Problem 1.1

The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

Solution 1.1

$$F = C_D \rho V^2 \frac{A}{2}$$

or

$$C_D = \frac{2F}{\rho V^2 A}, \text{ where}$$

$$F \doteq MLT^{-2}, \rho \doteq ML^{-3}, V \doteq LT^{-1}, A \doteq L^2$$

Thus,

$$C_D \doteq \frac{\left(MLT^{-2}\right)}{\left[\left(ML^{-3}\right)\left(LT^{-1}\right)^2\left(L^2\right)\right]} = M^0 L^0 T^0$$

Hence, C_D is dimensionless.

Problem 1.2

The Mach number is a dimensionless ratio of the velocity of an object in a fluid to the speed of sound in the fluid. For an airplane flying at velocity V in air at absolute temperature T, the Mach number Ma is,

$$\mathrm{Ma} = \frac{V}{\sqrt{kRT}},$$

where k is a dimensionless constant and R is the specific gas constant for air. Show that Ma is dimensionless.

Solution 1.2

We denote the dimension of temperature by θ and use Newton's second law to get $F = \dfrac{ML}{T^2}$.

Then

$$[M_a] = \frac{\left(\dfrac{L}{T}\right)}{\sqrt{(1)\left(\dfrac{FL}{M\theta}\right)\theta\left(\dfrac{ML}{T^2F}\right)}} = \frac{\left(\dfrac{L}{T}\right)}{\sqrt{\dfrac{L^2}{T^2}}}$$

or

$$[\mathrm{M_a}] = [1].$$

Problem 1.3

Verify the dimensions, in both the FLT and the MLT systems, of the following quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

Solution 1.3

a) volume $\doteq \underline{\underline{L^3}}$

b) acceleration $=$ time rate of change of velocity $\doteq \dfrac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

c) mass $\doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

mass $\doteq \underline{\underline{FL^{-1}T^2}}$

d) moment of inertia $\left(\text{area}\right) =$ second moment of area $\doteq \left(L^2\right)\left(L^2\right) \doteq \underline{\underline{L^4}}$

e) work $=$ force $\times$ distance $\doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2}$

work $\doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.4

Verify the dimensions, in both the FLT and the MLT systems, of the following quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

Solution 1.4

a) angular velocity $= \dfrac{\text{angular displacement}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

b) energy ~ capacity of body to do work

single work = force × distance ➔ energy $\doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2}$ ➔ energy $\doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

c) moment of inertia (area) = second moment of area $\doteq \left(L^2\right)\left(L^2\right) \doteq \underline{\underline{L^4}}$

d) power = rate of doing work $\doteq \dfrac{FL}{T} \doteq \underline{\underline{FLT^{-1}}} \doteq \left(MLT^{-2}\right)(L)\left(T^{-1}\right) \doteq \underline{\underline{ML^2T^{-3}}}$

e) pressure $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}} \doteq \left(MLT^{-2}\right)\left(L^{-2}\right) \doteq \underline{\underline{ML^{-1}T^{-2}}}$

Problem 1.5

Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

Solution 1.5

a) $\text{frequency} = \dfrac{\text{cycles}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$$\text{stress} \doteq \frac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

c) $\text{strain} = \dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \underline{\underline{L^0}}\,(\text{dimensionless})$

d) $\text{torque} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.6

If u is velocity, x is length, and t is time, what are the dimensions (in the MLT system) of **(a)** $\partial u / \partial t$, **(b)** $\partial^2 u / \partial x \partial t$, and **(c)** $\int (\partial u / \partial t) dx$?

Solution 1.6

a) $\dfrac{\partial u}{\partial t} \doteq \dfrac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

b) $\dfrac{\partial^2 u}{\partial x \partial t} \doteq \dfrac{LT^{-1}}{(L)(T)} \doteq \underline{\underline{T^{-2}}}$

c) $\int \dfrac{\partial u}{\partial t} \partial x \doteq \dfrac{\left(LT^{-1}\right)}{T}(\mathrm{L}) \doteq \underline{\underline{L^2 T^{-2}}}$

Problem 1.7

Verify the dimensions, in both the FLT system and the MLT system, of the following quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

Solution 1.7

a) $\text{acceleration} = \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$

c) $\text{moment of a force} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)L \doteq \underline{\underline{ML^2T^{-2}}}$

d) $\text{volume} = (\text{length})^3 \doteq \underline{\underline{L^3}}$

e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)L \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.8

If p is pressure, V is velocity, and ρ is fluid density, what are the dimensions (in the MLT system) of **(a)** p/ρ, **(b)** $pV\rho$, and **(c)** $p/\rho V^2$?

Solution 1.8

a) $$\frac{p}{\rho} \doteq \frac{FL^{-2}}{ML^{-3}} = \frac{MLT^{-2}L^{-2}}{ML^{-3}} = \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$$

b) $$pV\rho \doteq \left(ML^{-1}T^{-2}\right)\left(LT^{-1}\right)\left(ML^{-3}\right) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$$

c) $$\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{\left(ML^{-3}\right)\left(LT^{-1}\right)^2} \doteq M^0L^0T^0 \left(\underline{\underline{\text{dimensionless}}}\right)$$

Problem 1.9

If P is force and x is length, what are the dimensions (in the FLT system) of **(a)** dP / dx, **(b)** d^3P / dx^3, and **(c)** $\int P\ dx$?

Solution 1.9

a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \underline{\underline{FL^{-1}}}$

b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \underline{\underline{FL^{-3}}}$

c) $\int Pdx \doteq \underline{\underline{FL}}$

Problem 1.10

If V is velocity, ℓ is length, and ν is a fluid property (the kinematic viscosity) having dimensions of L^2T^{-1}, which of the following combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, and **(d)** $V/\ell\nu$?

Solution 1.10

a) $V\ell\nu \doteq \left(LT^{-1}\right)(L)\left(L^2T^{-1}\right) \doteq L^4T^{-2}$ $\left(\underline{\underline{\text{not dimensionless}}}\right)$

b) $\dfrac{V\ell}{\nu} \doteq \dfrac{\left(LT^{-1}\right)(L)}{\left(L^2T^{-1}\right)} \doteq L^0T^0$ $\left(\underline{\underline{\text{dimensionless}}}\right)$

c) $V^2\nu \doteq \left(LT^{-1}\right)^2\left(L^2T^{-1}\right) \doteq L^4T^{-3}$ $\left(\underline{\underline{\text{not dimensionless}}}\right)$

d) $\dfrac{V}{\ell\nu} \doteq \dfrac{\left(LT^{-1}\right)}{(L)\left(L^2T^{-1}\right)} \doteq L^{-2}$ $\left(\underline{\underline{\text{not dimensionless}}}\right)$

Problem 1.11

The momentum flux is given by the product $\dot{m}V$, where $\dot{m}$ is mass flow rate and V is velocity. If mass flow rate is given in units of mass per unit time, show that the momentum flux can be expressed in units of force.

Solution 1.11

$$[\dot{m}V] = \left(\frac{M}{T}\right)\left(\frac{L}{T}\right) = M\frac{L}{T^2}\left[\frac{FT^2}{ML}\right] = \underline{\underline{F}}$$

where $\frac{1}{g_c} = \left[\frac{FT^2}{ML}\right]$ comes from Newton's Second Law.

Problem 1.12

An equation for the frictional pressure loss Δp (inches H_2O) in a circular duct of inside diameter d (in.) and length L (ft) for air flowing with velocity V (ft/min) is

$$\Delta p = 0.027\left(\frac{L}{d^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82},$$

where V_0 is a reference velocity equal to 1000 ft/min. Find the units of the "constant" 0.027.

Solution 1.12

Solving for the constant gives

$$0.027 = \frac{\Delta p_L}{\left(\frac{L}{D^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82}}.$$

The units give

$$[0.027] = \frac{(\text{in. } H_2O)}{\left(\frac{\text{ft}}{\text{in.}^{1.22}}\right)\left(\frac{\frac{\text{ft}}{\text{min}}}{\frac{\text{ft}}{\text{min}}}\right)^{1.82}}$$

$$\boxed{[0.027] = \frac{\text{in. } H_2O \cdot \text{in.}^{1.22}}{\text{ft}}}$$

Problem 1.13

The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8\mu\ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ is a fluid property called viscosity $\left(FL^{-2}T\right)$, and ℓ is the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

Solution 1.13

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\frac{\left[L^4\right]\left[FL^{-2}\right]}{\left[FL^{-2}T\right]\left[L\right]}$$

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\left[L^3T^{-1}\right]$$

The constant is $\frac{\pi}{8}$ is dimensionless.

Yes. This is a general homogeneous equation
because it is valid in any consistent units system.

Problem 1.14

Show that each term in the following equation has units of lb/ft^3. Consider u as velocity, y as length, x as length, p as pressure, and μ as absolute viscosity.

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}.$$

Solution 1.14

$$\left[\frac{\partial p}{\partial x}\right] = \frac{\left[\dfrac{\text{lb}}{\text{ft}^2}\right]}{[\text{ft}]} \quad \text{or} \quad \boxed{\left[\frac{\partial p}{\partial x}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]},$$

and

$$\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}\cdot\text{sec}}{\text{ft}^2}\right] \frac{\left[\dfrac{\text{ft}}{\text{sec}}\right]}{\left[\text{ft}^2\right]} \quad \text{or} \quad \boxed{\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]}.$$

Problem 1.15

The pressure difference, Δp, across a partial blockage in an artery (called a stenosis) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ is the blood viscosity $(FL^{-2}T)$, ρ is the blood density $\left(ML^{-3}\right)$, D is the artery diameter, A_0 is the area of the unobstructed artery, and A_1 is the area of the stenosis. Determine the dimensions of the constants K_υ and K_u. Would this equation be valid in any system of units?

Solution 1.15

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1 \right]^2 \rho V^2$$

$$FL^{-2} \doteq [K_v] \frac{FT}{L^2} \frac{L}{T} \frac{1}{L} + [K_u] \left(\frac{L^2}{L^2} - 1 \right)^2 \left(\frac{FT^2}{L} \frac{1}{L^3} \right) \left(\frac{L}{T} \right)^2$$

$$FL^{-2} \doteq [K_v]\left(FL^{-2}\right) + [K_u]\left(FL^{-2}\right)$$

K_υ and K_u are <u>dimensionless</u> because each term in the equation must have the same dimensions. <u>Yes,</u> The equation would be valid in any consistent system of units.

Problem 1.16

Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_υ, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_\upsilon)^a (\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See the equation $c = \sqrt{\frac{E_\upsilon}{\rho}}$.)

Solution 1.16

Substituting $[c] = LT^{-1}$ $\quad [E_v] = FL^{-2}$ $\quad [\rho] = FL^{-4}T^2$ into the equation provided yields:

$$\left[LT^{-1}\right] = \left[\left(FL^{-2}\right)^a\right]\left[\left(FL^{-4}T^2\right)^b\right] = F^{a+b}L^{-2a-4b}T^{2b}$$

Dimensional homogeneity requires that the exponent of each dimension on both sides of the equal sign be the same.

F: $\quad 0 = a + b$
L: $\quad 1 = -2a - 4b$
T: $\quad -1 = 2b$

Therefore:

T: $\quad -1 = 2b \rightarrow b = -1/2$
F: $\quad a = -b \rightarrow a = 1/2$
L: $\quad 1 = -2a - 4b = -2(1/2) - 4(-1/2) = 1$ ✓

$$\boxed{a = \frac{1}{2}; \quad b = -\frac{1}{2}}$$

Yes, this is consistent with the standard formula for the speed of sound.

Problem 1.17

A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$Q = 3.09\ BH^{3/2}$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft^3/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

Solution 1.17

$$Q = 3.09\ BH^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L][L]^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L]^{\frac{5}{2}}$$

Since each term in the equation must have the same dimensions the constant

3.09 must have dimensions of $L^{\frac{1}{2}}T^{-1}$ and is therefore not dimensionless. $\underline{\underline{\text{No}}}$.

Since the constant has dimensions its value will change with a change in units.

$\underline{\underline{\text{No}}}$.

Problem 1.18

A commercial advertisement shows a pearl falling in a bottle of shampoo. If the diameter D of the pearl is quite small and the shampoo is sufficiently viscous, the drag $\mathcal{D}$ on the pearl is given by Stokes's law,

$$\mathcal{D} = 3\pi\mu VD,$$

where V is the speed of the pearl and μ is the fluid viscosity. Show that the term on the right side of Stokes's law has units of force.

Solution 1.18

$$[\mathcal{D}] = [3\pi\mu VD] = \left(\frac{M}{LT}\right)\left(\frac{L}{T}\right)L = M\frac{L}{T^2} = M\underbrace{\left[\frac{FT^2}{ML}\right]}_{\frac{1}{g_c}}\frac{L}{T^2} = \underline{\underline{F}}$$

where $\dfrac{1}{g_c} = \left[\dfrac{F\,T^2}{M\,L}\right]$ comes from Newton's Second Law.

Problem 1.20

Express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s^2, and **(e)** 0.0234 $\text{lb}\cdot\text{s/ft}^2$.

Solution 1.20

a) $$10.2\,\frac{\text{in.}}{\text{min}} = \left(10.2\,\frac{\text{in.}}{\text{min}}\right)\left(2.540\times10^{-2}\,\frac{\text{m}}{\text{in.}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 4.32\times10^{-3}\,\frac{\text{m}}{\text{s}} = \underline{\underline{4.32\,\frac{\text{mm}}{\text{s}}}}$$

b) $$4.81\,\text{slugs} = (4.81\,\text{slugs})\left(1.459\times10\,\frac{\text{kg}}{\text{slug}}\right) = \underline{\underline{70.2\,\text{kg}}}$$

c) $$3.02\,\text{lb} = (3.02\,\text{lb})\left(4.448\,\frac{\text{N}}{\text{lb}}\right) = \underline{\underline{13.4\,\text{N}}}$$

d) $$73.1\,\frac{\text{ft}}{\text{s}^2} = \left(73.1\,\frac{\text{ft}}{\text{s}^2}\right)\left(3.048\times10^{-1}\,\frac{\frac{\text{m}}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}\right) = \underline{\underline{22.3\,\frac{\text{m}}{\text{s}^2}}}$$

e) $$0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2} = \left(0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(4.788\times10\,\frac{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right) = \underline{\underline{1.12\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

Problem 1.21

Express the following quantities in BG units: **(a)** $14.2\ \mathrm{km}$, **(b)** $8.14\,\mathrm{N/m^3}$, **(c)** $1.61\,\mathrm{kg/m^3}$, **(d)** $0.0320\,\mathrm{N\cdot m/s}$, and **(e)** $5.67\,\mathrm{mm/hr}$.

Solution 1.21

a) $$14.2\ \mathrm{km} = \left(14.2\times10^3\ \mathrm{m}\right)\left(3.281\frac{\mathrm{ft}}{\mathrm{m}}\right) = \underline{\underline{4.66\times10^4\ \mathrm{ft}}}$$

b) $$8.14\frac{\mathrm{N}}{\mathrm{m^3}} = \left(8.14\frac{\mathrm{N}}{\mathrm{m^3}}\right)\left(6.366\times10^{-3}\frac{\frac{\mathrm{lb}}{\mathrm{ft^3}}}{\frac{\mathrm{N}}{\mathrm{m^3}}}\right) = \underline{\underline{5.18\times10^{-2}\frac{\mathrm{lb}}{\mathrm{ft^3}}}}$$

c) $$1.61\frac{\mathrm{kg}}{\mathrm{m^3}} = \left(1.61\frac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(1.940\times10^{-3}\frac{\frac{\mathrm{slugs}}{\mathrm{ft^3}}}{\frac{\mathrm{kg}}{\mathrm{m^3}}}\right) = \underline{\underline{3.12\times10^{-3}\frac{\mathrm{slugs}}{\mathrm{ft^3}}}}$$

d) $$0.0320\frac{\mathrm{N\cdot m}}{\mathrm{s}} = \left(0.0320\frac{\mathrm{N\cdot m}}{\mathrm{s}}\right)\left(7.376\times10^{-1}\frac{\frac{\mathrm{ft\cdot lb}}{\mathrm{s}}}{\frac{\mathrm{N\cdot m}}{\mathrm{s}}}\right) = \underline{\underline{2.36\times10^{-2}\frac{\mathrm{ft\cdot lb}}{\mathrm{s}}}}$$

e) $$5.67\frac{\mathrm{mm}}{\mathrm{hr}} = \left(5.67\times10^{-3}\frac{\mathrm{m}}{\mathrm{hr}}\right)\left(3.281\frac{\mathrm{ft}}{\mathrm{m}}\right)\left(\frac{1\ \mathrm{hr}}{3600\ \mathrm{s}}\right) = \underline{\underline{5.17\times10^{-6}\frac{\mathrm{ft}}{\mathrm{s}}}}$$

Problem 1.22

Express the following quantities in SI units: **(a)** 160 acres, **(b)** 15 gallons (U.S.), **(c)** 240 miles, **(d)** 79.1 hp, and **(e)** 60.3 °F.

Solution 1.22

a) $$160\,\text{acre} = (160\,\text{acre})\left(4.356\times10^{4}\,\frac{\text{ft}^2}{\text{acre}}\right)\left(9.290\times10^{-2}\,\frac{\text{m}^2}{\text{ft}^2}\right) = \underline{\underline{6.47\times10^{5}\,\text{m}^2}}$$

b) $$15\,\text{gallons} = (15\,\text{gallons})\left(3.785\,\frac{\text{liters}}{\text{gallon}}\right)\left(10^{-3}\,\frac{\text{m}^3}{\text{liter}}\right) = \underline{\underline{56.8\times10^{-2}\,\text{m}^3}}$$

c) $$240\,\text{mi} = (240\,\text{mi})\left(5280\,\frac{\text{ft}}{\text{mi}}\right)\left(3.048\times10^{-1}\,\frac{\text{m}}{\text{ft}}\right) = \underline{\underline{3.86\times10^{5}\,\text{m}}}$$

d) $$79.1\,\text{hp} = (79.1\,\text{hp})\left(550\,\frac{\frac{\text{ft}\cdot\text{lb}}{\text{s}}}{\text{hp}}\right)\left(1.356\,\frac{\text{J}}{\text{ft}\cdot\text{lb}}\right) = 5.90\times10^{4}\,\frac{\text{J}}{\text{s}}\times\frac{1\,\text{W}}{1\frac{\text{J}}{\text{s}}} = \underline{\underline{5.90\times10^{4}\,\text{W}}}$$

e) Relationship between units of temperature:

$$\text{K} = {}^\circ\text{C} + 273 = \frac{5}{9}\left({}^\circ\text{F} - 32\right) + 273$$

$$\frac{5}{9}\left(60.3^\circ\text{F} - 32\right) + 273 = \underline{\underline{289\,\text{K}}}$$

Problem 1.23

Water flows from a large drainage pipe at a rate of $1200\ \text{gal/min}$. What is this volume rate of flow in **(a)** m^3/s, **(b)** liters/min, and **(c)** ft^3/s?

Solution 1.23

a) $$\text{flowrate} = \left(1200\,\frac{\text{gal}}{\text{min}}\right)\left(6.309\times10^{-5}\,\frac{\frac{\text{m}^3}{\text{s}}}{\frac{\text{gal}}{\text{min}}}\right) = \underline{\underline{7.57\times10^{-2}\,\frac{\text{m}^3}{\text{s}}}}$$

b) Since $1\ \text{liter} = 10^{-3}\ \text{m}^3$,

$$\text{flowrate} = \left(7.57\times10^{-2}\,\frac{\text{m}^3}{\text{s}}\right)\left(\frac{10^3\ \text{liters}}{\text{m}^3}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) = \underline{\underline{4540\,\frac{\text{liters}}{\text{min}}}}$$

c) $$\text{flowrate} = \left(7.57\times10^{-2}\,\frac{\text{m}^3}{\text{s}}\right)\left(3.531\times10\,\frac{\frac{\text{ft}^3}{\text{s}}}{\frac{\text{m}^3}{\text{s}}}\right) = \underline{\underline{2.67\,\frac{\text{ft}^3}{\text{s}}}}$$

Problem 1.24

The universal gas constant R_0 is equal to $49,700\ \mathrm{ft}^2/\left(\mathrm{s}^2\cdot{}^\circ\mathrm{R}\right)$, or $8310\ \mathrm{m}^2/\left(\mathrm{s}^2\cdot\mathrm{K}\right)$. Show that these two magnitudes are equal.

Solution 1.24

$$R_0=\left(\frac{8310\,\mathrm{m}^2}{\mathrm{s}^2\cdot\mathrm{K}}\right)\left(\frac{3.281\,\mathrm{ft}}{1\,\mathrm{m}}\right)^2\left(\frac{(5/9)\,\mathrm{K}}{1^\circ\mathrm{R}}\right)=\underline{\underline{49,700\ \frac{\mathrm{ft}^2}{\left(\mathrm{sec}^2\cdot{}^\circ\mathrm{R}\right)}}}$$

Problem 1.25

Dimensionless combinations of quantities (commonly called dimensionless parameters) play an important role in fluid mechanics. Make up five possible dimensionless parameters by using combinations of some of the quantities listed in Table B.1 Physical Properties of Water (BG/EE Units).

