \times 1.0\mathrm{MJh^{-1}}$$

Then, by definition, the diffuse radiation is the same at all [upward facing] orientations, so

$$G_{dh} = G_d{}^* = G_{dc} = 0.5\mathrm{MJh^{-1}}$$

The beam component varies with cosθ as before, so

$G_b{}^* = G_{bh} / \cos\theta_z = 0.95\mathrm{MJh^{-1}}$ and $G_c{}^* = G_b{}^* / \cos\theta_c = 0.7\mathrm{MJh^{-1}}$.

Adding these,

the total irradiance in the beam direction is

G_t*= 0.95+0.5 =1.45 MJh^{-1}

And the total irradiance on the collector is

G_t*= 0.7+0.5 = 1.2 MJh^{-1}

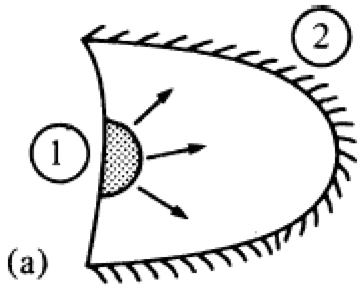
Comparing these to the values in parts (a) and (b), we see that the irradiance is, as expected, weaker in the beam direction (reduced from 1.9 MJ/h to 1.45MJ/h) and also on the collector (reduced from 1.4 MJ/h to 1.2 MJ/h) which is reasonable since the collector is oriented close to the beam direction.
Note that the input power to a focusing solar collector would be very much reduced is these conditions, since the diffuse irradiation cannot be focused.

Solution to Problem 2.4

Comment: the only part of this Problem that requires physical insight is the first line. The rest is simple algebra.

A short cut calculation is to take the first line as a special case of formula (C.17) of RER Appendix C.

Although the derivation of (C.17) is simple, see below, it is by no means obvious to most students, who wonder why T_2^4 (T_s^4 in this case) is coupled with ε_1 rather than with ε_2. The key to understanding is to realise that radiation is not only emitted directly (terms in σT^4) but also reflected and/or radiated back from the second body, as alluded to in RER sec. R3.5..



Step 1. The geometry is that of the diagram in Appendix C (copied above) above, but with $A_1 << A_2$ (the sky is effectively infinite in extent compared to any solar collector).

All the radiation emitted by the body [1] reaches the sky [2] and none is reflected back. Thus the sky behaves as a black body with $\alpha_2 = \varepsilon_2 = 1$ and the shape factor F_{12} =1 (see RER sec. R3.5.6). The shape factor in the reverse direction follows from RER eqn (3.43) :

$$F_{21} = (A_1 / A_2) F_{12}$$
$$= A_1 / A_2 \qquad \text{since } F_{12} = 1.$$

Body [1] behaves as a 'grey' body with $\varepsilon_1 = \alpha_1 = 1 - \rho_1$, as in RER sec R3.5.7.

We can now calculate the net radiative power reaching [2] from [1]

P_{12} = (power emitted from [1])
- (power emitted from [2]) x (fraction of this that reaches [1])
+ (power reaching [1] from [2]) x (fraction reflected back to [2])

$$= \sigma\varepsilon_1 T_1^4 A_1 - (\sigma\varepsilon_2 T_2^4 A_2)(A_1 / A_2) + (\sigma\varepsilon_2 A_1 T_2^4)(1-\varepsilon_1)$$
$$= \sigma\varepsilon_1 T_1^4 A_1 + (\sigma T_2^4 A_1)[-1 + (1-\varepsilon_1)]$$
$$= \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

which is eqn (C.17) of RER Appendix C. Step 2. The net radiative transfer Eqn (C.17) becomes for the case at hand:

$$P_r = A_1 \varepsilon_1 \sigma (T_1^4 - T_s^4) \qquad \text{(B1)}$$

We can change this into a linear form matching those for other forms of heat transfer, where heat is lost to the environment at ambient temperature T_a

$$P_r = A_1 h_r (T_1 - T_a)$$

by factorising the bracket in (B1) and multiplying by $1 = (T_1 - T_a)/(T_1 - T_a)$, thus:

$$\begin{aligned} P_r &= A_1\varepsilon_1\sigma(T_1^4 - T_s^4) \\ &= A_1\varepsilon_1\sigma(T_1^2 + T_s^2)(T_1^2 - T_s^2) \\ &= A_1\varepsilon_1\sigma(T_1^2 + T_s^2)(T_1 + T_s)(T_1 - T_s)\times\frac{(T_1 - T_a)}{(T_1 - T_a)} \end{aligned}$$

which is the required result.