Solution 1.25

Some possible examples:

$$\underline{\underline{\frac{\text{acceleration} \times \text{time}}{\text{velocity}}}} \doteq \frac{\left(LT^{-2}\right)(T)}{\left(LT^{-1}\right)} \doteq L^0 T^0$$

$$\underline{\underline{\text{frequency} \times \text{time}}} \doteq \left(T^{-1}\right)(T) \doteq T^0$$

$$\underline{\underline{\frac{(\text{velocity})^2}{\text{length} \times \text{acceleration}}}} \doteq \frac{\left(LT^{-1}\right)^2}{(L)\left(LT^{-2}\right)} \doteq L^0 T^0$$

$$\underline{\underline{\frac{\text{force} \times \text{time}}{\text{momentum}}}} \doteq \frac{(F)(T)}{\left(MLT^{-1}\right)} \doteq \frac{(F)(T)}{\left(FT^2L^{-1}\right)\left(LT^{-1}\right)} \doteq F^0 L^0 T^0$$

$$\underline{\underline{\frac{\text{density} \times \text{velocity} \times \text{length}}{\text{dynamic viscosity}}}} \doteq \frac{\left(ML^{-3}\right)\left(LT^{-1}\right)(L)}{ML^{-1}T^{-1}} \doteq M^0 L^0 T^0$$

Problem 1.26

An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is velocity, g is the acceleration of gravity, and ℓ is length. Determine the value of the Froude number for $V = 10$ ft/s, $g = 32.2\ \text{ft/s}^2$, and $\ell = 2$ ft. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

Solution 1.26

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10\ \dfrac{\text{ft}}{\text{s}}}{\sqrt{\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)(2\ \text{ft})}} = \underline{\underline{1.25}}$$

In SI units:

$$V = \left(10\frac{\text{ft}}{\text{s}}\right)\left(0.3048\frac{\text{m}}{\text{ft}}\right) = 3.05\frac{\text{m}}{\text{s}}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$\ell = (2\ \text{ft})\left(0.3048\frac{\text{m}}{\text{ft}}\right) = 0.610\ \text{m}$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05\ \dfrac{\text{m}}{\text{s}}}{\sqrt{\left(9.81\ \dfrac{\text{m}}{\text{s}^2}\right)(0.610\ \text{m})}} = \underline{\underline{1.25}}$$

The value of a dimensionless parameter is independent of the unit system.

Problem 1.28

A tank contains 500 kg of a liquid whose specific gravity is 2. Determine the volume of the liquid in the tank.

Solution 1.28

$m = \rho V = SG\rho_{H_2O}V$

Thus,

$$V = \frac{m}{\left(SG\rho_{H_2O}\right)} = \frac{500\text{ kg}}{\left((2)\left(999\,\dfrac{\text{kg}}{\text{m}^3}\right)\right)} = \underline{\underline{0.250\text{ m}^3}}$$

Problem 1.29

A stick of butter at 35°F measures $1.25\,\text{in.} \times 1.25\,\text{in.} \times 4.65\,\text{in.}$ and weighs 4 ounces. Find its specific weight.

Solution 1.29

$$\gamma = \frac{W}{\cancel{V}} = \frac{(4\,\text{oz})\left(\dfrac{1\,\text{lb}}{16\,\text{oz}}\right)}{(1.25\,\text{in.})^2(4.65\,\text{in.})\left(\dfrac{1\,\text{ft}}{12\,\text{in.}}\right)^3} = \underline{\underline{59.5\,\frac{\text{lb}}{\text{ft}^3}}}$$

Problem 1.30

Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter $\left(\text{g/m}^3\right)$. Assume that a cumulus cloud occupies a volume of 1 cubic kilometer, and its liquid water content is 0.2 g/m^3. **(a)** What is the volume of this cloud in cubic miles? **(b)** How much does the water in the cloud weigh in pounds?

Solution 1.30

a) $\text{Volume} = (1\text{ km})^3\left(\dfrac{1{,}000\text{ m}}{1\text{ km}}\right)^3 = 10^9\text{ m}^3\left(\dfrac{3.281\text{ ft}}{1\text{ m}}\times\dfrac{1\text{ m}}{5{,}280\text{ ft}}\right)^3 = \underline{\underline{0.240\text{ mi}^3}}$

b) $W = \gamma \times \text{Volume}$

$$\gamma = \rho g = \left(0.2\,\frac{\text{g}}{\text{m}^3}\right)\left(\frac{1\text{ kg}}{1{,}000\text{ g}}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot\text{m}}\right) = 1.962\times10^{-3}\,\frac{\text{N}}{\text{m}^3}$$

$$W = \left(1.962\times10^{-3}\,\frac{\text{N}}{\text{m}^2}\right)\left(10^9\text{ m}^3\right)\left(2.248\times10^{-1}\,\frac{\text{lb}}{\text{N}}\right) = \underline{\underline{4.4\times10^5\text{ lb}}}$$

Problem 1.31

A tank of oil has a mass of 25 slugs. **(a)** Determine its weight in pounds and in Newtons at the Earth's surface. **(b)** What would be its mass (in slugs) and its weight (in pounds) if located on the moon's surface where the gravitational attraction is approximately one-sixth that at the Earth's surface?

Solution 1.31

a) $\text{weight} = \text{mass} \times g$

$$= (25\text{ slugs})\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1\text{ lb}\cdot\text{s}^2}{1\text{ slug}\cdot\text{ft}}\right) = \underline{\underline{805\text{ lb}}}$$

$$= (25\text{ slugs})\left(\frac{14.59\text{ kg}}{1\text{ slug}}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot\text{m}}\right) = \underline{\underline{3580\text{ N}}}$$

b) $\text{mass} = \underline{\underline{25\text{ slugs}}}\,(\text{mass does not depend on gravitational attraction})$

$$\text{weight} = \frac{805\text{ lb}}{6} = \underline{\underline{134\text{ lb}}}$$

Problem 1.32

A certain object weighs 300 N at the Earth's surface. Determine the mass of the object (in kilograms) and its weight (in newtons) when located on a planet with an acceleration of gravity equal to 4.0 $\mathrm{ft/s^2}$.

Solution 1.32

$$\text{mass} = \frac{\text{weight}}{g} = \frac{300\ \mathrm{N}}{9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}} = \underline{\underline{30.6\ \mathrm{kg}}}$$

For $g = 4.0\frac{\mathrm{ft}}{\mathrm{s}^2}$,

$$\text{weight} = (30.6\ \mathrm{kg})\left(4.0\,\frac{\mathrm{ft}}{\mathrm{s}^2}\right)\left(0.3048\,\frac{\mathrm{m}}{\mathrm{ft}}\right)\left(\frac{1\ \mathrm{N}\cdot\mathrm{s}^2}{1\ \mathrm{kg}\cdot\mathrm{m}}\right) = \underline{\underline{37.3\ \mathrm{N}}}$$

Problem 1.33

The density of a certain type of jet fuel is 775 kg/m^3. Determine its specific gravity and specific weight.

Solution 1.33

$$SG = \frac{\rho}{\rho_{H_2O\,@\,4\,^\circ C}} = \frac{775\,\dfrac{\text{kg}}{\text{m}^3}}{1000\,\dfrac{\text{kg}}{\text{m}^3}} = \underline{\underline{0.775}}$$

$$\gamma = \rho g = \left(775\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^3}\right) = \underline{\underline{7.60\,\frac{\text{kN}}{\text{m}^3}}}$$

Problem 1.34

At 4°C a mixture of automobile antifreeze (50% water and 50% ethylene glycol by volume) has a density of 1064 kg/m^3. If the water density is 1000 kg/m^3, find the density of the ethylene glycol.

Solution 1.34

$$S = \frac{(\rho_{\text{mixture}})_{4°\text{C}}}{(\rho_{\text{water}})_{4°\text{C}}} = \frac{\left(\dfrac{m_{eg}+m_w}{\forall}\right)_{4°\text{C}}}{\left(\dfrac{m_w}{\forall}\right)_{4°\text{C}}} = \frac{m_{eg}+m_w}{m_w}$$

where m_w is the mass of the pure water in volume $\cancel{V}$ at 4°C.
Then

$$S = \frac{\rho_{eg}(0.5\forall)+\rho_w(0.5\forall)}{\rho_w \forall} = 0.5\left(\frac{\rho_{eg}+\rho_w}{\rho_w}\right).$$

The problem statement gives

$$S = \frac{\rho_{\text{mixture}}}{\rho_w} = \frac{1064\,\dfrac{\text{kg}}{\text{m}^3}}{1000\,\dfrac{\text{kg}}{\text{m}^3}} = 1.064$$

$$\rightarrow 1.064 = 0.5\left(\frac{\rho_{eg}+\rho_w}{\rho_w}\right)$$

$$\rightarrow \rho_{eg} = \rho_w\left(\frac{1.064}{0.5}-1\right) = \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1.064}{0.5}-1\right)$$

$$\boxed{\rho_{eg} = 1130\,\frac{\text{kg}}{\text{m}^3}}$$

<u>DISCUSSION</u> If the mixture were at some temperature T, then for equal volumes of mixture and 4°C water,

$$S = \frac{(m_{eg}+m_w)_T}{(m_w)_{4°\text{C}}} = \frac{0.5(\rho_{eg}+\rho_w)_T}{(\rho_w)_{4°\text{C}}}$$

Problem 1.35

A *hydrometer* is used to measure the specific gravity of liquids. For a certain liquid, a hydrometer reading indicates a specific gravity of 1.15.What is the liquid's density and specific weight? Express your answer in SI units.

Solution 1.35

$$SG = \frac{\rho}{\rho_{H_2O\,@4^\circ\mathrm{C}}}$$

$$1.15 = \frac{\rho}{1000\,\dfrac{\mathrm{kg}}{\mathrm{m}^3}}$$

$$\rho = (1.15)\left(1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right) = \underline{\underline{1150\,\frac{\mathrm{kg}}{\mathrm{m}^3}}}$$

$$\gamma = \rho g = \left(1150\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{1\,\mathrm{N}\cdot\mathrm{s}^2}{1\,\mathrm{kg}\cdot\mathrm{m}}\right) = \underline{\underline{11.3\,\frac{\mathrm{kN}}{\mathrm{m}^3}}}$$

Problem 1.36

An open, rigid-walled, cylindrical tank contains 4 ft^3 of water at 40 °F. Over a 24-hour period of time the water temperature varies from 40 to 90 °F. Make use of the data in Appendix B to determine how much the volume of water will change. For a tank diameter of 2 ft, would the corresponding change in water depth be very noticeable? Explain.

Solution 1.36

mass of water $= \forall \times \rho$

Amount ot mass is not a function of temperature.

$$\forall_{40^o} \times \rho_{40^o} = m = \forall_{90^o} \times \rho_{90^o}$$

From Table B.1 Physical Properties of Water (BG/EE Units)

$$\rho_{H_2O\,@40\,^oF} = 1.940\,\frac{\text{slugs}}{\text{ft}^3}$$

$$\rho_{H_2O\,@90\,^oF} = 1.931\,\frac{\text{slugs}}{\text{ft}^3}$$

Therefore,

$$\cancel{V}_{90^o} = \frac{\left(4\,\text{ft}^3\right)\left(1.940\,\dfrac{\text{slugs}}{\text{ft}^3}\right)}{1.931\,\dfrac{\text{slugs}}{\text{ft}^3}} = 4.0186\,\text{ft}^3$$

Thus, the increase in volume is: $\Delta\forall = 4.0186 - 4.000 = \underline{\underline{0.0186\,\text{ft}^3}}$

For a tank diameter of 2 ft: $\Delta\text{h} = \dfrac{\Delta\forall}{A} = \dfrac{0.0186\,\text{ft}^3}{\dfrac{\pi}{4}(2\,\text{ft})^2} = 5.92 \times 10^{-3}\,\text{ft} = 0.0710\,\text{in.}$

This small change in depth would not be very noticeable. $\underline{\underline{\text{No.}}}$

Note: A slightly different value for Δh will be obtained if specific weight of water is used rather than its density. This is due to the fact that there is some uncertainty in the fourth significant figure of these two values, and the solution is sensitive to this uncertainty.

Problem 1.38

A mountain climber's oxygen tank contains 1 lb of oxygen when he begins his trip at sea level where the acceleration of gravity is 32.174 ft/s^2. What is the weight of the oxygen in the tank when he reaches the top of Mt. Everest where the acceleration of gravity is 32.082 ft/s^2? Assume that no oxygen has been removed from the tank; it will be used on the descent portion of the climb.

Solution 1.38

$W = mg$

Let $(\ )_{sl}$ denote sea level and $(\ )_{MtE}$ denote the top of Mt. Everest

Thus,

$W_{sl} = 1\,lb = m_{sl}g_{sl}$

and

$W_{MtE} = m_{MtE}g_{MtE}$

However $m_{sl} = m_{MtE}$ so that since $m = \dfrac{W}{g}$,

$$m_{sl} = \frac{W_{sl}}{g_{sl}} = m_{MtE} = \frac{W_{MtE}}{g_{MtE}}$$

or

$$W_{MtE} = W_{sl}\frac{g_{MtE}}{g_{sl}} = 1\,\text{lb}\frac{32.082\,\frac{\text{ft}}{\text{s}^2}}{32.174\,\frac{\text{ft}}{\text{s}^2}} = \underline{\underline{0.9971\,\text{lb}}}$$

Problem 1.39

The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg, while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

Solution 1.39

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

$$\text{total weight} = \text{mass} \times g = (0.369\ \text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}}\right) = 3.62\ \text{N}$$

$$\text{weight of can} = 0.153\ \text{N}$$

$$\text{volume of fluid} = \left(355\times10^{-3}\ \text{L}\right)\left(\frac{1\ \text{m}^3}{1{,}000\ \text{L}}\right) = 355\times10^{-6}\ \text{m}^3$$

Therefore,

$$\gamma = \frac{3.62\ \text{N} - 0.153\ \text{N}}{355\times10^{-6}\ \text{m}^3} = \underline{\underline{9770\ \frac{\text{N}}{\text{m}^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\ \frac{\text{N}}{\text{m}^3}}{9.81\ \frac{\text{m}}{\text{s}^2}} = 996\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}\left(\frac{1\ \text{kg}\cdot\text{m}}{1\ \text{N}\cdot\text{s}^2}\right) = \underline{\underline{996\ \frac{\text{kg}}{\text{m}^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O\,@4\,^\circ C}} = \frac{996\ \frac{\text{kg}}{\text{m}^3}}{1000\ \frac{\text{kg}}{\text{m}^3}} = \underline{\underline{0.996}}$$

For water at 20 °C (see Table B.2 Physical Properties of Water [SI Units])

$$\gamma_{H_2O} = 9789\frac{\text{N}}{\text{m}^3}; \quad \rho_{H_2O} = 998.2\frac{\text{kg}}{\text{m}^3}; \quad SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

Problem 1.40

The variation in the density of water, ρ, with temperature, T, in the range $20\ °C \le T \le 50\ °C$, is given in the following table.

Density (kg/m^3)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2 T + c_3 T^2$ which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at $42.1\ °C$?

Solution 1.40

Fit the data to a second order polynomial using a standard curve-fitting program such as found in Excel. Thus,

$$\rho = \underline{\underline{1001 - 0.0533\mathrm{T} - 0.0041\mathrm{T}^2}}$$

As shown in the table below, ρ (predicted) from Eq.(1) is in good agreement with ρ (given).

T (°C)	ρ-predict (kg/m^3)	ρ-data (kg/m^3)
20	998.3	998.2
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.3	992.2
45	990.3	990.2
50	988.1	988.1

$$\text{At } T = 42.1\ °\text{C}: \ \rho = 1001 - 0.0533(42.1\ °\text{C}) - 0.0041(42.1\ °\text{C})^2 = \underline{\underline{991.5\,\frac{\text{kg}}{\text{m}^3}}}$$

Problem 1.41

If 1 cup of cream having a density of 1005 kg/m^3 is turned into 3 cups of whipped cream, determine the specific gravity and specific weight of the whipped cream.

Solution 1.41

Mass of cream, $m = \left(1005\,\frac{\text{kg}}{\text{m}^3}\right)\times\left(\forall_{\text{cup}}\right)$, were $\forall$ = volume.

Noting that the mass is the same in liquid and in "whipped" form,

$$\rho_{\substack{\text{whipped}\\\text{cream}}} = \frac{m_{\substack{\text{whipped}\\\text{cream}}}}{\forall_{3\text{ cups}}} = \frac{\left(1005\,\frac{\text{kg}}{\text{m}^3}\right)\forall_{cup}}{\forall_{3\,cups}} = \frac{1005\,\frac{\text{kg}}{\text{m}^3}}{3} = 335\,\frac{\text{kg}}{\text{m}^3}$$

$$SG = \frac{\rho_{\substack{\text{whipped}\\\text{cream}}}}{\rho_{H_2O\,@4\,^\circ C}} = \frac{335\,\frac{\text{kg}}{\text{m}^3}}{1000\,\frac{\text{kg}}{\text{m}^3}} = \underline{\underline{0.335}}$$

$$\gamma_{\substack{\text{whipped}\\\text{cream}}} = \rho_{\substack{\text{whipped}\\\text{cream}}}\times g = \left(335\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}}\right) = \underline{\underline{3,290\,\frac{\text{N}}{\text{m}^3}}}$$

Problem 1.42

With the exception of the 410 bore, the gauge of a shotgun barrel indicates the number of round lead balls, each having the bore diameter of the barrel, that together weigh 1 lb. For example, a shotgun is called a 12-gauge shotgun if a $\frac{1}{12}$-lb lead ball fits the bore of the barrel. Find the diameter of a 12-gauge shotgun in inches and millimeters. Lead has a specific weight of 0.411 lb/in^3.

Solution 1.42

$$\forall_{\text{ball}} = \frac{\text{weight}}{\gamma} = \frac{\frac{1}{12}\text{ lb}}{\left(0.411 \frac{\text{lb}}{\text{in.}^3}\right)} = 0.20276\,\text{in.}^3$$

For a sphere: $\forall = \dfrac{4\pi R^3}{3} \quad \rightarrow \quad R = \sqrt[3]{\dfrac{3\forall}{4\pi}}$

$$R = \sqrt[3]{\frac{3\left(0.20276\,\text{in.}^3\right)}{4\pi}} = 0.3644\,\text{in.}$$

$$\boxed{D = 0.729\text{ in.}}$$

$$D = 0.729\,\text{in.}\left(\frac{25.4\,\text{mm}}{\text{in.}}\right)$$

$$\boxed{D = 18.5\,\text{mm}}$$

Problem 1.44

A regulation basketball is initially flat and is then inflated to a pressure of approximately 24 lb/in^2 absolute. Consider the air temperature to be constant at 70 °F. Find the mass of air required to inflate the basketball. The basketball's inside radius is 4.67 in.

Solution 1.44

$$m = \frac{p\forall}{RT} = \frac{p}{RT}\left(\frac{4}{3}\pi R^3\right)$$

$$= \frac{\left(24\dfrac{\text{lb}}{\text{in.}^2}\right)\left(\dfrac{4\pi}{3}\right)(4.67\,\text{in.})^3\left(\dfrac{1\,\text{ft}}{12\,\text{in.}}\right)}{\left(1716\dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\,^\circ\text{R}}\right)(460+70)\,^\circ\text{R}}$$

$$\boxed{m = 0.000938\,\text{slug}}$$

Problem 1.45

Nitrogen is compressed to a density of 4 kg/m^3 under an absolute pressure of 400 kPa. Determine the temperature in degrees Celsius.

Solution 1.45

$$T = \frac{p}{\rho R} = \frac{400 \times 10^3 \ \frac{\text{N}}{\text{m}^2}}{\left(4 \ \frac{\text{kg}}{\text{m}^3}\right)\left(296.8 \ \frac{\text{J}}{\text{kg} \cdot \text{K}}\right)\left(\frac{1 \ \frac{\text{N} \cdot \text{m}}{\text{s}}}{1 \ \text{J}}\right)} = 337 \ \text{K}$$

$$T_C = T_K - 273 = 337 \ \text{K} - 273 = \underline{\underline{64 \ ^0\text{C}}}$$

Problem 1.46

The temperature and pressure at the surface of Mars during a Martian spring day were determined to be $-50°C$ and 900 Pa, respectively. **(a)** Determine the density of the Martian atmosphere for these conditions if the gas constant for the Martian atmosphere is assumed to be equivalent to that of carbon dioxide. **(b)** Compare the answer from part (a) with the density of the Earth's atmosphere during a spring day when the temperature is $18°C$ and the pressure 101.6 kPa (abs).

Solution 1.46

a) $$\rho_{\text{Mars}} = \frac{p}{RT} = \frac{900\,\dfrac{\text{N}}{\text{m}^2}}{\left(188.9\,\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\,\frac{\text{N}\cdot\text{m}}{\text{s}}}{1\,\text{J}}\right)\left[\left(-50\,^0C + 273\right)\text{K}\right]} = \underline{\underline{0.0214\,\frac{\text{kg}}{\text{m}^3}}}$$

b) $$\rho_{\text{Earth}} = \frac{p}{RT} = \frac{101.6\times 10^3\,\dfrac{\text{N}}{\text{m}^2}}{\left(286.9\,\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\,\frac{\text{N}\cdot\text{m}}{\text{s}}}{1\,\text{J}}\right)\left[\left(18\,^0C + 273\right)\text{K}\right]} = \underline{\underline{1.22\,\frac{\text{kg}}{\text{m}^3}}}$$

Thus, $$\frac{\rho_{\text{Mars}}}{\rho_{\text{Earth}}} = \frac{0.0214\,\dfrac{\text{kg}}{\text{m}^3}}{1.22\,\dfrac{\text{kg}}{\text{m}^3}} = 0.0175 = \underline{\underline{1.75\%}}$$

Problem 1.47

A closed tank having a volume of 2 ft^3 is filled with 0.30 lb of a gas. A pressure gage attached to the tank reads 12 psi when the gas temperature is 80 °F. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

Solution 1.47

Density of gas in tank $\rho = \dfrac{\text{weight}}{g \times \text{volume}} = \dfrac{0.30\ \text{lb}}{\left(32.2\ \dfrac{\text{ft}}{\text{s}^2}\right)\left(2\ \text{ft}^3\right)} = 4.66 \times 10^{-3}\ \dfrac{\text{slugs}}{\text{ft}^3}$

Since $\rho = \dfrac{p}{RT}$ with $p = (12 + 14.7)$ psia (atmospheric pressure assumed to be ≈ 14.7 psia) and with $T = (80\ °\text{F} + 460)\ °\text{R}$ it follows that

$$\rho = \frac{\left(26.7\ \dfrac{\text{lb}}{\text{in}^2}\right)\left(144\ \dfrac{\text{in}^2}{\text{ft}^2}\right)}{R(540)\,°\text{R}} = \frac{7.12}{R}\ \frac{\text{slugs}}{\text{ft}^3} \qquad (1)$$

From Table 1.7 Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (BG Units)

$R = 1.554 \times 10^3\ \dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°\text{R}}$ for oxygen … and … $R = 1.242 \times 10^4\ \dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°\text{R}}$ for helium.

Thus, from Eq.(1) if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3}\ \frac{\text{slugs}}{\text{ft}^3} = 4.58 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}$$

and for helium

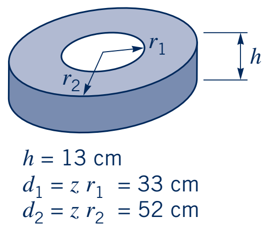
$$\rho = \frac{7.12}{1.242 \times 10^4}\ \frac{\text{slugs}}{\text{ft}^3} = 5.73 \times 10^{-4}\ \frac{\text{slugs}}{\text{ft}^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be oxygen.

Problem 1.48

Assume that the air volume in a small automobile tire is constant and equal to the volume between two concentric cylinders 13 cm high with diameters of 33 cm and 52 cm. The air in the tire is initially at 25°C and 202 kPa. Immediately after air is pumped into the tire, the temperature is 30°C and the pressure is 303 kPa. What mass of air was added to the tire? What would be the air pressure after the air has cooled to a temperature of 0 °C?

Solution 1.48



The mass of air added to the tire is the difference of the final mass of air m_f and the initial mass m_i. Assuming air is an ideal gas,

$$m_f - m_i = \left(\frac{p}{RT}\forall\right)_f - \left(\frac{p}{RT}\forall\right)_i = \frac{\forall}{R}\left(\frac{p_f}{T_f} - \frac{p_i}{T_i}\right).$$

Now

$$\forall = \pi\left(r_2^2 - r_1^2\right)h = \pi\left[(26\,\text{cm})^2 - (16.5\,\text{cm})^2\right](13\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^3$$

$$= 0.0165\,\text{m}^3.$$

$$m_f - m_i = \frac{\left(0.0165\,\text{m}^3\right)}{\left(287.0\,\dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{303\,\text{kPa}}{(273+30)\,\text{K}} - \frac{202\,\text{kPa}}{(273+25)\,\text{K}}\right]\left(\frac{1000\,\dfrac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)$$

$$\boxed{m_f - m_i = 0.0185\,\text{kg}}$$

Now consider the cooling process. The initial state will be 30 °C and 303 kPa. The final state will be 0 °C and p_f. Applying the ideal gas law to both states gives

$$\left(\frac{p}{RT}\forall\right)_i = \left(\frac{p}{RT}\forall\right)_f.$$

Since $\forall_f = \forall_i$,

$$p_f = p_i\left(\frac{T_f}{T_i}\right) = (303\,\text{kPa})\left(\frac{273+0}{273+30}\right) = \boxed{p_f = 273\,\text{kPa}}$$

Problem 1.49

A compressed air tank contains 5 kg of air at a temperature of $80°\text{C}$. A gage on the tank reads $300\ \text{kPa}$. Determine the volume of the tank.

Solution 1.49

$$\text{volume} = \frac{\text{mass}}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{(300+101)\times 10^3\ \dfrac{\text{N}}{\text{m}^3}}{\left(286.9\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\ \frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)\left[(80\ °\text{C}+273)\,\text{K}\right]} = 3.96\ \frac{\text{kg}}{\text{m}^3}$$

$$\text{volume} = \frac{5\ \text{kg}}{3.96\ \dfrac{\text{kg}}{\text{m}^3}} = \underline{\underline{1.26\ \text{m}^3}}$$

Problem 1.50

A rigid tank contains air at pressure of 90 psia and a temperature of 60 °F. By how much will the pressure increase as the temperature is increased to 110 °F?

Solution 1.50

$p = \rho RT$

For a rigid closed tank, the air mass and volume are constant so $\rho =$ constant. Thus, from the equation above (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (1)$$

where $p_1 = 90$ psia, $T_1 = 60\ °\text{F} + 460 = 520\ °\text{R}$,

and $T_2 = 110\ °\text{F} + 460 = 570\ °\text{R}$.

From Eq.(1)

$$p_2 = \frac{T_2}{T_1} p_1 = \left(\frac{570\ °\text{R}}{520\ °\text{R}}\right)(90\text{ psia}) = \underline{\underline{98.7\text{ psia}}}$$

Problem 1.51

The density of oxygen contained in a tank is 2.0 kg/m^3 when the temperature is 25 °C. Determine the gage pressure of the gas if the atmospheric pressure is 97 kPa.

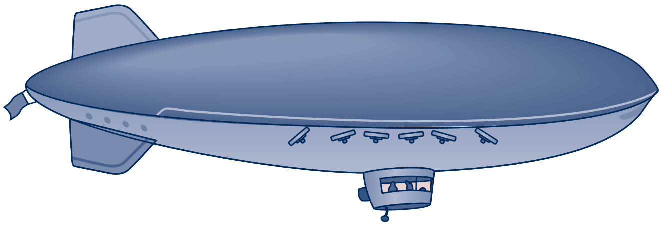
Solution 1.51

$$p = \rho RT = \left(2.0\ \frac{\text{kg}}{\text{m}^3}\right)\left(259.8\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left[(25\ °\text{C} + 273)\ \text{K}\right] = 155\ \text{kPa}\,(\text{abs})$$

$$p(\text{gage}) = P_{abs} - P_{atm} = 155\ \text{kPa} - 97\ \text{kPa} = \underline{\underline{58\ \text{kPa}}}$$

Problem 1.52

The helium-filled blimp shown in the figure below is used at various athletic events. Determine the number of pounds of helium within it if its volume is 68,000 ft^3 and the temperature and pressure are 80 °F and 14.2 psia, respectively.



Solution 1.52

$W = \gamma V$ where $V = 68000\ \text{ft}^3$ and $\gamma = \rho g = \left(\dfrac{p}{RT}\right) g$

Thus,

$$\gamma = \frac{14.2\ \dfrac{\text{lb}}{\text{in.}^2}\left(144\ \dfrac{\text{in.}^2}{\text{ft}^2}\right)}{\left(\left(1.242\times10^4\ \dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)(80+460)\ {}^\circ\text{R}\right)}\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right) =$$

$$= 9.82\times10^{-3}\ \frac{\text{slug}}{\text{ft}^2\cdot\text{s}^2}\left(\frac{1\ \text{lb}}{\left(\dfrac{\text{slug ft}}{\text{s}^2}\right)}\right) = 9.82\times10^{-3}\ \frac{\text{lb}}{\text{ft}^3}$$

Hence,

$$W = 9.82\times10^{-3}\ \frac{\text{lb}}{\text{ft}^3}\left(68000\ \text{ft}^3\right) = \underline{\underline{668\ \text{lb}}}$$

Problem 1.53

Develop a computer program for calculating the density of an ideal gas when the gas pressure in Pascals (abs), the temperature in degrees Celsius, and the gas constant in J/kg·K are specified. Plot the density of helium as a function of temperature from 0 °C to 200 °C and pressures of 50, 100, 150, and 200 kPa (abs).

Solution 1.53

For an ideal gas

$$p = \rho RT$$

so that

$$\rho = \frac{p}{RT}$$

where p is absolute pressure, R the gas constant, and T is absolute temperature. Thus, if the temperature is in °C then

$$T = °\mathrm{C} + 273.15\ \mathrm{K}$$

A spreadsheet (EXCEL) program for calculating ρ follows.

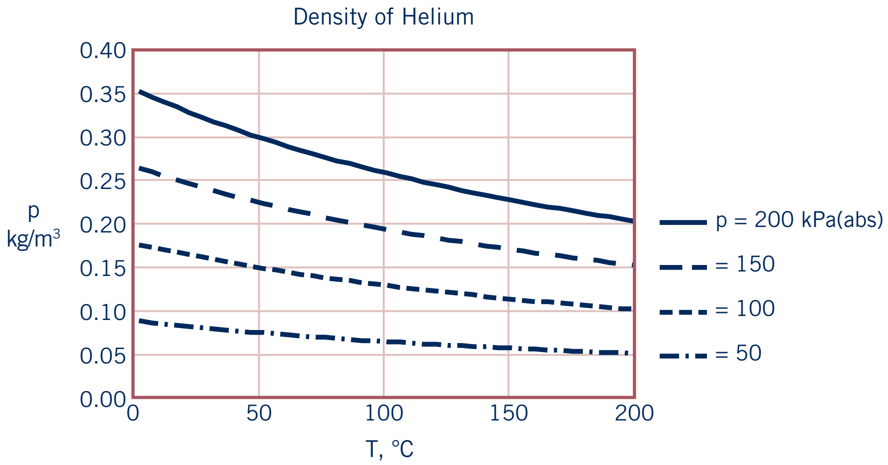
This program calculates the density of an ideal gas when the absolute pressure in Pascal, the temperature in degrees C, and the gas constant in J/kg·K are specified. To use, replace current values with desired values of temperature, pressure, and gas constant.				
A	B	C	D	
Pressure	Temperature	Gas constant	Destiny	
Pa	°C	J/kg·K	kg/m³	
1.01E+05	15	286.9	1.22	

Formula:
=A10/((B10+273.15)*C10)

Example: Calculate ρ for $p = 200$ kPa, temperature = 20 °C, and $R = 287\ \dfrac{\mathrm{J}}{\mathrm{kg \cdot K}}$.

This program calculates the density of an ideal gas when the absolute pressure in Pascal, the temperature in degrees Celsius, and the gas constant in J/kg·K are specified. To use, replace current values with desired values of temperature, pressure, and gas constant.			
A	B	C	D
Pressure	Temperature	Gas constant	Destiny
Pa	°C	J/kg·K	kg/m³
2.00E+05	20	287	2.38

The density of helium is plotted in the graph below.



Problem 1.55

For flowing water, what is the magnitude of the velocity gradient needed to produce a shear stress of $1.0\ \dfrac{N}{\text{m}^2}$?

Solution 1.55

$\tau = \mu \dfrac{du}{dy}$ where $\mu = 1.12 \times 10^{-3}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}$ and $\tau = 1.0\ \dfrac{\text{N}}{\text{m}^2}$

Thus,

$$\frac{du}{dy} = \frac{\tau}{\mu} = \frac{1.0\ \dfrac{\text{N}}{\text{m}^2}}{1.12 \times 10^{-3}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} = \underline{\underline{893\ \frac{1}{\text{s}}}}$$

Problem 1.56

Make use of the data in Appendix B to determine the dynamic viscosity of glycerin at $85\ °\text{F}$. Express your answer in both SI and BG units.

Solution 1.56

$$T_C = \frac{5}{9}\left(T_F - 32\right) = \frac{5}{9}\left(85\ °\text{F} - 32\right) = 29.4\ °\text{C}$$

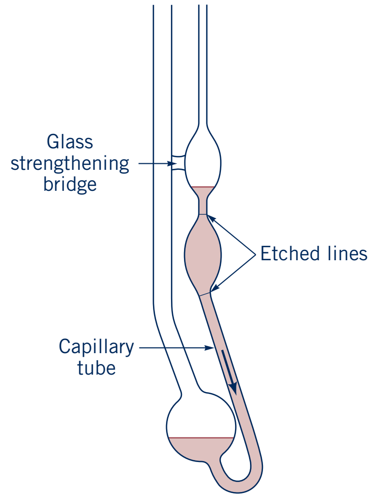
From the figure in Appendix B:

$$\mu\left(\text{glycerin at } 85\ °\text{F } \left(29.4\ °\text{C}\right)\right) \approx \underline{\underline{0.6\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}}\ (\text{SI units})$$

$$\mu \approx \left(0.6\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(2.089\times10^{-2}\ \frac{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}\right) \approx \underline{\underline{1.3\times10^{-2}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}}\ (\text{BG units})$$

Problem 1.57

One type of *capillary-tube viscometer* is shown in the figure below. For this device the liquid to be tested is drawn into the tube to a level above the top etched line. The time is then obtained for the liquid to drain to the bottom etched line. The kinematic viscosity, ν, in m^2/s is then obtained from the equation $\nu = KR^4 t$ where K is a constant, R is the radius of the capillary tube in mm, and t is the drain time in seconds. When glycerin at 20 °C is used as a calibration fluid in a particular viscometer, the drain time is 1430 s. When a liquid having a density of 970 kg/m^3 is tested in the same viscometer the drain time is 900 s. What is the dynamic viscosity of this liquid?



Solution 1.57

$$\nu = KR^4 t$$

For glycerin @ 20 °C $\nu = 1.19\times10^{-3}\ \dfrac{m^2}{s^2} = \left(KR^4\right)(1430\ s)$

$$KR^4 = 8.32\times10^{-7}\ \frac{m^2}{s^2}$$

For unknown liquid with $t = 900$ s

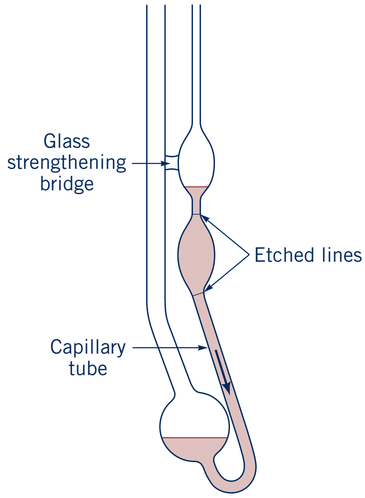
$$\nu = \left(8.32\times10^{-7}\ \frac{m^2}{s^2}\right)(900\ s) = 7.49\times10^{-4}\ \frac{m^2}{s^2}$$

By definition: $\nu \equiv \dfrac{\mu}{\rho} \rightarrow \quad \mu = \left(970\ \dfrac{kg}{m^3}\right)\left(7.49\times10^{-4}\ \dfrac{m^2}{s}\right) = 0.727\ \dfrac{kg}{m\cdot s}\times\dfrac{1\ N\cdot s^2}{1\ kg\cdot m}$

$$\underline{\underline{\mu = 0.727\ \frac{N\cdot s}{m^2}}}$$

Problem 1.58

The viscosity of a soft drink was determined by using a capillary tube viscometer shown in the figure. For this device the kinematic viscosity, ν, is directly proportional to the time, t, that it takes for a given amount of liquid to flow through a small capillary tube. That is, $\nu = Kt$. The following data were obtained from regular pop and diet pop. The corresponding measured specific gravities are also given. Based on these data, by what percent is the absolute viscosity, μ, of regular pop greater than that of diet pop?



	Regular pop	Diet pop
t(s)	377.8	300.3
SG	1.044	1.003

Solution 1.58

$$\% \text{ greater} = \left[\frac{\mu_{reg} - \mu_{diet}}{\mu_{diet}}\right] \times 100 = \left[\frac{\mu_{reg}}{\mu_{diet}} - 1\right] \times 100$$

By definition $\nu \equiv \dfrac{\mu}{\rho}$, and $\rho \equiv (SG)\rho_{H_2O\ @4\ °C}$.

Given $\nu = kt$:

$$\% \text{ greater} = \left[\frac{(\nu\rho)_{reg}}{(\nu\rho)_{diet}} - 1\right] \times 100 = \left[\frac{(t \times SG)_{reg}}{(t \times SG)_{diet}} - 1\right] \times 100 = \left[\frac{(377.8 \text{ s})(1.044)}{(300.3 \text{ s})(1.003)} - 1\right] \times 100$$

$$\% \text{ greater} = \underline{\underline{31.0\ \%}}$$

Problem 1.59

The viscosity of a certain fluid is 5×10^{-4} poise. Determine its viscosity in both SI and BG units.

Solution 1.59

From Appendix E, $1 \text{ poise} = 10^{-1} \dfrac{\text{N} \cdot \text{s}}{\text{m}^2}$.

Thus, $\mu = \left(5 \times 10^{-4} \text{ poise}\right)\left(\dfrac{10^{-1} \dfrac{\text{N} \cdot \text{s}}{\text{m}^2}}{1 \text{ poise}}\right) = \underline{\underline{5 \times 10^{-5} \dfrac{\text{N} \cdot \text{s}}{\text{m}^2}}}.$

From Table 1.4 Conversion Factors from SI Units to BG and EE Units (end paper)

$$\mu = \left(5 \times 10^{-5} \frac{\text{N} \cdot \text{s}}{\text{m}^2}\right)\left(\frac{2.089 \times 10^{-2} \dfrac{\text{lb} \cdot \text{s}}{\text{ft}^2}}{1 \dfrac{\text{N} \cdot \text{s}}{\text{m}^2}}\right) = \underline{\underline{10.4 \times 10^{-7} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}}}$$

Problem 1.60

The kinematic viscosity and specific gravity of a liquid are $3.5 \times 10^{-4}\ \dfrac{\mathrm{m}^2}{\mathrm{s}}$ and 0.79, respectively. What is the dynamic viscosity of the liquid in SI units?

Solution 1.60

$$\mu = \nu \rho$$

$$\rho = (SG)\left(\rho_{H_2O\ @\ 4\ °\mathrm{C}}\right)$$

$$\mu = \left(3.5 \times 10^{-4}\ \frac{\mathrm{m}^2}{\mathrm{s}}\right)\left(0.79 \times 10^3\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right) = 0.277\ \frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}} \times \frac{1\ \mathrm{N}\cdot\mathrm{s}^2}{1\ \mathrm{kg}\cdot\mathrm{m}} = \underline{\underline{0.277\ \frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{m}^2}}}$$

Problem 1.61

A liquid has a specific weight of 59 lb/ft^3 and a dynamic viscosity of 2.75 $\text{lb}\cdot\text{s/ft}^2$. Determine its kinematic viscosity.

Solution 1.61

By definition: $\nu \equiv \dfrac{\mu}{\rho}$, and $\rho = \dfrac{\gamma}{g}$,

$$\nu = \frac{\mu g}{\gamma} = \frac{\left(2.75\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(32.2\ \dfrac{\text{ft}}{\text{s}^2}\right)}{59\ \dfrac{\text{lb}}{\text{ft}^3}} = \underline{\underline{1.50\ \frac{\text{ft}^2}{\text{s}}}}$$

Problem 1.62

The kinematic viscosity of oxygen at $20°C$ and a pressure of 150kPa (abs) is 0.104 stokes. Determine the dynamic viscosity of oxygen at this temperature and pressure.

Solution 1.62

$$\nu \equiv \frac{\mu}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{150\times10^{3}\ \dfrac{\text{N}}{\text{m}^2}}{\left(259.8\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left[\left(20\ °\text{C}+273\right)\text{K}\right]} = 1.97\ \frac{\text{kg}}{\text{m}^3}$$

$$\nu = 0.104\ \text{stokes} = 0.104\ \frac{\text{cm}^2}{\text{s}}$$

$$\mu = \nu\rho = \left(0.104\ \frac{\text{cm}^2}{\text{s}}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2\left(1.97\ \frac{\text{kg}}{\text{m}^3}\right)$$

$$= 2.05\times10^{-5}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}\times\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \underline{\underline{2.05\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

Problem 1.63

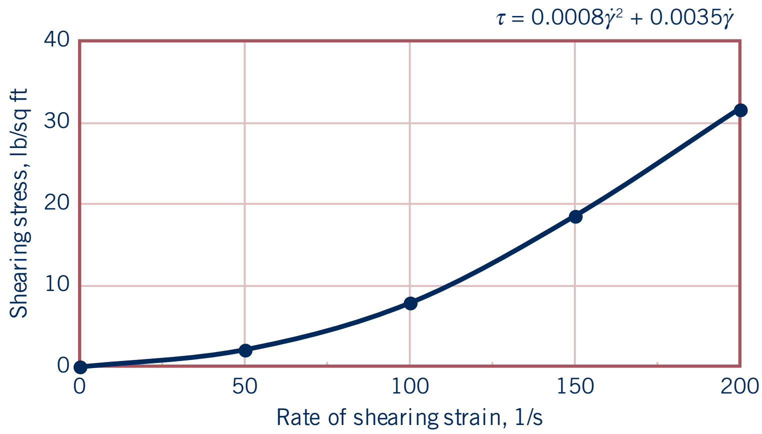
Fluids for which the shearing stress, τ, is not linearly related to the rate of shearing strain, $\dot{\gamma}$, are designated as non-Newtonian fluids. Such fluids are commonplace and can exhibit unusual behavior. Some experimental data obtained for a particular non-Newtonian fluid at 80 °F are shown below.

$\tau(\text{lb/ft}^2)$	0	2.11	7.82	18.5	31.7
$\dot{\gamma}(s^{-1})$	0	50	100	150	200

Plot these data and fit a second-order polynomial to the data using a suitable graphing program. What is the apparent viscosity of this fluid when the rate of shearing strain is 70 s^{-1}? Is this apparent viscosity larger or smaller than that for water at the same temperature?