Solution to Problem 2.5

(a) RER eqn (2.11) gives the angle between the solar beam and the local vertical, i.e. the zenith angle θ_z:

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\omega\cos\delta \qquad (2.11)$$

At sunrise and sunset $\theta_z = 90^o$, so cos $\theta_z = 0$ and (4.11) above becomes

$$\cos\omega = -\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} = -\tan\phi\tan\delta \qquad (C)$$

After solar noon, hour angle ω is positive (in the sign convention of RER sec2.5.1), and so at sunset ω is the positive solution of (C), i.e.

at sunset $\omega = \omega_1 = \cos^{-1}(-\tan\phi\tan\delta)$.

Since the hour angle ω increases at 15 deg/ hour, it follows that the time in hours between solar noon and sunset is $N_1 = \omega_1/15$. By symmetry, the time between sunrise and sunset is

$$N = 2N_1 = \frac{2}{15}\cos^{-1}(-\tan\phi\tan\delta)$$

Which is RER eqn (2.7) as required.

(b) At northern mid-summer $\delta = +23.5^o$ and at northern midwinter $\delta = -23.5^o$ (see RER Figure 2.5). Hence we obtain the following table of numerical results. Note the significant differences in the periods of daylight.

	Northern Midsummer $\delta = +23.5^o$	Northern Midwinter $\delta = -23.5^o$
Latitude ϕ = +12^o	N= 12.7 h	N= 11.3 h
Latitude ϕ = +60^o	N= 18.5 h	N= 5.4 h

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Solution to problem 2.6

(a) Irradiance on horizontal (above atmosphere) is:

$$G'_{oh} = G_0^* \cos\theta_z \qquad (2.24)$$

Here the prime indicates that the quantity refers to the approximated circular Earth orbit .

But from RER eqn (2.11)

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\omega\cos\delta$$

So in (2.24) $G'_{oh} = G_0^* \cos\theta_z = G_0^*(\sin\phi\sin\delta + \cos\phi\cos\omega\cos\delta)$

The daily insolation is (from RER eqn (2.6)):

$$H_{0h}' = \int_{t=-t_S/2}^{t=+t_S/2} G_{oh}'.dt \qquad \text{(H1)}$$

Where t_s is the length of the day.
We convert this integral to being in terms of ω by using
RER eqn (2.4), written in the form:

$$\omega = k(t - t_{\text{noon}}) + \omega_{eq} + (\psi - \psi_{\text{zone}}) \qquad (2.4)$$

with k=15°/h . Since within any one day the last two terms of (2.4) are effectively constant, for small changes $\delta t = \delta\omega / k$

So in (H1) $H_{0h}' = \int_{t=-t_S/2}^{t=+t_S/2} G_{oh}'.dt == \int_{\omega=-\omega_S}^{\omega=+\omega_S} G_{oh}'.d\omega / k$

where ω_s is the hour angle at sunset.

Hence $H'_{0h} = \int_{\omega=-\omega_S}^{\omega=+\omega_S} G_0^*(\sin\phi\sin\delta + \cos\phi\cos\omega\cos\delta) d\omega / k$

$$\begin{aligned} H_{0h}' &= \left(\frac{G_0^*}{k}\right)(\sin\phi\sin\delta)(2\omega_s) + \left(\frac{G_0^*}{k}\right)(\cos\phi\cos\delta)\left[\sin\omega\right]_{-\omega_s}^{+\omega_s} \\ &= t_s G_0^* \sin\phi\sin\delta + \left(\frac{G_0^*}{k}\right)(\cos\phi\cos\delta)(2\sin\omega_s) \end{aligned} \qquad \text{(H2)}$$

With ω measured in radians for the integration.

Since $t = +t_s/2$ when $\omega = +\omega_s$ (radians) and since 180° = π radians, it follows that

$$\frac{2}{k} = \frac{t_s}{\omega_s} = \frac{t_s}{\omega_s} \times \frac{180^\circ}{\pi \text{ radians}}$$

Hence eqn (H2) becomes

$$H_{oh}' = t_s G_0^* [\sin\phi\sin\delta + \left(\frac{180^\circ}{\pi \text{ rad}}\right)\left(\frac{1}{\omega_s}\right)(\cos\phi\cos\delta)(\sin\omega_s)] \qquad (2.24)$$

As required.