Solution 1.63

Rate of shearing strain, 1/s	Shearing stress, lb/sq ft
0	0
50	2.11
100	7.82
150	18.5
200	31.7



From the graph $\tau = 0.0008\dot{\gamma}^2 + 0.0035\dot{\gamma}$ where τ is the shearing stress in $\dfrac{\text{lb}}{\text{ft}^2}$ and $\dot{\gamma}$ is the rate of shearing strain in s^{-1}. Fitting a second-order polynomial to the data yields:

$$\mu_{apparent} = \frac{d\tau}{d\dot{\gamma}} = (2)(0.0008)\dot{\gamma} + 0.0035$$

At $\dot{\gamma} = 70\ \text{s}^{-1}$

$$\mu_{apparent} = (2)\left(0.0008\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}^2}\right)\left(70\ \text{s}^{-1}\right) + 0.0035\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2} = \underline{\underline{0.116\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}}$$

From Table B.1 Physical Properties of Water (BG/EE Units)

$\mu_{H_2O\ @\ 80\ °\text{F}} = 1.791\times10^{-5}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}$. Water is a Newtonian fluid so this value is independent of $\dot{\gamma}$. Thus, the viscosity of the non-Newtonian fluid when sheared at a rate of 70 s^{-1} is much <u>larger</u> than the viscosity of water at 80 °F.

Problem 1.64

Water flows near a flat surface and some measurements of the water velocity, u, parallel to the surface, at different heights, y, above the surface are obtained. At the surface $y = 0$. After an analysis of the data, the lab technician reports that the velocity distribution in the range $0 < y < 0.1$ ft is given by the equation

$u = 0.81 + 9.2y + 4.1 \times 10^3 y^3$

with u in ft/s when y is in ft. **(a)** Do you think that this equation would be valid in any system of units? Explain. **(b)** Do you think this equation is correct? Explain.

Solution 1.64

(a)

$$\mathrm{u} = 0.81 + 9.2y + 4.1 \times 10^3 y^3$$

$$\left[\mathrm{LT}^{-1}\right] \doteq [0.81] + [9.2][\mathrm{L}] + \left[4.1 \times 10^3\right]\left[\mathrm{L}^3\right]$$

Each term in the equation must have the same dimensions. Thus, the constant 0.81 must have dimensions of LT^{-1}, 9.2 dimensions of T^{-1}, and 4.1×10^3 dimensions of $\mathrm{L}^{-2}\mathrm{T}^{-1}$. Since the constants in the equation have dimensions their values will change with a change in units. No.

(b) Equation cannot be correct since at $y = 0 \quad \mathrm{u} = 0.81\dfrac{\mathrm{ft}}{\mathrm{s}}$, a non-zero value which would violate the "no-slip" condition. Not correct.

Problem 1.65

Calculate the Reynolds numbers for the flow of water and for air through a 4-mm-diameter tube, if the mean velocity is 3m/s and the temperature is $30°\text{C}$ in both cases. Assume the air is at standard atmospheric pressure.

Solution 1.65

For water at $30\ °\text{C}$ (from Table B.2 Physical Properties of Water [SI Units]):

$$\rho = 995.7\ \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 7.975\times10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\left(995.7\ \dfrac{\text{kg}}{\text{m}^3}\right)\left(3\dfrac{\text{m}}{\text{s}}\right)(0.004\ \text{m})}{7.975\times10^{-4}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} \times \frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}} = \underline{\underline{15000}}$$

For air at $30\ °\text{C}$ (from the Physical Properties of Air at Standard Atmospheric Pressure [SI Units]):

$$\rho = 1.165\ \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 1.86\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\left(1.165\ \dfrac{\text{kg}}{\text{m}^3}\right)\left(3\dfrac{\text{m}}{\text{s}}\right)(0.004\ \text{m})}{1.86\times10^{-5}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} \times \frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}} = \underline{\underline{752}}$$

Problem 1.66

SAE 30 oil at $30°F$ flows through a 2-in.-diameter pipe with a mean velocity of 5 ft/s. Determine the value of the Reynolds number.

Solution 1.66

$$\rho = 1.77 \ \frac{\text{slugs}}{\text{ft}^3}$$

$$\mu = 8.0 \times 10^{-3} \ \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\left(1.77 \ \dfrac{\text{slugs}}{\text{ft}^3}\right)\left(5 \dfrac{\text{ft}}{\text{s}}\right)\left(\dfrac{2}{12} \ \text{ft}\right)}{8.0 \times 10^{-3} \ \dfrac{\text{lb} \cdot \text{s}}{\text{ft}^2}} \times \frac{1 \, \text{lb} \cdot \text{s}^2}{1 \, \text{slug} \cdot \text{ft}} = \underline{\underline{184}}$$

Problem 1.67

For air at standard atmospheric pressure, the values of the constants that appear in the Sutherland equation $\mu = \dfrac{CT^{3/2}}{T+S}$ are $C = 1.458\times10^{-6}\ \text{kg}/\left(\text{m}\cdot\text{s}\cdot\text{K}^{1/2}\right)$ and $S = 110.4$ K. Use these values to predict the viscosity of air at $10\ °\text{C}$ and $90\ °\text{C}$ and compare with values given in the table Physical Properties of Air at Standard Atmospheric Pressure (SI Units).

Solution 1.67

$$\mu = \frac{CT^{\frac{3}{2}}}{T+S} = \frac{\left(1.458\times10^{-6}\ \dfrac{\text{kg}}{\text{m}\cdot\text{s}\cdot\text{K}^{1/2}}\right)T^{3/2}}{T+110.4\ \text{K}}$$

For $T = 10\ °\text{C} \rightarrow \text{T}=10+273.15\ \text{K} = 283.15\ \text{K}$,

$$\mu = \frac{\left(1.458\times10^{-6}\ \dfrac{\text{kg}}{\text{m}\cdot\text{s}\cdot\text{K}^{1/2}}\right)(283.15\ \text{K})^{\frac{3}{2}}}{283.15\ \text{K}+110.4\ \text{K}}$$

$$\mu = 1.765\times10^{-5}\ \frac{\text{kg}}{\text{m}\cdot\text{s}\cdot}\text{K}^{3/2-1/2-1}\times\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \underline{\underline{1.765\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

From the table, $\mu = 1.76\times10^{-5}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}$.

For $T = 90\ °\text{C} = 90\ °\text{C}+273.15 = 363.15$ K,

$$\mu = \frac{\left(1.458\times10^{-6}\right)(363.15)^{3/2}}{363.15+110.4} = \underline{\underline{2.13\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

From the table, $\mu = 2.14\times10^{-5}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}$

Problem 1.68

Use the values of viscosity of air given in the table of Physical Properties of Air at Standard Atmospheric Pressure (SI Units) at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants C and S, which appear in the Sutherland equation $\mu = \dfrac{CT^{3/2}}{T+S}$.

Compare your results with the values given in Problem 1.67.

(Hint: Rewrite the equation in the form $\dfrac{T^{3/2}}{\mu} = \left(\dfrac{1}{C}\right)T + \dfrac{S}{C}$ and plot $\dfrac{T^{3/2}}{\mu}$ versus T. From the slope and intercept of this curve, C and S can be obtained.)

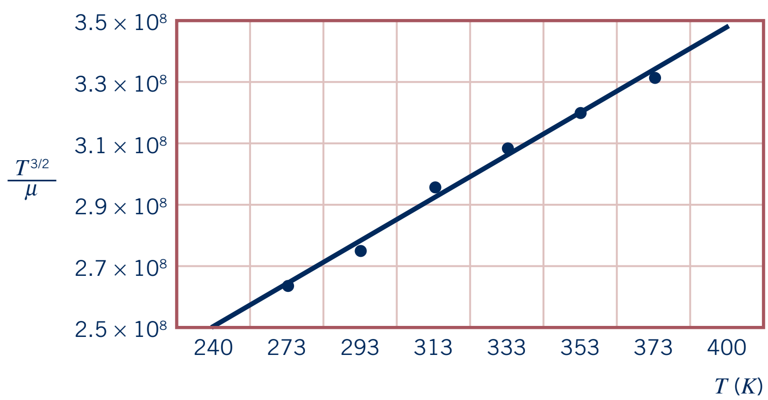
Solution 1.68

Equation $\mu = \dfrac{CT^{3/2}}{T+S}$ can be written in the form $\dfrac{T^{\frac{3}{2}}}{\mu} = \left(\dfrac{1}{C}\right)T + \dfrac{S}{C}$ (1)

Entering the specified temperatures and the corresponding viscosities from the table yields the 1st and 3rd columns in the following table. The 2nd column is a conversion of the temperatures to an absolute scale and the 4th column contains the values of the LHS of Eq. 1 for these temperatures and viscosities.

T (°C)	T (K)	$\mu\left(\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}\right)$	$\dfrac{T^{3/2}}{\mu}\left[\dfrac{K^{3/2}}{\left(\dfrac{\text{kg}}{\text{m}\cdot\text{s}}\right)}\right]$
0	273.15	1.71×10^{-5}	2.640×10^{8}
20	293.15	1.82×10^{-5}	2.758×10^{8}
40	313.15	1.87×10^{-5}	2.963×10^{8}
60	333.15	1.97×10^{-5}	3.087×10^{8}
80	353.15	2.07×10^{-5}	3.206×10^{8}
100	373.15	2.17×10^{-5}	3.322×10^{8}

Plotting $\dfrac{T^{3/2}}{\mu}$ versus. T yields:



A polynomial of order one, which is a straight line, would be a reasonable fit for the data. Using Excel to determine the constants for a fit of the data to a straight line given by

$$y = bx + a$$

where $x = T$, $y = \dfrac{T^{3/2}}{\mu}$, $b = \dfrac{1}{C}$, and $a = \dfrac{S}{C}$.

yields: $y = 6.969 \times 10^5 x + 7.441 \times 10^7$.

Therefore: $\dfrac{1}{C} = b = 6.969 \times 10^5$

$$\underline{\underline{C = 1.43 \times 10^{-6} \ \frac{\text{kg}}{\left(\text{m} \cdot \text{s} \cdot \text{K}^{3/2}\right)}}}$$

And $\dfrac{S}{C} = a = 7.441 \times 10^7$

$$\underline{\underline{S = 107\ \text{K}}}$$

These values for C and S are in good agreement with values given in Problem 1.67.

Problem 1.69

The viscosity of a fluid plays a very important role in determining how a fluid flows. The value of the viscosity depends not only on the specific fluid but also on the fluid temperature. Some experiments show that when a liquid, under the action of a constant driving pressure, is forced with a low velocity, V, through a small horizontal tube, the velocity is given by the equation $V = K/\mu$. In this equation K is a constant for a given tube and pressure, and μ is the dynamic viscosity. For a particular liquid of interest, the viscosity is given by Andrade's equation

$$\mu = De^{\frac{B}{T}}$$

with $D = 5\times10^{-7}\ \text{lb}\cdot\text{s/ft}^2$ and $B = 4000\ °\text{R}$. By what percentage will the velocity increase as the liquid temperature is increased from $40\ °\text{F}$ to $100\ °\text{F}$? Assume all other factors remain constant.

Solution 1.69

$$V_{40°} = \frac{K}{\mu_{40°}} \qquad (1)$$

$$V_{100°} = \frac{K}{\mu_{100°}} \qquad (2)$$

$$\%\text{ increase in } V = \left[\frac{V_{100°} - V_{40°}}{V_{40°}}\right]\times100 = \left[\frac{V_{100°}}{V_{40°}} - 1\right]\times100$$

and from Eq. (1) and (2)

$$\%\text{ increase in } V = \left[\frac{\frac{K}{\mu_{100°}}}{\frac{K}{\mu_{40°}}} - 1\right]\times100 = \left[\frac{\mu_{100°}}{\mu_{40°}} - 1\right]\times100 \qquad (3)$$

From Andrade's equation

$$\mu_{40°} = 5\times10^{-7}e^{\frac{4000}{(40\ °\text{F}+460)}}$$

and

$$\mu_{100°} = 5\times10^{-7}e^{\frac{4000}{(100\ °\text{F}+460)}}$$

Thus, from Eq. (3)

$$\%\text{ increase in } V = \left[\frac{5\times10^{-7}e^{\frac{4000}{500}}}{5\times10^{-7}e^{\frac{4000}{560}}} - 1\right]\times100 = \underline{\underline{136\ \%}}$$

Problem 1.70

Use the value of the viscosity of water given in Table B.2 Physical Properties of Water (SI Units) at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants D and B which appear in Andrade's equation $\mu = De^{B/T}$. Calculate the value of the viscosity at 50 °C and compare with the value given in the table above. (*Hint:* Rewrite the equation in the form $\ln \mu = (B)\frac{1}{T} + \ln D$ and plot $\ln \mu$ versus $1/T$. From the slope and intercept of this curve, B and D can be obtained. If a nonlinear curve-fitting program is available, the constants can be obtained directly from Andrade's equation $\mu = De^{B/T}$ without rewriting the equation.)

Solution 1.70

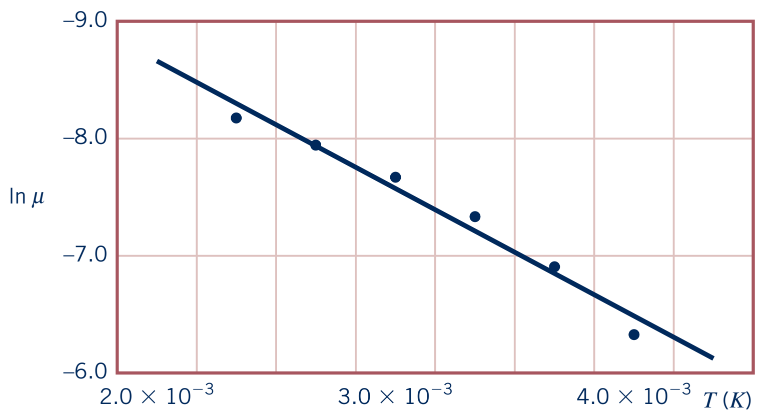
Equation $\mu = De^{B/T}$ can be written in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D \qquad (1)$$

and with data from the table in the problem:

T (°C)	T (K)	$\frac{1}{T\ (\mathrm{K})}$	$\mu\left(\frac{\mathrm{N\cdot s}}{\mathrm{m}^2}\right)$	$\ln \mu$
0	273.15	3.661×10^{-3}	1.787×10^{-3}	−6.327
20	293.15	3.411×10^{-3}	1.002×10^{-3}	−6.906
40	313.15	3.193×10^{-3}	6.529×10^{-4}	−7.334
60	333.15	3.002×10^{-3}	4.665×10^{-4}	−7.670
80	353.15	2.832×10^{-3}	3.547×10^{-4}	−7.944
100	373.15	2.680×10^{-3}	2.818×10^{-4}	−8.174

A plot of $\ln \mu$ versus $\frac{1}{T}$ is shown below:



Although there appears to be a slight curvature to the data in the semi-log plot, it also appears to be reasonably well approximated by a straight line as would be expected for data that follows a n exponential law. Using an exponential law ($y = ae^{bx}$) fit in Excel, (which is the same as fitting a straight-line on a semi-log plot), yields:

$$\underline{\underline{D = a = 1.767 \times 10^{-6}\ \frac{\mathrm{N \cdot s}}{\mathrm{m}^2}}}$$

and

$$\underline{\underline{B = b = 1.870 \times 10^3\ \mathrm{K}}}$$

so that

$$\mu = 1.767 \times 10^{-6} e^{\frac{1870}{T}}$$

At 50 °C (323.15 K), $\mu = 1.767 \times 10^{-6} e^{\frac{1870}{323.15}} = \underline{\underline{5.76 \times 10^{-4}\ \frac{\mathrm{N \cdot s}}{\mathrm{m}^2}}}$

From the table in the problem, $\mu = 5.468 \times 10^{-4}\ \frac{\mathrm{N \cdot s}}{\mathrm{m}^2}$

Problem 1.71

For a certain liquid $\mu = 7.1\times10^{-5}\ \text{lb}\cdot\text{s}/\text{ft}^2$ at $40\ °\text{F}$ and $\mu = 1.9\times10^{-5}\ \text{lb}\cdot\text{s}/\text{ft}^2$ at $150\ °\text{F}$. Make use of these data to determine the constants D and B, which appear in Andrade's equation $\mu = De^{B/T}$. What would be the viscosity at $80\ °\text{F}$?

Solution 1.71

$\mu = De^{B/T}$

At $T = (40\ °\text{F} + 459.67) = 499.67\ °\text{R},\ \mu = 7.1\times10^{-5}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}$, and

at $T = (150\ °\text{F} + 459.67) = 609.67\ °\text{R},\ \mu = 1.9\times10^{-5}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}$.

Take the logarithm of both sides of the equation $\mu = De^{D/T}$ to yield

$$\ln\mu = B\left(\frac{1}{T}\right) + \ln D \qquad (1)$$

Substitute above values of μ and T into Eq. (1) to give

$$\ln\left(7.1\times10^{-5}\right) = B\left(\frac{1}{499.67}\right) + \ln D \qquad (2)$$

$$\ln\left(1.9\times10^{-5}\right) = B\left(\frac{1}{609.67}\right) + \ln D \qquad (3)$$

and solve Eqs. (2) and (3) simultaneously for B and D.

Subtract Eq. (3) from Eq. (2) to give

$$\ln\left(\frac{7.1\times10^{-5}}{1.9\times10^{-5}}\right) = B\left(\frac{1}{499.67} - \frac{1}{609.67}\right) \rightarrow \underline{\underline{B = 3650\ \text{K}.}}$$

Substitute this value of B into Eq. (2) to yield

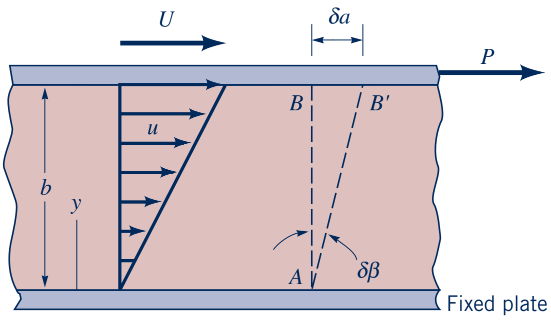
$$\ln\left(7.1\times10^{-5}\right) = 3650\left(\frac{1}{499.67}\right) + \ln D \rightarrow \underline{\underline{D = 4.77\times10^{-8}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}.}}$$

At $T = 80\ °\text{F} + 459.67 = 539.67\ °\text{R}$

$$\mu = 4.77\times10^{-8}\,e^{\frac{3650}{539.67}} = \underline{\underline{4.13\times10^{-5}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}}$$

Problem 1.72

For a parallel plate arrangement of the type shown in the figure below it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.



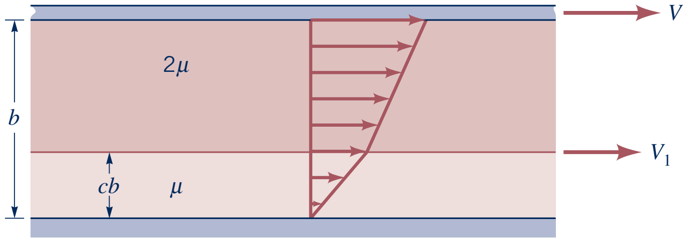
Solution 1.72

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b}$$

$$\mu = \frac{\tau}{\left(\frac{U}{b}\right)} = \frac{150\ \frac{\text{N}}{\text{m}^2}}{\left(\frac{1\ \frac{\text{m}}{\text{s}}}{0.002\ \text{m}}\right)} = \underline{\underline{0.300\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

Problem 1.73

Two flat plates are oriented parallel above a fixed lower plate as shown in the figure below. The top plate, located a distance b above the fixed plate, is pulled along with speed V. The other thin plate is located a distance cb, where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1, which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom. Plot the ratio V_1/V as a function of c for $0 < c < 1$.



Solution 1.73

For constant speed, V_1, of the middle plate, the net force on the plate is 0, Hence, $F_{top} = F_{bottom}$ where $F = \tau A$. Thus, the shear stress on the top and bottom of the plate must be equal.

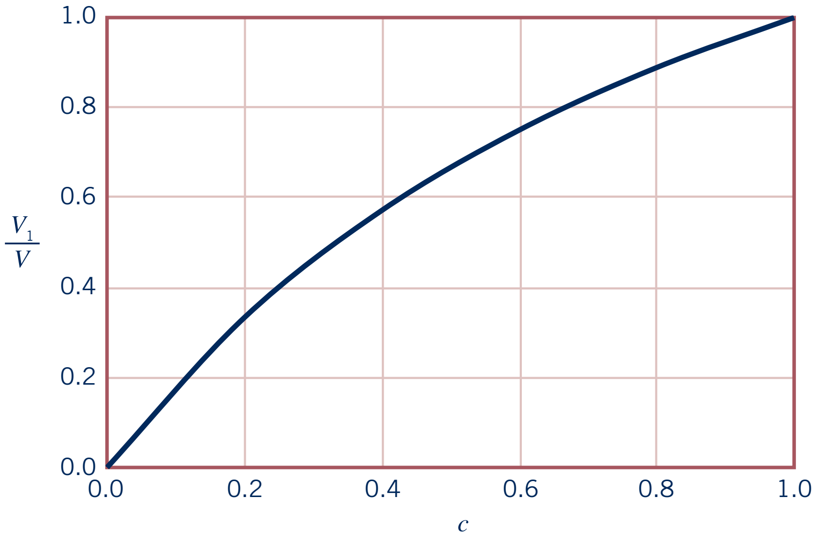
$$\tau_{top} = \tau_{bottom} \text{ where } \tau = \mu \frac{du}{dy} \qquad (1)$$

For the bottom fluid $\dfrac{du}{dy} = \dfrac{V_1}{cb}$, while for the top fluid $\dfrac{du}{dy} = \dfrac{(V - V_1)}{b - cb}$.

Hence, from Eqn. (1), $(2\mu)\dfrac{(V - V_1)}{b(1-c)} = (\mu)\dfrac{V_1}{cb}$

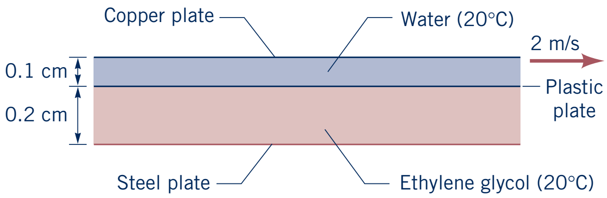
$$2cV - 2cV_1 = V_1 - cV_1 \rightarrow \frac{V_1}{V} = \underline{\underline{\frac{2c}{c+1}}}$$

Note: $c = 0 \rightarrow \dfrac{V_1}{V} = 0,\ c = \dfrac{1}{2} \rightarrow \dfrac{V_1}{V} = \dfrac{2}{3},\ c = 1 \rightarrow \dfrac{V_1}{V} = 1$



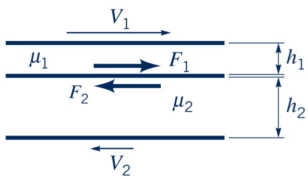
Problem 1.74

Three large plates are separated by thin layers of ethylene glycol and water, as shown in the figure below. The top plate moves to the right at 2 m/s. At what speed and in what direction must the bottom plate be moved to hold the center plate stationary?



Solution 1.74

The center plate is stationary if $F_1 = F_2$ (see image). Assuming Newtonian fluids and thin layers,



$$F = \mu\left(\frac{du}{dy}\right)_{\substack{\text{center}\\\text{plate}}} \cong \mu\frac{V}{h}$$

so

$$\mu_1\frac{V_1}{h_1} = \mu_2\frac{V_2}{h_2}$$

or

$$V_2 = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{h_2}{h_1}\right)V_1 = \left(\frac{\mu_w}{\mu_{eg}}\right)\left(\frac{h_{eg}}{h_w}\right)V_1.$$

From the liquid properties table: $\mu_{eg} = 1.99\times10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ and $\mu_w = 1.00\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}$.

$$V_2 = \left(\frac{1.00\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{1.99\times10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}\right)\left(\frac{0.2\,\text{cm}}{0.1\,\text{cm}}\right)\left(2\,\frac{\text{m}}{\text{s}}\right)$$

$$\boxed{V_2 = 0.201\,\frac{\text{m}}{\text{s}},\ \text{left.}}$$

Problem 1.75

There are many fluids that exhibit non-Newtonian behavior. For a given fluid, the distinction between Newtonian and non-Newtonian behavior is usually based on measurements of shear stress and rate of shearing strain. Assume that the viscosity of blood is to be determined by measurements of shear stress, τ, and rate of shearing strain, $\frac{du}{dy}$, obtained from a small blood sample tested in a suitable viscometer. Based on the data given below, determine if the blood is a Newtonian or non-Newtonian fluid. Explain how you arrived at your answer.

$\tau(\mathrm{N/m^2})$	0.04	0.06	0.12	0.18	0.30	0.52	1.12	2.10
$du/dy\ (\mathrm{s^{-1}})$	2.25	4.50	11.25	22.5	45.0	90.0	225	450

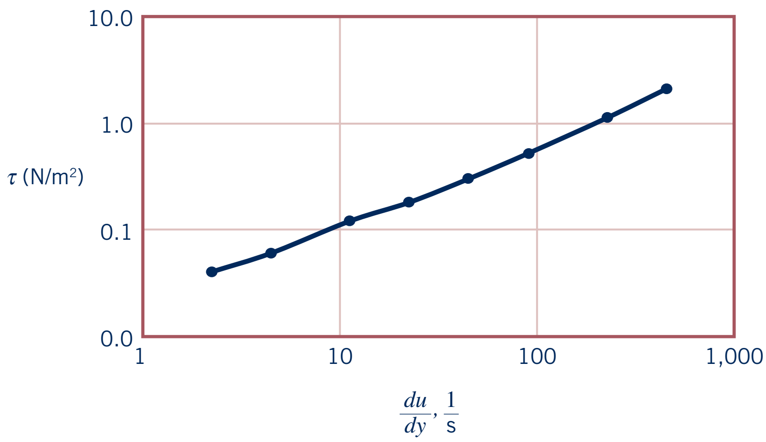
Solution 1.75

For a Newtonian fluid the ratio of τ to $\frac{du}{dy}$ is a constant. For the data given:

$\frac{\tau}{du/dy}\left(\mathrm{N\cdot s/m^2}\right)$	0.0178	0.0133	0.0107	0.0080	0.0067	0.0058	0.0050	0.0047

The ratio is not a constant but decreases as the rate of shearing strain increases. Thus, this fluid (blood) is a non-Newtonian fluid.

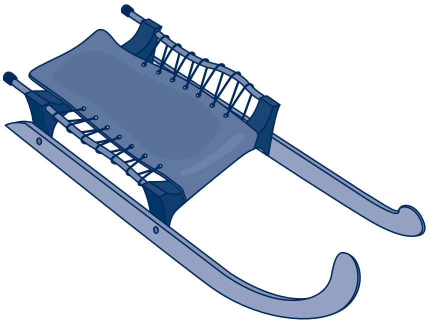
NOTE: The behavior of many non-Newtonian fluids can be well approximated by a power-law relationship. If that is true for this fluid, on a log-log plot the relationship between shear stress and strain rate should be a straight line.



It appears that for this sample, the blood indeed is well represented by a power-law.

Problem 1.76

The sled shown in the figure below slides along on a thin horizontal layer of water between the ice and the runners. The horizontal force that the water puts on the runners is equal to 1.2 lb when the sled's speed is 50 ft/s. The total area of both runners in contact with the water is $0.08\ \text{ft}^2$, and the viscosity of the water is $3.5\times10^{-5}\ \text{lb}\cdot\text{s}/\text{ft}^2$. Determine the thickness of the water layer under the runners. Assume a linear velocity distribution in the water layer.



Solution 1.76

F (force) $= \tau A$

$\tau = \mu \dfrac{dv}{dy} = \mu \dfrac{V}{d}$ where d = thickness of water layer.

Thus,

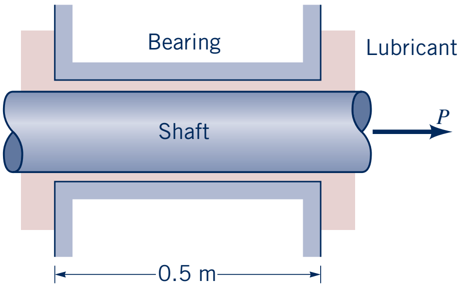
$$F = \mu \frac{V}{d} A$$

and

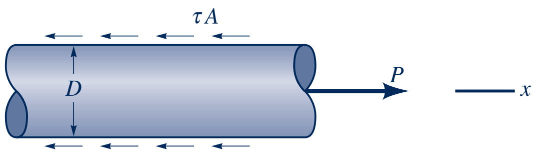
$$d = \frac{\mu V A}{F} = \frac{\left(3.5\times10^{-5}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(50\ \dfrac{\text{ft}}{\text{s}}\right)\left(0.08\ \text{ft}^2\right)}{1.2\ \text{lb}} = \underline{\underline{11.7\times10^{-4}\ \text{ft}}}$$

Problem 1.77

A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in the figure below. The lubricant that fills the 0.3-mm gap between the shaft and bearing is oil having a kinematic viscosity of $8.0 \times 10^{-4}\ \text{m}^2/\text{s}$ and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of $3\ \text{m/s}$. Assume the velocity distribution in the gap is linear.



Solution 1.77



$$\sum F_x = 0$$

Thus,

$$P = \tau A$$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and $\tau = \mu \dfrac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \dfrac{V}{b}$

so that

$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell) = \left(\nu \rho \frac{V}{b}\right)(\pi D \ell)$$

Since $\mu = \nu\rho = \nu(SG)\left(\rho_{H_2O\ @\ 4\ °\text{C}}\right)$,

$$P = \frac{\left(8.0 \times 10^{-4}\ \frac{\text{m}^2}{\text{s}}\right)\left(0.91 \times 10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(3\frac{\text{m}}{\text{s}}\right)(\pi)(0.025\ \text{m})(0.5\ \text{m})}{(0.0003\ \text{m})}$$

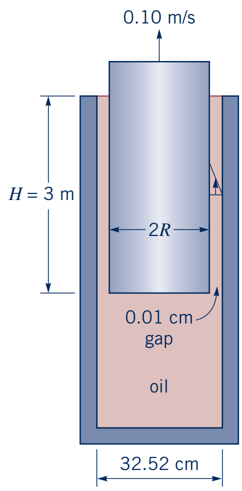
$$P = 286 \frac{\text{k}\cdot\text{m}}{\text{s}^2} \times \frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \underline{\underline{286\ \text{N}}}$$

Problem 1.78

A hydraulic lift in a service station has a 32.50-cm-diameter ram that slides in a 32.52-cm-diameter cylinder. The annular space is filled with SAE 10 oil at 20°C. The ram is traveling upward at the rate of 0.10 m/s. Find the frictional force when 3.0 m of the ram is engaged in the cylinder.

Solution 1.78

Modeling the oil as a Newtonian fluid:



$$\tau = \mu \frac{du}{dy}$$

linear velocity profile aross gap.

$$\frac{du}{dy} = \frac{\left(0.10\frac{\text{m}}{\text{s}}\right)}{(0.01\,\text{cm})\left(\dfrac{\text{m}}{100\,\text{cm}}\right)} = 1000\frac{1}{\text{s}}.$$

$$\mu = 0.123\,\text{Pa}\cdot\text{s at } 20\,^{\text{o}}\text{C}$$

$$\tau = \left(0.123\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(1000\frac{1}{\text{s}}\right) = 123\frac{\text{N}}{\text{m}^2}$$

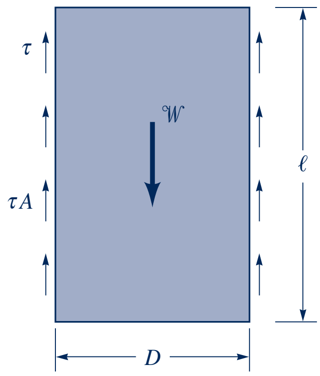
Therfore,
$$F = \tau A = \tau(2\pi RH) = \left(123\frac{\text{N}}{\text{m}^2}\right)2\pi(16.25\text{ cm})(3\text{ m})\left(\frac{1\text{ m}}{100\text{ cm}}\right)$$

$$\boxed{F = 377\text{ N}}$$

Problem 1.79

A piston having a diameter of 5.48 in. and a length of 9.50 in. slides downward with a velocity V through a vertical pipe. The downward motion is resisted by an oil film between the piston and the pipe wall. The film thickness is 0.002 in., and the cylinder weighs 0.5 lb. Estimate V if the oil viscosity is $0.016\ \text{lb}\cdot\text{s/ft}^2$. Assume the velocity distribution in the gap is linear.

Solution 1.79



Constant velocity $\rightarrow \sum F_{\text{vertical}} = 0$

$$W = \tau A = \tau \pi D \ell$$

Linear velocity profile $\rightarrow$ Newtonian fluid $\rightarrow \tau = \mu \dfrac{du}{dy} = \mu \dfrac{(\text{velocity})}{(\text{film thickness})} = \mu \dfrac{V}{\delta}$

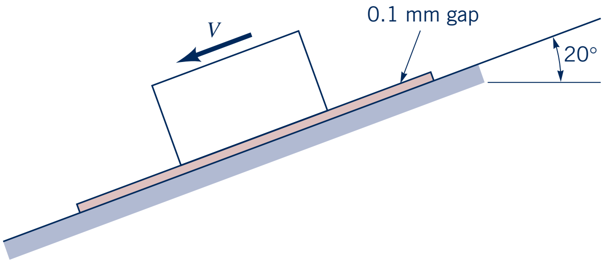
Substitution yields:

$$W = \left(\mu \frac{V}{\delta}\right)(\pi D \ell)$$

$$V = \frac{W\delta}{\pi D \ell \mu} = \frac{(0.5\ \text{lb})\left(\dfrac{0.002}{12}\ \text{ft}\right)}{\pi\left(\dfrac{5.48}{12}\ \text{ft}\right)\left(\dfrac{9.50}{12}\ \text{ft}\right)\left(0.016\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)} = \underline{\underline{0.00459\ \frac{\text{ft}}{\text{s}}}}$$

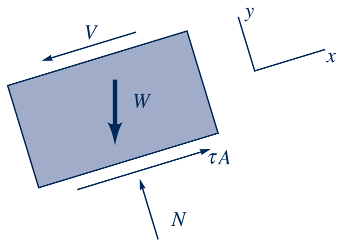
Problem 1.80

A 10-kg block slides down a smooth inclined surface as shown in the figure below. Determine the terminal velocity of the block if the 0.1-mm gap between the block and the surface contains SAE 30 oil at 60 °F. Assume the velocity distribution in the gap is linear, and the area of the block in contact with the oil is $0.1\,\text{m}^2$.



Solution 1.80

Draw free body diagram to help resolve forces:



Constant velocity $\rightarrow \sum F_x = 0$

$$W \sin\theta = \tau A$$

Linear velocity profile $\rightarrow$ Newtonian fluid $\rightarrow \tau = \mu \dfrac{du}{dy} = \mu \dfrac{(\text{velocity})}{(\text{film thickness})} = \mu \dfrac{V}{b}$

where b is film thickness. Substitution yields:

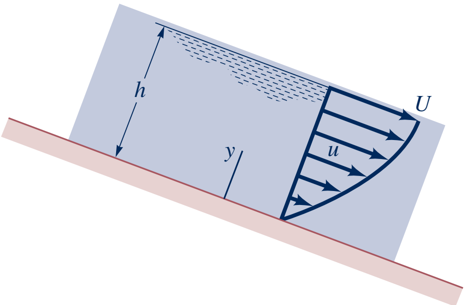
$$W \sin\theta = mg\sin\theta = \mu \frac{V}{b} A$$

$$V = \frac{bmg\sin\theta}{\mu A} = \frac{(0.0001\text{ m})(10\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\sin 20°)}{\left(0.38\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(0.1\text{ m}^2\right)} \times \frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}}$$

$$\underline{\underline{V = 0.0883\,\frac{\text{m}}{\text{s}}}}$$

Problem 1.81

A layer of water flows down an inclined fixed surface with the velocity profile shown in the figure below. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for $U = 2\ \text{m/s}$ and $h = 0.1\ \text{m}$.



$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$$

Solution 1.81

Enforcing the no-slip boundary condition at the solid surface:

$$\tau = \mu \frac{du}{dy} = \mu \frac{d}{dy}\left[U\left(2\frac{y}{h} - \frac{y^2}{h^2}\right)\right] = \mu U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$$

Thus, at the fixed surface $(y = 0)$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2D}{h}$$

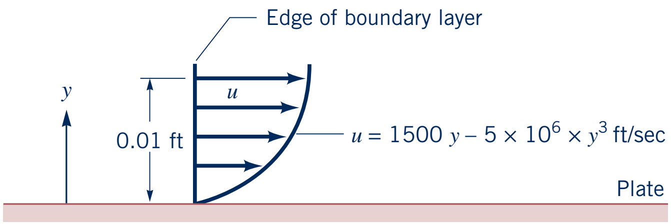
Thus

$$\tau_{y=0} = \mu U \frac{2}{h} = \left(1.12 \times 10^{-3}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(2\ \frac{\text{m}}{\text{s}}\right)\frac{2}{0.1\ \text{m}}$$

$$= 4.48 \times 10^{-2}\ \frac{\text{N}}{\text{m}^2} \text{ acting in direction of flow}$$

Problem 1.82

Oil (absolute viscosity = $0.0003\,\text{lb}\cdot\text{s/ft}^2$, density = $50\,\text{lbm/ft}^3$) flows in the boundary layer, as shown in the figure below. The plate is 1 ft wide perpendicular to the paper. Calculate the shear stress at the plate surface.



Solution 1.82

Assuming a newtonian fluid, the shear stress on the plate by the oil is

$$\tau = \mu\left(\frac{du}{dy}\right)_{y=0}$$

For the given velocity profile:

$$u = \frac{1500}{\text{sec}}y - \frac{5\times10^6}{\text{sec}\cdot\text{ft}^2}y^3 \;\rightarrow\; \frac{du}{dy} = \frac{1500}{\text{sec}} - \frac{15\times10^6\,y^2}{\text{sec}\cdot\text{ft}^2} \;\rightarrow\; \left(\frac{du}{dy}\right)_{y=0} = 1500\frac{1}{\text{sec}}$$

Substitution yields:

$$\tau = \left(\frac{0.0003\,\text{lb}\cdot\text{sec}}{\text{ft}^2}\right)\left(1500\frac{1}{\text{sec}}\right) = \underline{\underline{0.45\frac{\text{lb}}{\text{s}}}}$$

Problem 1.83

Standard air flows past a flat surface, and velocity measurements near the surface indicate the following distribution:

$y\,(\text{ft})$	0.005	0.01	0.02	0.04	0.06	0.08
$u\,(\text{ft/s})$	0.74	1.51	3.03	6.37	10.21	14.43

The coordinate y is measured normal to the surface and u is the velocity parallel to the surface. **(a)** Assume the velocity distribution is of the form $u = C_1 y + C_2 y^3$ and use a standard curve-fitting technique to determine the constants C_1 and C_2.
(b) Make use of the results of part (a) to determine the magnitude of the shearing stress at the wall ($y = 0$) and at $y = 0.05$ ft.

Solution 1.83

(a) Use nonlinear regression program to obtain coefficients C_1 and C_2. The program produces least squares estimates of the parameters of a nonlinear model. For the data given,

$$\underline{\underline{C_1 = 153\ \text{s}^{-1}}} \text{ and } \underline{\underline{C_2 = 4350\ \text{ft}^{-2}\text{s}^{-1}}}$$

(b) For a Newtonian fluid:

$$\tau = \mu \frac{du}{dy} = \mu \frac{d}{dy}\left(C_1 y + 3C_2 y^2\right) = \mu C_1 + 6\mu C_2 y$$

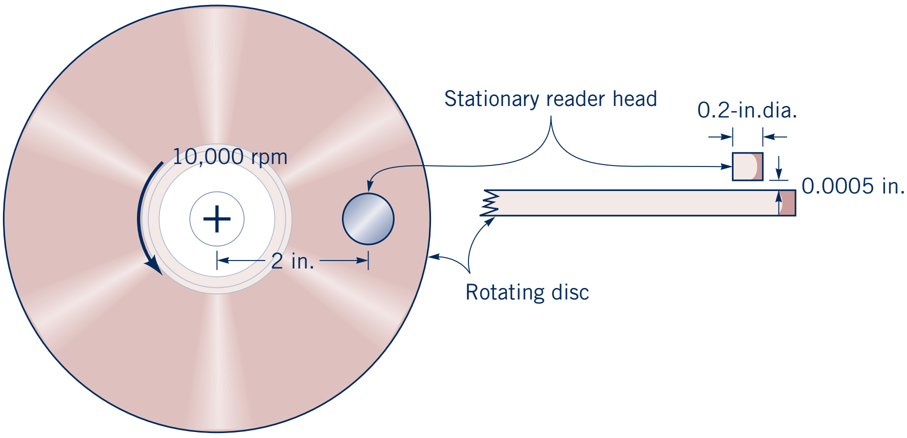
At the locations specified:

$$y = 0 \rightarrow \tau = \mu C_1 = \left(3.74\times10^{-7}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(153\ \frac{1}{\text{s}}\right) = \underline{\underline{5.72\times10^{-5}\ \frac{\text{lb}}{\text{ft}^2}}}$$

$$y = 0.05\text{ ft} \rightarrow \tau = \left(3.74\times10^{-7}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left[153\ \frac{1}{\text{s}} + 3\left(4350\ \frac{1}{\text{ft}^2\cdot\text{s}}\right)(0.05\text{ ft})^2\right] = \underline{\underline{6.94\times10^{-5}\ \frac{\text{lb}}{\text{ft}^2}}}$$

Problem 1.84

A new computer drive is proposed to have a disc, as shown in the figure below. The disc is to rotate at 10,000 rpm, and the reader head is to be positioned 0.0005 in. above the surface of the disc. Estimate the shearing force on the reader head as a result of the air between the disc and the head.



Solution 1.84

F = shear force on head = τA

Assuming a uniform and linear velocity profile:

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b} = \frac{\mu}{b}\omega R$$

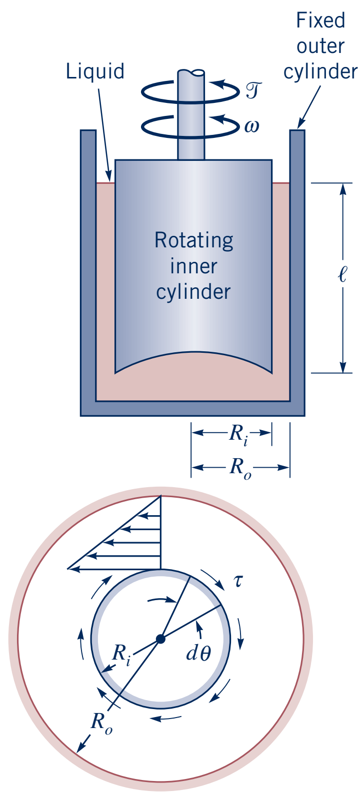
$$\tau = \left(\frac{3.74\times10^{-7}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{\dfrac{0.0005}{12}\ \text{ft}} \right) \left(10000\ \frac{\text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \right) \left(\frac{2}{12}\ \text{ft} \right) = 1.57\ \frac{\text{lb}}{\text{ft}^2}$$

$$F = \tau A = \left(1.57\ \frac{\text{lb}}{\text{ft}^2} \right) \frac{\pi}{4} \left(\frac{0.2}{12}\ \text{ft} \right)^2 = \underline{\underline{3.43\times10^{-4}\ \text{lb}}}$$

Problem 1.85

The space between two 6-in.-long concentric cylinders is filled with glycerin (viscosity $= 8.5 \times 10^{-3}\ \text{lb} \cdot \text{s/ft}^2$). The inner cylinder has a radius of 3 in. and the gap width between cylinders is 0.1 in. Determine the torque and the power required to rotate the inner cylinder at 180 rev/min. The outer cylinder is fixed. Assume the velocity distribution in the gap to be linear.