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(b) Numerically, in midsummer (δ = +23.5°) at latitude ϕ = +48°, RER eqn (2.7) gives

$$t_s = (2/15)\cos^{-1}(-\tan\phi\tan\delta) = +15.8\text{h}$$

Hence

$$\omega_s = \tfrac{1}{2} \times 15.8\text{h} \times 15^\circ/\text{h} = 119^\circ \text{ and so } \sin\omega_s = 0.875$$

Feeding these angles into (2.24), gives the term in [ ] =0.555, and so

$$H'_{oh} = (G_0^* t_s) \times 0.555$$
$$= (1367 \frac{\text{Jm}^{-2}}{\text{s}})(15.8\text{h}) \times \frac{3600\text{s}}{\text{h}} \times 0.555$$
$$= 43.1 \text{ MJm}^{-2}$$

In midsummer (22 June), day number is $n = 172$.

Hence, using RER eqn (4.25) and the value below this for e', the extraterrestial irradiance is,

$$H_{0h} = [1 + e' \cos(360^0 n / 365^o)] H_{0h}'$$
$$= [0.967] \times 43.1\text{MJm}^{-2} = 41.6\text{MJm}^{-2}$$

But according to RER Fig.2.7 in mid-summer the daily insolation at 48^o at the Earth's surface under a clear sky (i.e. the maximum of the 48^o curve of Fig.2.7) is about 28.5 MJm^{-2}. This implies that the clearness index of RER eqn (2.19) is

$$K_T = H_h / H_{0h} = 28.5 / 41.6 = 0.68$$

Which is a not unreasonable value, considering the atmospheric absorption indicated by the difference in irradiation between air mass m=0 and m=1 of RER Fig.2.15. (See also the discussion of RER sec2.8.5).

Similarly in midwinter (δ= -23.5^o, n =358), following the analysis as above, we find

$t_s = 8.15$h , ω_s =61.1^o , $\sin\omega_s$ = 0.875,

[term] =0.394, and

$H'_{0h} = 15.8$MJ

Whereas, according to RER Fig.2.7, the clear-sky surface daily insolation in mid-winter is the minimum of the 48^o curve, i.e. 6.5MJ. This implies a clearness index of K_T = 0.41. That this is less than K_T in mid-summer is to be expected, because at latitude 48^o in mid-winter, even at mid-day, the sunshine passes through air mass $m \approx 2$.

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Solution to Problem 2.7

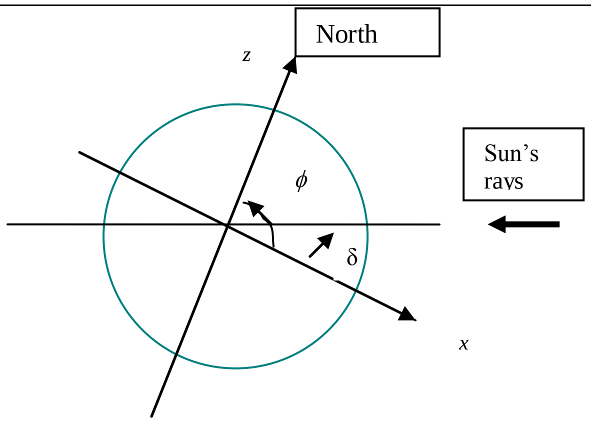
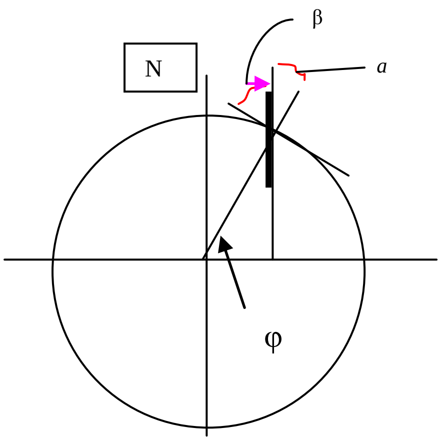
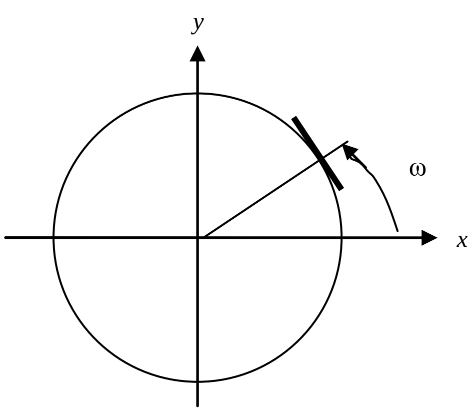
Comment: To plot all the relevant 3-dimensional lines on one 2-dimensional diagram is very confusing. It is easier to separate the various planes, as below.