Solution 1.85



top view

(ℓ ~ cylinder length)

The torque on a sector of the cylinder surface corresponding to an included angle of dθ is:

$$dT = dFR_i = \tau dAR_i = \tau\left[\left(R_i d\theta\right)\ell\right]R_i = \tau R_i^{\,2}\ell d\theta$$

Integrating around the cylinder:

$$T = \tau R_i^2 \ell \int_0^{2\pi} d\theta = 2\pi\tau R_i^2 \ell$$

For a linear velocity distribution in the gap

$$\tau = \mu \frac{R_i \omega}{R_0 - R_i}$$

$$T = 2\pi\left(\mu \frac{R_i\omega}{R_0 - R_i}\right)R_i^2 \ell = 2\pi \frac{R_i^3 \ell \mu \omega}{R_0 - R_i}$$

$$\omega = \left(180\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\,\text{min}}{60\ \text{s}}\right) = 6\pi\ \frac{\text{rad}}{\text{s}}$$

$$T = \frac{2\pi\left(\frac{3}{12}\ \text{ft}\right)^3\left(\frac{6}{12}\ \text{ft}\right)\left(8.5\times10^{-3}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(6\pi\ \frac{\text{rad}}{\text{s}}\right)}{\left(\frac{0.1}{12}\ \text{ft}\right)}$$

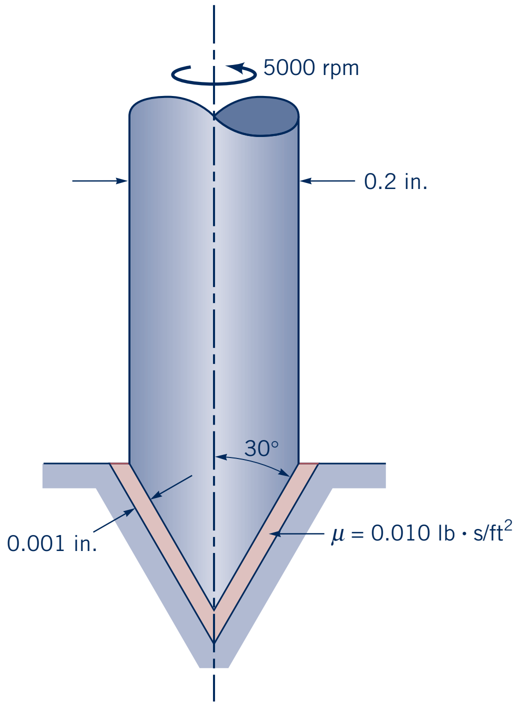
$$T = \underline{\underline{0.944\ \text{ft}\cdot\text{lb}}}$$

To compute power requied:

$$\text{power} = T \times \omega = \left(0.944\ \text{ft}\cdot\text{lb}\right)\left(6\pi\ \frac{\text{rad}}{\text{s}}\right) = \underline{\underline{17.8\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}}$$

Problem 1.86

A pivot bearing used on the shaft of an electrical instrument is shown in the figure below. An oil with a viscosity of $\mu = 0.010\ \text{lb}\cdot\text{s}/\text{ft}^2$ fills the 0.001-in. gap between the rotating shaft and the stationary base. Determine the frictional torque on the shaft when it rotates at 5000 rpm.

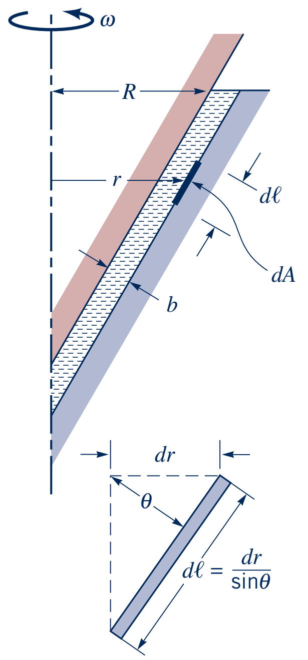


Solution 1.86

Let dT = torque on area element dA, where $dA = 2\pi r d\ell = \dfrac{2\pi r dr}{\sin\theta}$

Thus,

$$dT = rdF = r\tau dA \text{ where } \tau = \mu\frac{du}{dy} = \mu\frac{\omega r}{b}$$



so that,

$$dT = r\left(\mu\frac{\omega r}{b}\right)\left(\frac{2\pi r dr}{\sin\theta}\right) = \frac{2\pi\mu\omega}{b\sin\theta}r^3 dr$$

Hence,

$$T = \int dT = \frac{2\pi\mu\omega}{b\sin\theta}\int_{r=0}^{r=R} r^3 dr = \frac{\pi\mu\omega}{2b\sin\theta}R^4 \qquad (1)$$

Now,

$$R = 0.1\text{ in.}, b = 0.001\text{ in.}, \mu = 0.010\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}, \theta = 30\text{ deg, and}$$

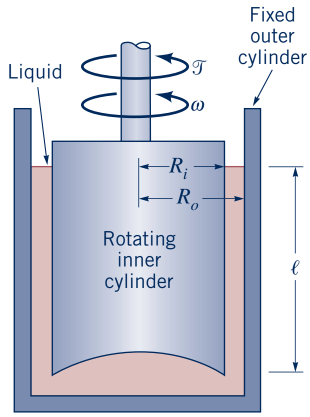
$$\omega = 5000\frac{\text{rev}}{\text{min}}\left(\frac{\text{min}}{60\text{ s}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right) = 524\frac{\text{rad}}{\text{s}}$$

Thus, from Eq. (1),

$$T = \frac{\pi\left(0.010\dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(524\dfrac{\text{rad}}{\text{s}}\right)}{2\left(\dfrac{0.001}{12}\text{ ft}\right)\sin 30^\circ}\left(\frac{0.1}{12}\text{ ft}\right)^4 = \underline{\underline{9.53\times10^{-4}\text{ ft}\cdot\text{lb}}}$$

Problem 1.87

The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in the figure below. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque T required to develop ω is measured and the viscosity is calculated from these two measurements. **(a)** Develop an equation relating μ, ω, T, ℓ, R_0, and R_i. Neglect end effects and assume the velocity distribution in the gap is linear. **(b)** The following torque-angular velocity data were obtained with a rotating cylinder viscometer of the type discussed in part (a).



Torque (ft · lb)	13.1	26.0	39.5	52.7	64.9	78.6
Angular velocity (rad/s)	1.0	2.0	3.0	4.0	5.0	6.0

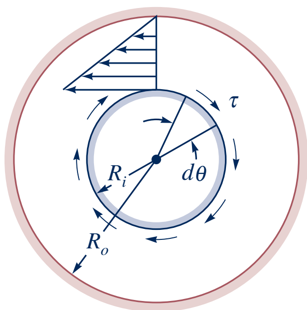
For this viscometer $R_0 = 2.50$ in. , $R_i = 2.45$ in. , and $\ell = 5.00$ in. Make use of these data and a standard curve-fitting program to determine the viscosity of the liquid contained in the viscometer.

Solution 1.87

(a) Torque, dT, on infinitesimal-size axial strip on the inner cylinder surface due to shearing stress:

$$dT = R_i \tau dA = R_i \tau \left(R_i d\theta\right) \ell = R_i^2 \ell \tau d\theta$$

Integration yields: $T = R_i^2 \ell \tau \int_0^{2\pi} d\theta = 2\pi R_i^2 \ell \tau$



top view

(ℓ ~ cylinder length)

For a linear velocity distribution in the gap:

$$\tau = \mu \frac{R_i \omega}{R_0 - R_i}$$

$$\underline{\underline{T = \frac{2\pi R_i^3 \ell \mu \omega}{R_0 - R_i}}} \qquad (1)$$

(b) For a fixed geometry and a given viscosity, Eq. (1) is of the form

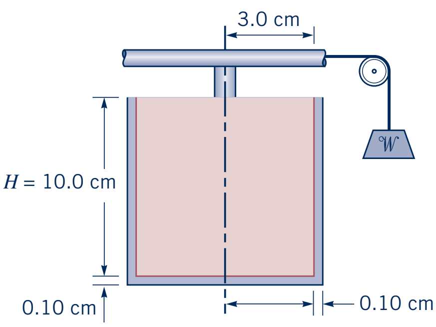
$$y = bx \quad \text{where: } y = \mathrm{T}, \quad x = \omega, \quad b = \frac{2\pi R_i^3 \ell \mu}{R_0 - R_i}$$

Entering the data provided into Excel: $b = 13.08\ \mathrm{ft \cdot lb \cdot s}$

Solving for viscosity: $\mu = \dfrac{(b)(R_0 - R_i)}{2\pi R_i^3 \ell} = \dfrac{(13.08\ \mathrm{ft \cdot lb \cdot s})\left(\dfrac{2.50 - 2.45}{12}\ \mathrm{ft}\right)}{2\pi\left(\dfrac{2.45}{12}\ \mathrm{ft}\right)^3\left(\dfrac{5.00}{12}\ \mathrm{ft}\right)} = \underline{\underline{2.45\ \dfrac{\mathrm{lb \cdot s}}{\mathrm{ft}^2}}}$

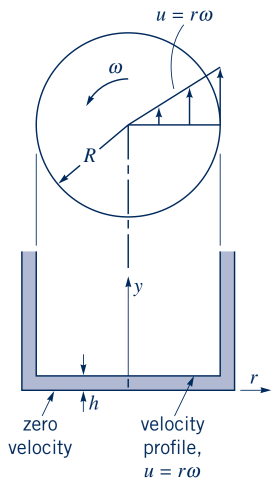
Problem 1.88

The concentric cylinder viscometer shown in the figure below has a cylinder height of $10.0\ \text{cm}$, a cylinder radius of $3.0\ \text{cm}$, and a uniform gap between the cylinder and the container (bottom and sides) of $0.10\ \text{cm}$. The pulley has a radius of $3.0\ \text{cm}$. Determine the weight required to produce a constant rotational speed of 30 rpm if the gap is filled with: (a) water, (b) gasoline, (c) glycerin.



Solution 1.88

Resisting torque is due to shear stress acting on cylinder surfaces. Assuming a linear velocity profile across the narrow gaps, the torque on the cylinder wall is:



$$T_1 = (\tau A)R = \left(\mu \left.\frac{du}{dy}\right|_w\right)(\pi DH)R = \mu\left(\frac{\omega R}{h}\right)(2\pi RH)R = \frac{2\pi\mu\omega HR^3}{h}$$

The velocity at the cylinder bottom is a function of radial position. The infinitesimal torque acting on an annular ring of differential width is:

$$dT_2 = (\tau dA)r = \left(\mu\frac{\omega r}{h}\right)(2\pi r dr)r = \frac{2\pi\mu\omega}{h}r^3 dr$$

$$T_2 = \frac{2\pi\mu\omega}{h}\int_0^R r^3 dr = \frac{\pi\mu\omega}{2h}R^4$$

Total torque: $T = T_1 + T_2 = 2\pi\mu\omega R^3\left(\frac{H}{h}\right) + \frac{\pi\mu\omega}{2h}R^4 = \frac{\pi\mu\omega R^4}{h}\left(\frac{2H}{R}+\frac{1}{2}\right)$

Neglecting friction: $WR_p = T \rightarrow W = \frac{\pi\mu\omega R^4}{hR_p}\left(\frac{2H}{R}+\frac{1}{2}\right)$

$$W = \frac{\pi\mu\left(30\frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\times\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\times\frac{1\ \cancel{\text{min}}}{60\ \text{sec}}\right)(3\ \text{cm})^{\cancel{4}\,2}}{(0.1\ \cancel{\text{cm}})(3\ \cancel{\text{cm}})}\left(\frac{2(10\ \cancel{\text{cm}})}{3\ \cancel{\text{cm}}}+\frac{1}{2}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = \left(1.91\frac{\text{m}^2}{\text{s}}\right)\mu$$

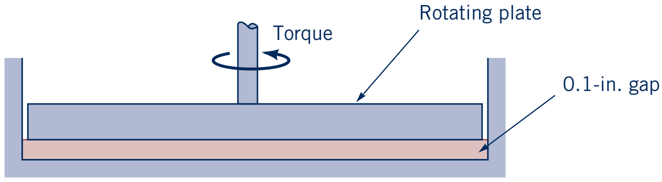
$$(a)\quad \text{water} \rightarrow W = \left(1.910\frac{\text{m}^2}{\text{s}}\right)\left(1.12\times10^{-3}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right) = \boxed{2.14\times10^{-3}\ \text{N}}$$

$$(b)\quad \text{gasoline} \rightarrow W = \left(1.910\frac{\text{m}^2}{\text{s}}\right)\left(3.10\times10^{-4}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right) = \boxed{5.92\times10^{-4}\ \text{N}}$$

$$(c)\quad \text{glycerine} \rightarrow W = \left(1.910\frac{\text{m}^2}{\text{s}}\right)\left(1.50\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right) = \boxed{2.87\ \text{N}}$$

Problem 1.89

A 12-in.-diameter circular plate is placed over a fixed bottom plate with a 0.1-in. gap between the two plates filled with glycerin as shown in the figure below. Determine the torque required to rotate the circular plate slowly at 2 rpm. Assume that the velocity distribution in the gap is linear and that the shear stress on the edge of the rotating plate is negligible.



Solution 1.89

As shown, considering an annular ring of differential width

$$dT = r\tau dA = r\tau 2\pi r dr$$

Integration yields: $T = 2\pi \int_0^R r^2 \tau dr$

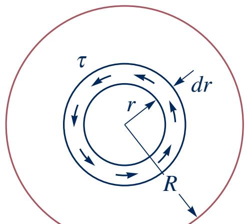
For the annular strip: $\tau = \mu \dfrac{r\omega}{\delta}$

Thus, $T = \dfrac{2\pi\mu\omega}{\delta} \int_0^R r^3 dr = \dfrac{2\pi\mu\omega}{\delta}\left(\dfrac{R^4}{4}\right)$

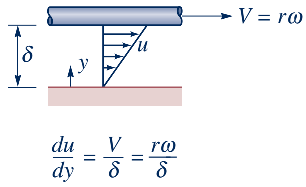
Using the data specified:

$$T = \frac{2\pi\left(0.0313\,\dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(2\,\dfrac{\text{rev}}{\text{min}}\right)\left(2\pi\,\dfrac{\text{rad}}{\text{rev}}\right)\left(\dfrac{1\,\text{min}}{60\,\text{s}}\right)\left(\dfrac{6}{12}\,\text{ft}\right)^4}{\left(\dfrac{0.1}{12}\,\text{ft}\right)(4)}$$

$$\boxed{T = 0.0772\ \text{ft}\cdot\text{lb}}$$



Stresses acting on bottom of plate



Velocity distribution

Problem 1.91

Some measurements on a blood sample at 37 °C (98.6 °F) indicate a shearing stress of $0.52\ \text{N}/\text{m}^2$ for a corresponding rate of shearing strain of $200\ \text{s}^{-1}$. Determine the apparent viscosity of the blood and compare it with the viscosity of water at the same temperature.

Solution 1.91

$$\tau = \mu \frac{du}{dy} = \mu \dot{\gamma}$$

$$\mu_{blood} = \frac{\tau}{\dot{\gamma}} = \frac{0.52 \dfrac{\text{N}}{\text{m}^2}}{200 \dfrac{1}{\text{s}}} = \underline{\underline{26.0 \times 10^{-4} \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

From Table B.2 Physical Properties of Water (SI Units)

$$@30\,°\text{C}\ \ \mu_{H_2O} = 7.975 \times 10^{-4} \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$@40\,°\text{C}\ \ \mu_{H_2O} = 6.529 \times 10^{-4} \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

Thus, with linear interpolation, $\mu_{H_2O}(37\,°\text{C}) = 6.96 \times 10^{-4} \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}$

and

$$\frac{\mu_{blood}}{\mu_{H_2O}} = \frac{26.0 \times 10^{-4} \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}{6.96 \times 10^{-4} \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} = \underline{\underline{3.74}}$$

Problem 1.93

A sound wave is observed to travel through a liquid with a speed of 1500 m/s. The specific gravity of the liquid is 1.5. Determine the bulk modulus for this fluid.

Solution 1.93

$c = \sqrt{\dfrac{E_N}{\rho}}$, where $\rho = SG\rho_{H_2O}$ and $SG = 1.5$

Thus,

$$E_N = c^2\rho = c^2 SG\rho_{H_2O}$$

$$= \left(1500\,\frac{\text{m}}{\text{s}}\right)^2 (1.5)\left(999\,\frac{\text{kg}}{\text{m}^3}\right)$$

$$= 3.37\times10^9\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2\text{m}^2}$$

or

$$E_N = \underline{\underline{3.37\times10^9\,\frac{\text{N}}{\text{m}^2}}}$$

Problem 1.94

A rigid-walled cubical container is completely filled with water at 40 °F and sealed. The water is then heated to 100 °F. Determine the pressure that develops in the container when the water reaches this higher temperature. Assume that the volume of the container remains constant and the value of the bulk modulus of the water remains constant and equal to 300,000 psi.

Solution 1.94

Since the water mass remains constant,

$$\rho_{40^\circ}\forall = \rho_{100^\circ}\left(\forall + \Delta\forall\right)$$

where $\forall$ is volume and $\Delta\forall$ is change in volume if water were unconstrained during heating.

Thus, $\dfrac{\Delta\forall}{\forall} = \dfrac{\rho_{40^\circ}}{\rho_{100^\circ}} - 1$

From the table Physical Properties of Water (BG/EE Units)

$\rho_{40^\circ} = 1.940\,\dfrac{\text{slugs}}{\text{ft}^3}$ and $\rho_{100^\circ} = 1.927\,\dfrac{\text{slugs}}{\text{ft}^3}$ so that

$$\frac{\Delta\forall}{\forall} = \frac{1.940\,\dfrac{\text{slugs}}{\text{ft}^3}}{1.927\,\dfrac{\text{slugs}}{\text{ft}^3}} - 1 = 0.00675$$

From the equation $E_\vee = -\dfrac{dp}{\dfrac{\Delta\forall}{\forall}}$ it follows with $d\forall \approx \Delta\forall$ and $dp \approx \Delta p$ that the change in pressure required to compress the water back to its original volume is

$$\Delta p = -\left(300000\ \text{psi}\right)\left(-0.00675\right)$$

$$= \underline{\underline{2.03 \times 10^3\ \text{psi}}}$$

Problem 1.95

Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1 %.

Solution 1.95

$$E_v = -\forall \frac{dp}{d\forall} \approx -\forall \frac{\Delta p}{\Delta \forall}$$

Thus,

$$\Delta p \approx -\frac{E_v \Delta \forall}{\forall} = -\left(4.14 \times 10^6 \frac{\text{lb}}{\text{in.}^2}\right)(-0.001)$$

$$\Delta p \approx \underline{\underline{4.14 \times 10^3 \text{ psi}}}$$

Problem 1.96

A 1m^3 volume of water is contained in a rigid container. Estimate the change in the volume of the water when a piston applies a pressure of 35 MPa.

Solution 1.96

$$E_v = -\forall \frac{dp}{d\forall} \approx -\forall \frac{\Delta p}{\Delta \forall}$$

Thus,

$$\Delta\forall \approx -\frac{\forall \Delta p}{E_v} = -\frac{\left(1\ \text{m}^3\right)\left(35\times10^6\ \dfrac{\text{N}}{\text{m}^2}\right)}{2.15\times10^9\ \dfrac{\text{N}}{\text{m}^2}} = -0.0163\ \text{m}^3$$

or <u>decrease</u> in volume $\approx \underline{\underline{0.0163\ \text{m}^3}}$

Problem 1.97

Determine the speed of sound at 20 °C in **(a)** air, **(b)** helium, and **(c)** natural gas (methane). Express your answer in m/s.

Solution 1.97

$c = \sqrt{kRT}$

With $T = 20\,°\mathrm{C} + 273 = 293\,\mathrm{K}$:

$$\text{(a) For air, } c = \sqrt{(1.40)\left(286.9\frac{\mathrm{J}}{\mathrm{kg\cdot K}}\right)(293\,\mathrm{K})\left(\frac{1\,\mathrm{N\cdot m}}{1\,\mathrm{J}}\right)\left(\frac{1\,\mathrm{kg\cdot m}}{1\,\mathrm{N\cdot s^2}}\right)} = \underline{\underline{343\frac{\mathrm{m}}{\mathrm{s}}}}$$

$$\text{(b) For helium, } c = \sqrt{(1.66)\left(2077\frac{\mathrm{J}}{\mathrm{kg\cdot K}}\right)(293\,\mathrm{K})\left(\frac{1\,\mathrm{N\cdot m}}{1\,\mathrm{J}}\right)\left(\frac{1\,\mathrm{kg\cdot m}}{1\,\mathrm{N\cdot s^2}}\right)} = \underline{\underline{1010\frac{\mathrm{m}}{\mathrm{s}}}}$$

$$\text{(c) For natural gas, } c = \sqrt{(1.31)\left(518.3\frac{\mathrm{J}}{\mathrm{kg\cdot K}}\right)(293\,\mathrm{K})\left(\frac{1\,\mathrm{N\cdot m}}{1\,\mathrm{J}}\right)\left(\frac{1\,\mathrm{kg\cdot m}}{1\,\mathrm{N\cdot s^2}}\right)} = \underline{\underline{446\frac{\mathrm{m}}{\mathrm{s}}}}$$

Problem 1.98

Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 25 psi. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process to be isothermal and the local atmospheric pressure to be 14.7 psi.

Solution 1.98

For isothermal compression, $\frac{p}{\rho} = \text{constant} = \frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$ where $i \sim$ initial state and $f \sim$ final state.

Thus, $p_f = \frac{\rho_f}{\rho_i} p_i$

Because the mass of air is constant: $\rho = \frac{\text{mass}}{\text{volume}} \rightarrow \frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3.$

Therefore,

$$p_f = (3)\left[(25 + 14.7)\,\text{psi}\,(\text{abs})\right] = 119\,\text{psi}\,(\text{abs})$$

or

$$p_f\,(\text{gage}) = (119 - 14.7)\,\text{psi} = \underline{\underline{104\,\text{psi}\,(\text{gage})}}$$

Problem 1.99

Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 25 psi. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process takes place without friction and without heat transfer (isentropic process) and the local atmospheric pressure to be 14.7 psi.

Solution 1.99

For isentropic compression, $\frac{p}{\rho^k} = \text{constant} = \frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$ where $i \sim$ initial state and $f \sim$ final state.

Thus $p_f = \left(\frac{\rho_f}{\rho_i}\right) p_i$

Because the amount of mass is constant: $\rho = \frac{\text{mass}}{\text{volume}} \rightarrow \frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3$

Therefore,

$$p_f = (3)^{1.40}\left[(25+14.7)\,\text{psi}\,(\text{abs})\right] = 184.8\,\text{psi}\,(\text{abs})$$

or

$$p_f\,(\text{gage}) = 184.8 - 14.7 = \underline{\underline{170\,\text{psi}\,(\text{gage})}}$$

Problem 1.100

Carbon dioxide at 30 °C and 300 kPa absolute pressure expands isothermally to an absolute pressure of 165 kPa. Determine the final density of the gas.

Solution 1.100

For isothermal compression, $\frac{p}{\rho} = \text{constant} = \frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$ where $i \sim$ initial state and $f \sim$ final state.

Thus, $\rho_f = \frac{p_f}{p_i}\rho_i$

Also, $\rho_i = \frac{p_i}{RT_i} = \frac{300\times 10^3 \frac{\text{N}}{\text{m}^2}}{\left(188.9\frac{\text{J}}{\text{kg}\cdot\text{K}}\times\frac{1\,\text{N}\cdot\text{m}}{1\,\text{J}}\right)\left[(30\,°\text{C}+273)\,\text{K}\right]} = 5.24\,\frac{\text{kg}}{\text{m}^3}$

Therefore,

$$\rho_f = \left(\frac{165\,\text{kPa}}{300\,\text{kPa}}\right)\left(5.24\frac{\text{kg}}{\text{m}^3}\right) = \underline{\underline{2.88\,\frac{\text{kg}}{\text{m}^3}}}$$

Problem 1.101

Natural gas at 70 °F and standard atmospheric pressure of 14.7 psi (abs) is compressed isentropically to a new absolute pressure of 70 psi. Determine the final density and temperature of the gas.

Solution 1.101

For isentropic compression, $\frac{p}{\rho^k} = \text{constant} = \frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$ where $i \sim$ initial state and $f \sim$ final state.

Therefore $\rho_f = \left(\frac{p_f}{p_i}\right)^{\frac{1}{k}} \rho_i$

$$\text{Also, } \rho_i = \frac{p_i}{RT_i} = \frac{\left(14.7\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(3.099\times10^3\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)\left[(70\,{}^\circ\text{F}+460)\,{}^\circ\text{R}\right]} = 1.29\times10^{-3}\,\frac{\text{slugs}}{\text{ft}^3}$$

$$\text{Therefore, } \rho_f = \left[\frac{70\text{ psia}}{14.7\text{ psia}}\right]^{\frac{1}{1.31}}\left(1.29\times10^{3}\,\frac{\text{slugs}}{\text{ft}^3}\right) = \underline{\underline{4.25\times10^{-3}\,\frac{\text{slugs}}{\text{ft}^3}}}$$

Using the ideal gas model:

$$T_f = \frac{p_f}{\rho_f R} = \frac{\left(70\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^3}\right)}{\left(4.25\times10^{-3}\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(3.099\times10^3\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)} = 765\,{}^\circ\text{R} = (765-460)\,{}^\circ\text{F} = \underline{\underline{305\,{}^\circ\text{F}}}$$

Problem 1.102

A compressed air tank in a service station has a volume of 10 ft^3. It contains air at 70°F and 150 psia. How many tubeless tires can it fill to 44.7 psia at 70 °F if each tire has a volume of 1.5 ft^3 and the compressed air tank is not refilled? The tank air temperature remains constant at 70 °F because of heat transfer through the tank's large surface area.

Solution 1.102

Modelling the air as an ideal gas, the mass of air m_t that can be put into each tire is found from:

$$m_{\text{tire}} = (p\forall)_f - (p\forall)_i = \left(\frac{p\forall}{RT}\right)_f - \left(\frac{p\forall}{RT}\right)_i = \frac{\forall}{RT}\left(p_f - p_i\right)$$

$$m_{\text{tire}} = \frac{\left(1.5\,\text{ft}^3\right)\left(44.7-14.7\right)\dfrac{\text{lb}}{\text{in.}^2}\left(12\dfrac{\text{in.}}{\text{ft}}\right)^2}{\left(1716\dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)\left(530\,{}^\circ\text{R}\right)} = 0.007125\,\text{slug}.$$

Air in the tank can be put into the tires until the tank air pressure drops to $44.7\dfrac{\text{lb}}{\text{in.}^2}$ absolute. The mass of air m_T that can be taken out of the tank and put into the tires is

$$m_T = (p\forall)_i - (p\forall)_f = \frac{\forall}{RT}\left(p_i - p_f\right)$$

$$= \frac{\left(10\,\text{ft}^3\right)\left(150-44.7\right)\dfrac{\text{lb}}{\text{in.}^2}\left(12\dfrac{\text{in.}}{\text{ft}}\right)^2}{\left(1716\dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)\left(530\,{}^\circ\text{R}\right)} = 0.1667\,\text{slug}.$$

The number of tires that can be filled is

$$No. = \frac{0.1667\,\text{slug}}{0.007125\,\text{slug}} = 23.4 \quad \text{or} \qquad \boxed{No. = 23\,\text{tires}}$$

Problem 1.103

A regulation basketball is initially flat and is then inflated to a pressure of approximately 24 lb/in^2 absolute. Consider the air temperature to be constant at 70 °F. Find the mass of air required to inflate the basketball. The basketball's inside radius is 4.67 in.

Solution 1.103

Modelling air as an ideal gas and looking up the gas constant for air:

$$m = \rho\forall = \frac{p\forall}{RT} = \frac{p}{RT}\left(\frac{4}{3}\pi R_i^3\right)$$

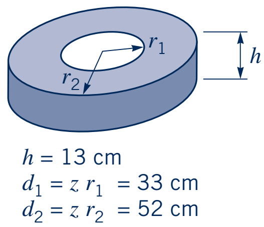
$$= \frac{\left(24\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{4\pi}{3}\right)(4.67\,\text{in.})^3\left(\frac{\text{ft}}{12\,\text{in.}}\right)}{\left(1716\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)(460+70)\,{}^\circ\text{R}}$$

$$\boxed{m = 0.000938\,\text{slug.}}$$

Problem 1.104

Assume that the air volume in a small automobile tire is constant and equal to the volume between two concentric cylinders 13 cm high with diameters of 33 cm and 52 cm. The air in the tire is initially at 25 °C and 202 kPa. Immediately after air is pumped into the tire, the temperature is 30 °C and the pressure is 303 kPa. What mass of air was added to the tire? What would be the air pressure after the air has cooled to a temperature of 0 °C?

Solution 1.104



The mass of air added to the tire is the difference of the final mass of air m_f and the initial mass m_i. Assuming air is an ideal gas,

$$m_f - m_i = \left(\frac{p\forall}{RT}\right)_f - \left(\frac{p\forall}{RT}\right)_i = \frac{\forall}{R}\left(\frac{p_f}{T_f} - \frac{p_i}{T_i}\right).$$

Using specified data:

$$\forall = \pi\left(r_2^2 - r_1^2\right)h = \pi\left[(26\,\text{cm})^2 - (16.5\,\text{cm})^2\right](13\,\text{cm})\left(\frac{\text{m}}{100\,\text{cm}}\right)^3 = 0.0165\,\text{m}^3.$$

$$m_f - m_i = \frac{\left(0.0165\,\text{m}^3\right)}{\left(287.0\,\dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{303\,\text{kPa}}{(273+30)\,\text{K}} - \frac{202\,\text{kPa}}{(273+25)\,\text{K}}\right]\left(\frac{1000\,\dfrac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)$$

$$\boxed{m_f - m_i = 0.0185\,\text{kg}}$$

Now consider the cooling process. The initial state will be 30°C and 303 kPa. The final state will be 0°C and p_f. Applying the ideal gas law to both states gives

$$\left(\frac{p\forall}{RT}\right)_i = \left(\frac{p\forall}{RT}\right)_f.$$

Since $\forall_f = \forall_i$,

$$p_f = p_i\left(\frac{T_f}{T_i}\right) = (303\,\text{kPa})\left(\frac{273+0}{273+30}\right) = \boxed{p_f = 273\,\text{kPa}}$$

Problem 1.105

Develop a computer program for calculating the final gage pressure of gas when the initial gage pressure, initial and final volumes, atmospheric pressure, and the type of process (isothermal or isentropic) are specified. Use BG units. Check your program against the results obtained for Problem 1.98.

Solution 1.105

For compression or expansion, $\dfrac{p}{\rho^n} = \text{constant}$,

where $n = 1$ for an isothermal process, and $n =$ specific heat ratio for an isentropic process.

Therefore: $\dfrac{p_i}{\rho_i^n} = \dfrac{p_f}{\rho_f^n}$ where $i \sim$ initial state and $f \sim$ final state, so that

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^n p_i \qquad (1)$$

Because the amount of mass is constant: $m = \rho\forall \rightarrow \rho_i \forall_i = \rho_f \forall_f \rightarrow \dfrac{\rho_f}{\rho_i} = \dfrac{\forall_i}{\forall_f}$

Recognizing that the pressure in Eq. 1 must be absolute pressure:

$$p_{fg} + p_{atm} = \left(\frac{\forall_i}{\forall_f}\right)^n \left(p_{ig} + p_{atm}\right) \qquad (2)$$

where the subscript g refers to gage pressure. Equation (2) can be written as

$$p_{fg} = \left(\frac{V_i}{V_f}\right)^n \left(p_{ig} + p_{atm}\right) - p_{atm} \qquad (3)$$

A spreadsheet (EXCEL) program for calculating the final gage pressure follows.

This program calculates the final gage pressure of an ideal gas when the initial gage pressure in psi, the initial volume, the final volume, the atmospheric pressure in psia, and the type of process (isothermal or isentropic) is specified. To use, replace current values and let k=1 for isothermal process or k=specific heat for isentropic process.						
A	B	C	D	E	F	
Initial gage pressure	Initial volume	Final volume	Atmospheric pressure		Final gage pressure	
P_{ig}(psi)	V_i	V_f	P_{atm}(psia)	k	P_{fg}(psia)	
25	1	0.3333	14.7	1	104.4	Row 10

Formula:
=((B10/C10)^E10)*(A10+D10)-D10

Data from Problem 1.98 are included in the above table, giving a final gage pressure of 104.4 psi.

Problem 1.106

Often the assumption is made that the flow of a certain fluid can be considered as incompressible flow if the density of the fluid changes by less than 2 %. If air is flowing through a tube such that the air pressure at one section is 9.0 psi and at a downstream section it is 8.6 psi at the same temperature, do you think that this flow could be considered an incompressible flow? Support your answer with the necessary calculations. Assume standard atmospheric pressure.

Solution 1.106

Modelling the air as an ideal gas undergoing an isothermal process:

$$p = \rho RT \rightarrow \quad \frac{p_1}{\rho_1} = \frac{p_2}{\rho_2} \rightarrow \quad \frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}$$

$$\%\,\text{density change} = \frac{\rho_1 - \rho_2}{\rho_1} \times 100$$

$$= \left(1 - \frac{\rho_2}{\rho_1}\right) \times 100 = \left(1 - \frac{p_2}{p_1}\right) \times 100 = \left[1 - \frac{(8.6 + 14.7)\,\text{psia}}{(9.0 + 14.7)\,\text{psia}}\right] \times 100$$

$$= 1.69\,\% < 2\,\%$$

$\underline{\underline{\text{Yes}}}$. This process is well modelled as an incompressible flow.

Problem 1.107

An important dimensionless parameter concerned with very high-speed flow is the *Mach number*, defined as V/c, where V is the speed of the object such as an airplane or projectile, and c is the speed of sound in the fluid surrounding the object. For a projectile traveling at 800 mph through air at 50 °F and standard atmospheric pressure, what is the value of the Mach number?

Solution 1.107

$$\text{Mach number} = \frac{V}{c}$$

From the table of Physical Properties of Air at Standard Atmospheric Pressure (BG/EE Units)

$$c_{\text{air@50°F}} = 1106 \frac{\text{ft}}{\text{s}}$$

$$\text{Mach number} = \frac{(800\ \text{mph})\left(5280 \frac{\text{ft}}{\text{mi}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)}{1106 \frac{\text{ft}}{\text{s}}} = \underline{\underline{1.06}}$$

Problem 1.108

The "power available in the wind" of velocity V through an area A is

$$\dot{W} = \frac{1}{2}\rho A V^3,$$

where ρ is the air density (0.075 lbm/ft^3). For an 18-mph wind, find the wind area A that will supply a power of 4 hp.

Solution 1.108

Solving for the area A and using appropriate unit conversion factors:

$$A = \frac{2\dot{W}}{\rho V^3} = \frac{2(4\,\text{hp})\left(\dfrac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right)}{\left(0.075\dfrac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(18\dfrac{\text{mi}}{\text{hr}}\right)^3\left(\dfrac{5280\ \text{ft}}{1\ \text{mi}}\times\dfrac{1\ \text{hr}}{3600\ \text{s}}\right)^3},$$

$$= 3.17\frac{\text{ft}\cdot\text{lb}\cdot\text{s}^2}{\text{lb}_\text{m}}\left(\frac{1\,\text{lb}_\text{m}\cdot 32.2\ \text{ft}}{1\,\text{lb}\cdot\text{s}^2}\right)$$

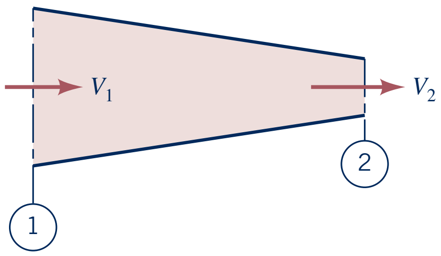
$$\boxed{A = 102\,\text{ft}^2}$$

Problem 1.109

Air enters the converging nozzle shown in the figure below at $T_1 = 70\,°\mathrm{F}$ and $V_1 = 50\,\mathrm{ft/s}$. At the exit of the nozzle, V_2 is given by

$$V_2 = \sqrt{V_1^2 + 2c_p\left(T_1 - T_2\right)},$$

where $c_p = 187\,\mathrm{ft \cdot lb/lbm \cdot °F}$ and T_2 is the air temperature at the exit of the nozzle. Find the temperature T_2 for which $V_2 = 1000\,\mathrm{ft/s}$.



Solution 1.109

Solving for T_2 and inserting the values specified: q_c gives

$$T_2 = T_1 - \frac{V_2^2 - V_1^2}{2c_p} = 70\,^\circ\mathrm{F} - \frac{\left(1000\,\frac{\mathrm{ft}}{\mathrm{s}}\right)^2 - \left(50\,\frac{\mathrm{ft}}{\mathrm{s}}\right)^2}{2\left(187\,\frac{\mathrm{ft \cdot lb}}{\mathrm{lbm} \cdot {}^\circ\mathrm{F}}\right)\left(\frac{1\,\mathrm{lb_m} \cdot 32.2\,\mathrm{ft}}{1\,\mathrm{lb} \cdot \mathrm{s}^2}\right)}$$

$$\boxed{T_2 = -12.8\,^\circ\mathrm{F}}$$

Problem 1.110

This water jet is a blast Usually, liquids can be treated as incompressible fluids. However, in some applications the *compressibility* of a liquid can play a key role in the operation of a device. For example, a water pulse generator using compressed water has been developed for use in mining operations. It can fracture rock by producing an effect comparable to a conventional explosive such as gunpowder. The device uses the energy stored in a water-filled accumulator to generate an ultrahigh-pressure water pulse ejected through a 10-to 25-mm-diameter discharge value. At the ultrahigh pressures used (300 to 400 MPa, or 3000 to 4000 atmospheres), the water is compressed (i.e., the volume reduced) by about 10% to 15%. When a fast-opening valve within the pressure vessel is opened, the water expands and produces a jet of water that upon impact with the target material produces an effect similar to the explosive force from conventional explosives. Mining with the water jet can eliminate various hazards that arise with the use of conventional chemical explosives, such as those associated with the storage and use of explosives and the generation of toxic gas by-products that require extensive ventilation. (See Problem 1.110.)

By what percent is the volume of water decreased if its pressure is increased to an equivalent to 3000 atmospheres (44,100 psi)?

Solution 1.110

$$E_V = -\frac{dp}{\frac{d\forall}{\forall}} \approx -\frac{\Delta p}{\frac{\Delta\forall}{\forall}}$$

$$\frac{\Delta\forall}{\forall} \approx -\frac{\Delta p}{E_V} = -\frac{44100\,\text{psia} - 14.7\,\text{psia}}{3.12\times 10^5\,\text{psia}} = -0.141$$

Thus, % decrease in volume $= \underline{\underline{14\,\%}}$

Problem 1.111

During a mountain climbing trip it is observed that the water used to cook a meal boils at 90 °C rather than the standard 100 °C at sea level. At what altitude are the climbers preparing their meal? See Table of Physical Properties of Water (SI Units) and Table of Properties of the U.S. Standard Atmosphere (SI Units) for data needed to solve this problem.

Solution 1.111

Water boils when the vapor pressure of the liquid is the same as atmospheric pressure.

From the water property table, at $T = 90\,°\text{C}$, $p_v = 7.01 \times 10^4 \dfrac{\text{N}}{\text{m}^2}(\text{abs})$.

From standard atmosphere table, $p = 7.01 \times 10^4 \dfrac{\text{N}}{\text{m}^2}(\text{abs}) \rightarrow \boxed{\text{altitude} = 3000\text{ m}}$

Problem 1.112

When a fluid flows through a sharp bend, low pressures may develop in localized regions of the bend. Estimate the minimum absolute pressure (in psi) that can develop without causing cavitation if the fluid is water at 160 °F.

Solution 1.112

Cavitation may occur when the local pressure equals the vapor pressure. For water at 160 °F (from Table of Physical Properties of Water [BG/EE Units])

$$T = 160\ ^{\circ}\mathrm{F} \rightarrow \quad p_v = 4.74\,\mathrm{psi\ (abs)} \rightarrow \quad \boxed{\text{minimum pressure} = 4.74\ \mathrm{psia}}$$

Problem 1.113

A partially filled closed tank contains ethyl alcohol at 68 °F. If the air above the alcohol is evacuated, what is the minimum absolute pressure that develops in the evacuated space?

Solution 1.113

Minimum pressure = vapor pressure = $\underline{\underline{0.85 \text{ psi (abs)}}}$

Problem 1.114

Estimate the minimum absolute pressure (in Pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20 °C.

Solution 1.114

Cavitation may occur when the section pressure at the pump inlet equals the vapor pressure

For carbon tetrachloride at 20 °C, $p_v = 13\ \text{kPa}\,(\text{abs})$.

Thus, minimum pressure = $\underline{\underline{13\ \text{kPa (abs)}}}$

Problem 1.115

When water at 70 °C flows through a converging section of pipe, the pressure decreases in the direction of flow. Estimate the minimum absolute pressure that can develop without causing cavitation. Express your answer in both BG and SI units.

Solution 1.115

Cavitation may occur in the converging section of pipe when the pressure equals the vapor pressure. From the Table of Physical Properties of Water (SI Units) for water at 70 °C, $p_v = 31.2 \text{ kPa (abs)}$.

Therefore,

$$\text{minimum pressure} = \underline{\underline{31.2 \text{ kPa (abs)}}}$$

$$= \left(31.2\times10^3 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{1.450\times10^{-4} \text{ psi}}{1 \frac{\text{N}}{\text{m}^2}}\right)$$

$$= \underline{\underline{4.52 \text{ psia}}}$$

Problem 1.116

At what atmospheric pressure will water boil at 35 °C? Express your answer in both SI and BG units.

Solution 1.116

The vapor pressure of water at 35°C is 5.81 kPa (abs) (from Table of Physical Properties of Water [SI Units] using linear interpolation).

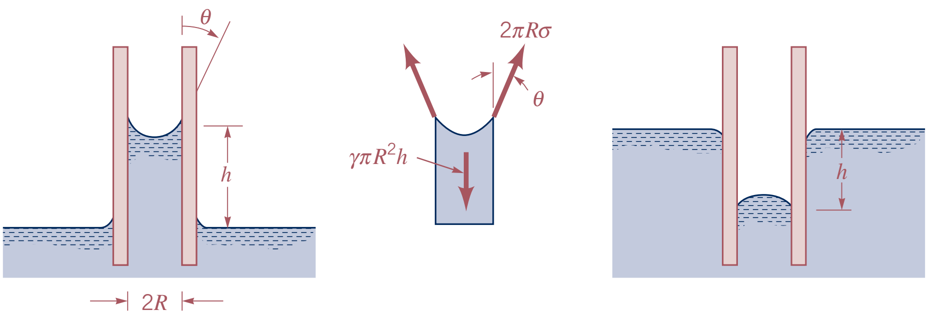
Thus, if water boils at this temperature the atmospheric pressure must be equal to $\underline{\underline{5.81\,\mathrm{kPa\,(abs)}}}$ in SI units. In BG units,

$$\left(5.81\times 10^{3}\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(\frac{1.450\times 10^{-4}\ \mathrm{psi}}{1\,\dfrac{\mathrm{N}}{\mathrm{m}^2}}\right)=\underline{\underline{0.842\,\mathrm{psi\,(abs)}}}$$

Problem 1.118

When a 2-mm-diameter tube is inserted into a liquid in an open tank, the liquid is observed to rise 10 mm above the free surface of the liquid. The contact angle between the liquid and the tube is zero, and the specific weight of the liquid is $1.2 \times 10^4 \text{ N/m}^3$. Determine the value of the surface tension for this liquid.