Throughout, as hinted in the problem, we use an (x,y,z) co-ordinate system centred on the centre of the Earth, with z-axis towards the north pole, and the sun in the x-z plane (i.e. with y =0).

The following 3 diagrams are all drawn for the northern hemisphere case $(\phi > 0)$.

As given in the Problem, collector azimuth $\gamma = 0$, and collector slope $\beta = \phi$.

Diagram (A) : the solar beam . In plane y =0	Unit vector in direction of sun is (in x,y,z co-ordinates) $\hat{\mathbf{s}}$ = ($\cos\delta$, 0 , $\sin\delta$)

	[These are the 'direction cosines']
Diagram (B) : in plane containing polar axis and the normal to the collector. 	We work through this diagram to establish that the plane of the collector (thick line) is normal to that of the equator. Angles β and ϕ are given as indicated. Then at top of diagram, complementary angle *a* is 90-β . Therefore the angle opposite it (i.e. the top angle in the triangle) is also 90-β . But in the triangle, the bottom left angle ϕ equals β (given). Hence the remaining (bottom right) angle in the triangle is $180-(90-\beta) - \beta = 90^{\circ}$ Hence a unit vector normal to the collector has zero component in the *z*-direction
Diagram C: seen from 'above' i.e. with the collector projected on the x-y plane 	Hour angle ω is the angle the Earth has rotated since solar noon. For simplicity, we show the case ω>0 (i.e. in the afternoon). Since the Sun is in the x-z plane, it is clear from diagram C that the x and y components of the unit vector normal to the collector are (cosω, sinω).

In summary:

from Diagram A, a unit vector in direction of the sun is

$\hat{\mathbf{s}} = (\cos\delta, 0, \sin\delta)$

From diagrams B and C, a unit vector normal to the collector is

$\hat{\mathbf{c}} = (\cos\omega, \sin\omega, 0)$.

Therefore the angle between the solar beam and the normal to the collector is given by

$$\cos\theta = \hat{\mathbf{s}}.\hat{\mathbf{c}} = \cos\omega\cos\delta + 0.\sin\omega + 0.\sin\delta$$
$$= \cos\omega\cos\delta$$

as required.

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Solution to Problem 2.8

This Solution addresses Problem 2.8 formulated as follows:
Is the energy in outgoing long-wave radiation from the Earth equal to that in the incoming short-wave radiation from the Sun. Why?

All the numbers needed to address this question are in RER Fig2.12a , which shows the relevant energy flows as annual averages in $W.m^{-2}$.

Interpretation 1: at the "top of the atmosphere"

The top of Fig 2.12a represents "the top of the atmosphere". We see there:
Downgoing (incoming, 'short-wave') solar radiation : 342 $W.m^{-2}$
Upgoing thermal ('long-wave') radiation : 235 $W.m^{-2}$

The reason these are not equal is the short-wave "reflected solar radiation " of 107 $W.m^{-2}$ shown at far left of Fig.2.12a.
But reassuringly the net energy balance of the Earth to space is zero (i.e. the Earth is in thermal equilibrium with the Sun , since

342-107=235.

Interpretation 2: near the Earth's surface

Incoming short-wave RFD (shown by the fat downward arrow flowing downwards from the 342 $W.m^{-2}$ at the top of the atmosphere) : = (342-77-67)= (168+30) = 198 $W.m^{-2}$
Outgoing long-wave radiation (shown by fat upwards arrow second from right at the surface) = (350+40)=390 $W.m^{-2}$

These are way out of balance because of the 'back radiation' from greenhouse gases and clouds and the upward heat fluxes by latent heat (shown as 'evapo-transpiration' of 78 $W.m^{-2}$) and sensible heat (free convection , shown as 'thermals' of 24 $W.m^{-2}$) .

But the surface of the Earth is in thermal balance , since
Net heat carried upwards = (30+ 24+78+390-324)= 198 $W.m^{-2}$.