Solution 1.118



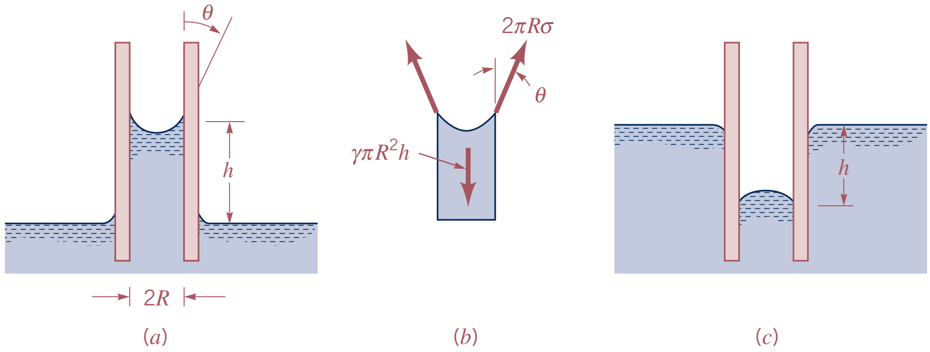
The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube a distance $h = \dfrac{2\sigma \cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified contact angle, $\theta = 0$:

$$\sigma = \frac{\gamma h R}{2\cos\theta} = \frac{1.2\times10^4 \dfrac{\text{N}}{\text{m}^3}\left(10\times10^{-3}\text{ m}\right)\left(\dfrac{2\times10^{-3}\text{ m}}{2}\right)}{2\cos 0} = \underline{\underline{0.060\frac{\text{N}}{\text{m}}}}$$

Problem 1.119

A soda straw with an inside diameter of $0.125\ \text{in.}$ is inserted into a pan of water at $60\ °\text{F}$. The water in the straw rises to a height of $0.150\ \text{in.}$ above the water surface in the pan. Determine the value of θ, the contact angle of the water with the straw (see the figure below).



Effect of capillary section in small tubes. (*a*) Rise of column for a liquid that wets the tube, (*b*) Free-body diagram for calculating column height. (*c*) Depression of column for a nonwetting liquid.

Solution 1.119

Consider the free body diagram of the liquid column inside the tube as shown in the figure. If the liquid column is in static equilibrium:

$$+\uparrow \sum F = 0 \rightarrow 2\pi R\sigma\cos\theta = \gamma\pi R^2 h$$

$$\cos\theta = \frac{\gamma\pi R^2 h}{2\pi R\sigma} = \frac{\gamma R h}{2\sigma}$$

$$\theta = \cos^{-1}\left(\frac{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{0.0625}{12}\,\text{ft}\right)\left(\frac{0.150}{12}\,\text{ft}\right)}{2\left(5.04\times10^{-3}\,\frac{\text{lb}}{\text{ft}}\right)}\right)$$

$$\boxed{\theta = 1.16\ \text{radians} = 66.2^{\circ}}$$

Problem 1.120

Small droplets of carbon tetrachloride at 68 °F are formed with a spray nozzle. If the average diameter of the droplets is 200 μm, what is the difference in pressure between the inside and outside of the droplets?

Solution 1.120

From the force balance on a half-droplet presented in the chapter:

$$\Delta p = \frac{2\sigma}{R}$$

Looking in the properties table for carbon tetrachloride at 68 °F, $\sigma = 2.69 \times 10^{-2} \frac{\text{N}}{\text{m}}$.

Substitution yields:

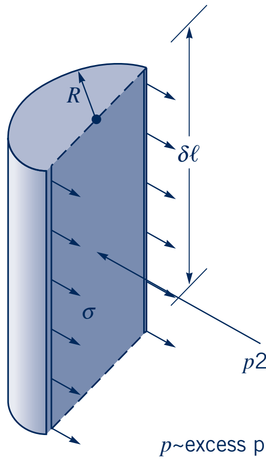
$$\Delta p = \frac{2\left(2.69 \times 10^{-2} \frac{\text{N}}{\text{m}}\right)}{100 \times 10^{-6} \text{ m}} = \underline{\underline{538 \text{ Pa}}}$$

Problem 1.121

A 12-mm-diameter jet of water discharges vertically into the atmosphere. Due to surface tension, the pressure inside the jet will be slightly higher than the surrounding atmospheric pressure. Determine this difference in pressure.

Solution 1.121

Considering the free body diagram of one half of a short length of jet, $\delta\ell$, equilibrium requires:



$$p(2R\delta\ell) = \sigma(2\delta\ell)$$

$$p = \frac{\sigma}{R}$$

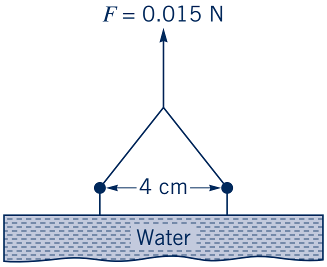
$$= \frac{7.34\times10^{-2}\ \frac{\text{N}}{\text{m}}}{\frac{1}{2}\left(\frac{12}{1000}\ \text{m}\right)}$$

$$= \underline{\underline{12.2\ \text{Pa}}}$$

Problem 1.122

A method used to determine the surface tension of a liquid is to determine the force necessary to raise a wire ring through the air–liquid interface, as shown in the figure below. What is the value of the surface tension if a force of 0.015 N is required to raise a 4-cm-diameter ring? Consider the ring weightless, as a tensiometer (used to measure the surface tension) "zeroes" out the ring weight.

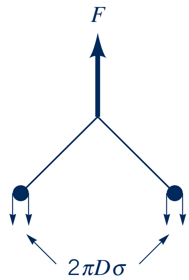
Solution 1.122



A free body diagram of the ring and supporting wires is shown on the right and gives

$$F = 2\pi D\sigma$$

$$\sigma = \frac{F}{2\pi D} = \frac{0.015\text{ N}}{2\pi\left(\frac{4}{100}\text{ m}\right)} = \underline{\underline{5.97\times 10^{-2}\ \frac{\text{N}}{\text{m}}}}$$

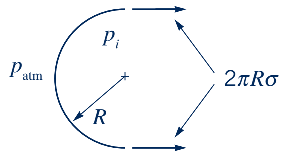


Problem 1.123

Calculate the pressure difference between the inside and outside of a spherical water droplet having a diameter of $\frac{1}{32}$ in. and a temperature of 50°F.

Solution 1.123

A force balance on the outside surface of the drop gives



$$\overset{\rightarrow}{+}\sum F = 0,$$

$$p_{atm}\pi R^2 - p_i \pi R^2 + 2\pi R\sigma = 0$$

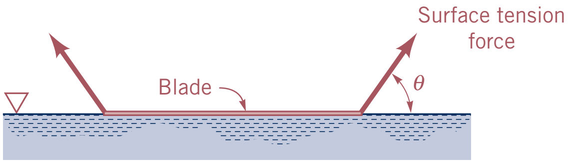
$$p_i - p_{atm} = \frac{2\sigma}{R}$$

For water at 50°F, $\sigma = 5.09\times10^{-3}\ \frac{\text{lb}}{\text{ft}}$ so

$$p_i - p_{atm} = \frac{2\left(5.09\times10^{-3}\ \frac{\text{lb}}{\text{ft}}\times\frac{1\ \text{ft}}{12\ \text{in.}}\right)}{\frac{1}{32}\ \text{in.}} = \underline{\underline{0.0271\ \text{psi}}}$$

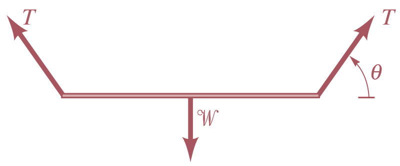
Problem 1.124

Surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in the figure below. **(a)** The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. **(b)** The mass of the single-edge blade is 2.6×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



Solution 1.124

(a) water ➔ $\sigma = 7.34 \times 10^{-2} \dfrac{\text{N}}{\text{m}}$



$$\sum F_{\text{vertical}} = T \sin\theta - W = 0$$

$$\sin\theta = \frac{W}{T} = \frac{mg}{\sigma L}$$

$$\theta = \sin^{-1}\left(\frac{(0.64 \text{ kg})\left(9.81 \dfrac{\text{m}}{\text{s}^2}\right)}{\left(7.34 \times 10^{-2} \dfrac{\text{N}}{\text{m}}\right)(0.206 \text{ m})}\right)$$

$$\boxed{\theta = 0.415 \text{ radians} = 24.5^{\circ}}$$

(b) For the single-edge blade

$$W = m_{\text{blade}} \times g = \left(2.61 \times 10^{-3} \text{ kg}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 0.0256 \text{ N}$$

and

$$T \sin\theta = (\sigma L)\sin\theta = \left(7.34 \times 10^{-2} \frac{\text{N}}{\text{m}}\right)(0.154 \text{ m})\sin\theta$$

$$= (0.0113 \text{ N})\sin\theta$$

For static equilibrium:

$$\sin\theta = \frac{0.0256 \text{ N}}{0.0113 \text{ N}} > 1, \quad \text{but } \sin\theta \le 1 \rightarrow \boxed{\text{single-edge blade cannot float}}$$

Problem 1.125

Explain how sweat soldering of copper pipe works from a fluid mechanics viewpoint.

Solution 1.125

Solder for sweat soldering copper pipe is an alloy with a melting point below that of copper. The copper parts are typically heated using a gas torch to a temperature below the melting point of copper but above the melting point of the solder. When the solder is "touched" to the joint, it melts. To form a good quality joint between a copper pipe and fittings, or between fittings, capillary action must draw liquid solder into the small gap to between the two parts to fill the gap and the solder must bond with the copper surface.

From "a fluid mechanics viewpoint," the flux used for sweat soldering of copper pipe reduces the surface tension of the liquefied solder, reducing the contact angle at the solder-copper interface, thereby producing a stronger capillary action that more effectively draws the liquid solder into the joint to fill it with solder.

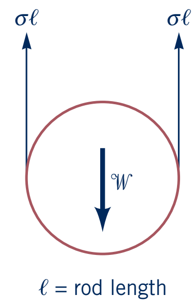
From "a chemical and mechanical viewpoint," at the elevated temperatures occurring during the soldering process, oxides quickly form on the surface of copper and interfere with the bonding process. Therefore, even after mechanical cleaning of the parts, flux acts as a reducing agent to remove oxides from the surface of the copper, facilitating a stronger bond between the solder and the copper.

Problem 1.126

Under the right conditions, it is possible, due to surface tension, to have metal objects float on water. Consider placing a short length of a small diameter steel ($\gamma = 490\,\text{lb}/\text{ft}^3$) rod on a surface of water. What is the maximum diameter that the rod can have before it will sink? Assume that the surface tension forces act vertically upward. *Note:* A standard paper clip has a diameter of 0.036 in. Partially, unfold a paper clip and see if you can get it to float on water. Do the results of this experiment support your analysis?

Solution 1.126

In order for rod to float, (see free body diagram):



$$2\sigma\ell \geq W \geq \left(\frac{\pi}{4}\right)\left(D^2\right)\ell\gamma_{\text{steel}}$$

Thus, for the limiting case

$$D_{\max}^2 = \frac{2\sigma\ell}{\left(\frac{\pi}{4}\right)\ell\gamma_{\text{steel}}} = \frac{8\sigma}{\pi\gamma_{\text{steel}}}$$

so that

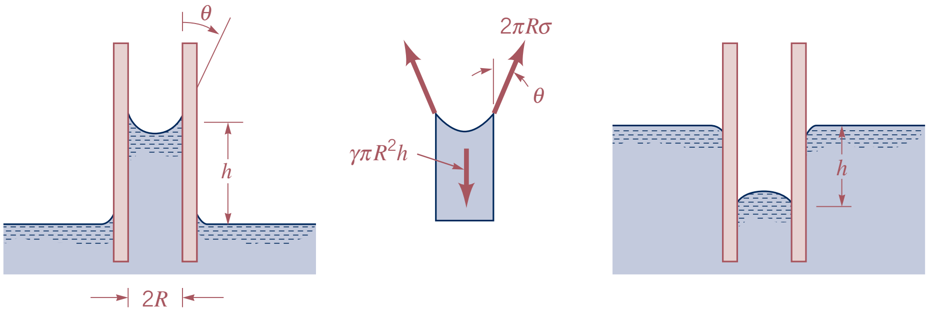
$$D_{\max} = \left[\frac{8\left(5.03\times10^{-3}\,\frac{\text{lb}}{\text{ft}}\right)}{\pi\left(490\,\frac{\text{lb}}{\text{ft}^3}\right)}\right]^{\frac{1}{2}} = 5.11\times10^{-3}\,\text{ft} = \underline{\underline{0.0614\,\text{in.}}}$$

Since a standard steel paper clip has a diameter of 0.036 in., which is less than 0.0614 in., it should float. A simple experiment will verify this. <u>Yes</u>.

Problem 1.127

An open, clean glass tube, having a diameter of 3 mm, is inserted vertically into a dish of mercury at 20 °C. How far will the column of mercury in the tube be depressed?

Solution 1.127



The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube a distance $h = \dfrac{2\sigma\cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified information:

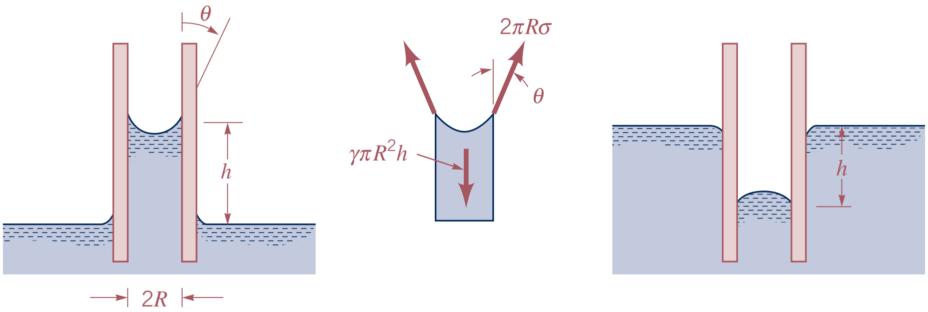
$$h = \frac{2\left(4.66\times10^{-1}\,\dfrac{\text{N}}{\text{m}}\right)\cos 130°}{\left(133\times10^{3}\,\dfrac{\text{N}}{\text{m}^3}\right)(0.0015\,\text{m})} = -3.00\times10^{-3}\ \text{m}$$

Thus, column will be depressed $\underline{\underline{3.00\ \text{mm}}}$

Problem 1.128

An open, clean glass tube ($\theta = 0°C$) is inserted vertically into a pan of water. What tube diameter is needed if the water level in the tube is to rise one tube diameter (due to surface tension)?

Solution 1.128



The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube a distance $h = \dfrac{2\sigma\cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified information:

$$2R = \frac{2\sigma\cos 0^{o}}{\gamma R}$$

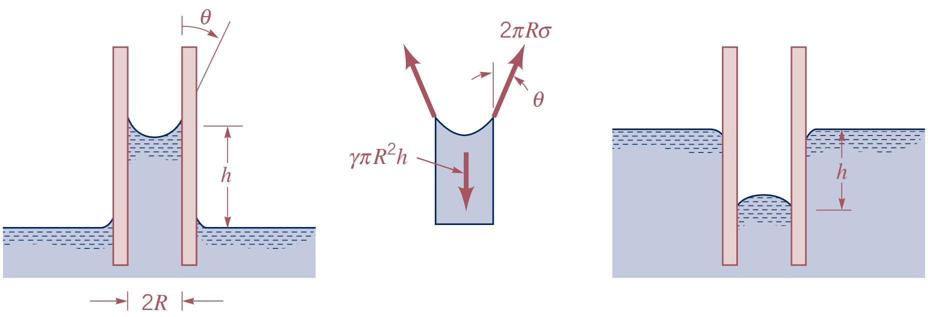
$$R^2 = \frac{\sigma}{\gamma} = \frac{5.03\times10^{-3}\,\dfrac{\text{lb}}{\text{ft}}}{62.4\,\dfrac{\text{lb}}{\text{ft}^3}} = 8.98\times10^{-3}\text{ ft}$$

$$\text{diameter} = 2R = \underline{\underline{1.80\times10^{-2}\text{ ft}}}$$

Problem 1.129

Determine the height that water at 60 °F will rise due to capillary action in a clean, $\frac{1}{4}$-in.-diameter tube. What will be the height if the diameter is reduced to 0.01 in.?

Solution 1.129



The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube a distance $h = \dfrac{2\sigma\cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified information:

$$h = \frac{2\left(5.03\times10^{-1}\,\dfrac{\text{lb}}{\text{ft}}\right)\left(\cos 0^{\circ}\right)}{\left(62.37\,\dfrac{\text{lb}}{\text{ft}^3}\right)\left(\dfrac{0.125}{12}\,\text{ft}\right)} = 1.55\times10^{-3}\,\text{ft}\left(\frac{12\,\text{in.}}{\text{ft}}\right) = \underline{\underline{0.186\text{ in.}}}$$

Similarly, (for $R = 0.005\,\text{in.}$): $h = \left(0.186\text{ in.}\right)\left(\dfrac{0.125\text{ in.}}{0.005\text{ in.}}\right) = \underline{\underline{4.65\text{ in.}}}$

Problem 1.130

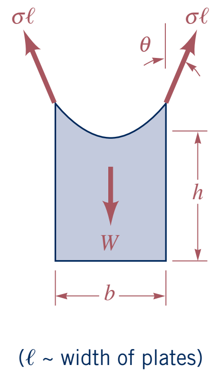
Two vertical, parallel, clean glass plates are spaced a distance of 2 mm apart. If the plates are placed in water, how high will the water rise between the plates due to capillary action?

Solution 1.130

For equilibrium in the vertical direction,

$$W = \gamma h b \ell = 2(\sigma \ell \cos\theta)$$

$$h = \frac{2\sigma\cos\theta}{\gamma b}$$



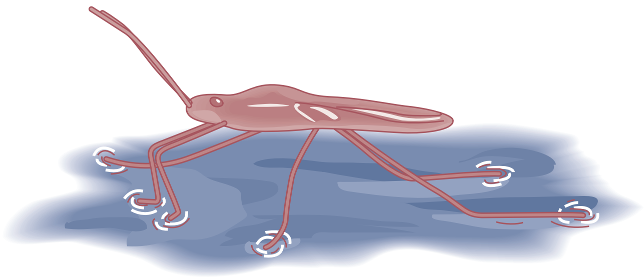
Thus, (for $\theta = 0$)

$$h = \frac{2\left(7.34\times10^{-2}\ \frac{\text{N}}{\text{m}}\right)(1)}{\left(9.80\times10^{3}\ \frac{\text{N}}{\text{m}^3}\right)(0.002\ \text{m})} = 7.49\times10^{-3}\ \text{m} = \underline{\underline{7.49\ \text{mm}}}$$

Problem 1.131

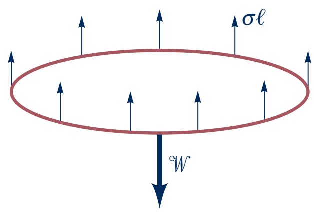
Walking on water Water striders are insects commonly found on ponds, rivers, and lakes that appear to "walk" on water. A typical length of a water strider is about 0.4 in., and they can cover 100 body lengths in one second. It has long been recognized that it is *surface tension* that keeps the water strider from sinking below the surface. What has been puzzling is how they propel themselves at such a high speed. They can't pierce the water surface or they would sink. A team of mathematicians and engineers from the Massachusetts Institute of Technology (MIT) applied conventional flow visualization techniques and high-speed video to examine in detail the movement of the water striders. They found that each stroke of the insect's legs creates dimples on the surface with underwater swirling vortices sufficient to propel it forward. It is the rearward motion of the vortices that propels the water strider forward. To further substantiate their explanation, the MIT team built a working model of a water strider, called Robostrider, which creates surface ripples and underwater vortices as it moves across a water surface. Waterborne creatures, such as the water strider, provide an interesting world dominated by surface tension. (See Problem 1.131.)

(a) The water strider bug shown in the figure below is supported on the surface of a pond by surface tension acting along the interface between the water and the bug's legs. Determine the minimum length of this interface needed to support the bug. Assume the bug weighs 10^{-4} N and the surface tension force acts vertically upwards. **(b)** Repeat part (a) if surface tension were to support a person weighing 750N.



Solution 1.131

$W \sim$ weight
$\sigma \sim$ surface tension
$\ell \sim$ length of interface



For equilibrium, $W = \sigma\ell$

(a) $$\ell = \frac{W}{\sigma} = \frac{10^{-4}\ \text{N}}{7.34\times10^{-2}\ \frac{\text{N}}{\text{m}}} = 1.36\times10^{-3}\ \text{m}\left(\frac{1000\ \text{mm}}{1\ \text{m}}\right) = \underline{\underline{1.36\ \text{mm}}}$$

(b) $$\ell = \frac{W}{\sigma} = \frac{750\ \text{N}}{7.34\times10^{-2}\ \frac{\text{N}}{\text{m}}} = \underline{\underline{1.02\times10^{4}\ \text{m}}}\left(6.34\ \text{mi !!}\right)$$