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Solution to Problem 2.9

(a) Having ice on top keeps the water beneath cool, because (i) it reflects back most of the incoming solar radiation (80% according to the question), which would otherwise warm that water, and (ii) the water beneath may lose a proportion of any extra heat that does get to it by melting a little of the ice above it (only 'a little' since the latent heat of ice is very high). (The proportion depends on the speed of currents in the water, among other factors).
With no ice on top of it, the water temperature will increase significantly because (i) a much larger fraction of the incoming solar radiation is absorbed (80% since only 20% is reflected), and (ii) there is less ice in a position to take up latent heat (none on top though there may be some at the side).

(b) Thus where ice has melted more than 'normal' in summer, the open water the increased area of open water reflects less solar radiation heat than the ice than the ice that would 'normally' be there, and so that are up more than 'normal'. Because the water is warmer than 'normal' in summer, it takes longer to freeze in autumn. Consequently less sea-ice (i.e. floating ice) is formed than in a usual winter, leaving an even greater area of open water, which in turn warms even more that the year before (other things being equal). In short, there is a 'positive feedback' which tends to decrease the area of sea ice in summer still further, and thus to lead to ever increasing warming in that region. Hence both the observed and projected temperature increases are greatest in the Arctic region, as all the ice there is floating. (Most of the ice in the Antarctic is on top of land, so this effect is less marked there..)

<u>Extra information for interest</u>
Quantitiative observed and projected increases in mean surface temperature according to IPCC Working Group 1 (2013) are given in the following table :

	Global average increase (^{o}C)	Arctic region increase (^{o}C)
Observed average temperature increase (1901-2012)	0.8	1.5-2.5
Projected temperature increase (1981-2000) to 2081-2100 (RCP4.5scenario/ RCP6.0 scenario)	2.0/ 2.5	4.5/ 6.0

Solution to Problem 2.10

(a) The bulk of the ocean can expand only upwards, since coastal volumes are <<1% of the ocean as a whole. Thus, its vertical expansion Δz is governed by the volumetric coefficient of expansion, so

$\Delta z / z = \Delta V / V = \beta \Delta T$ and thus for a temperature increase of ΔT =1^{o}C=1K (i.e roughly the observed increase in surface temperature)

$$\Delta z = (4\times10^{3}\text{m})\times(3\times10^{-4}\text{K}^{-1})\times(1.0\text{K}) = 1.2\text{m}$$

This is much less than the observed sea level rise principally because it takes a very long time (centuries) for a temperature increase at the surface to spread through the whole ocean.

<u>Extra information for interest</u>

Indeed if this took place purely by diffusion, the time taken would be

$$\tau \sim z^2 / \kappa \approx (4\times10^3 \text{m})^2 / (0.1\times10^{-6} \text{m}^2.\text{s}^{-1})$$
$$= 1.6\times10^{14} \text{s}\times(1\text{y} / 3.14\times10^7 \text{s}) \sim 5\times10^6 \text{years}$$

using data in RER Appendices A and B.
In fact even slow-moving ocean currents carry heat faster than this (including vertically) and so IPCC WG1 (2007) estimate the time frame required as a few centuries.

(B) Archimedes principle. The sea ice is floating already and thus displacing its own weight of water. Therefore its melting into liquid water does not increase the average sea level.

(C) Volume of Greenland ice sheet is

$$V_G = (0.5\text{km})\times(2\times10^6 \text{km}^2) = \frac{1.0\times10^6 \text{km}^3}{2.6\times10^8 \text{km}^2}$$
$$= 4\times10^{-3} \text{km} = 4\text{m}$$

Ocean covers about 50% of Earth's surface area, so total area of ocean is

$$A_o = 0.5\times4\pi R_E{}^2 = 2\times3.14\times(6.4\times10^3 \text{km})^2 = 2.6\times10^8 \text{km}^2$$

The volume V_G of water spread over an area A_o , will have a depth

$$\Delta z = V_G / A_o = \frac{1.0\times10^6 \text{km}^3}{2.6\times10^8 km^2} = 4\times10^{-3} \text{km} = 4\text{m}$$

So , on these approximate figures, melting the Greenland ice sheet would raise the global sea level by 4m

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Solution to Problem 2.11

For an outline of the projected impacts of climate change on biological and social systems see Box 17.1 and Box 17.5 in RER chapter 17. Note particularly the strong effect of the amount of fossil fuel to be used in the future.

For more detail (but at a global or regional level) the prime source is the reports by IPCC Working Group 2. The most recent such report is part of the IPCC's Fifth Assessment Report of 2013-2014, and is available at www.ipcc.ch .
In particular Table 1 in Box SPM2 of the *Summary for Policy Makers* gives a summary of the main risks for each of the 7 regions of the Earth.