Solutions Manual for Climate Mathematics:

Theory and Applications

SAMUEL S.P. SHEN AND RICHARD C.J. SOMERVILLE

Samuel S.P. Shen
San Diego State University, and
Scripps Institution of Oceanography, University of California, San Diego
and
Richard C.J. Somerville
Scripps Institution of Oceanography, University of California, San Diego

About this solutions manual

This solutions manual is solely for personal use by instructors and is password protected. Please use these resources responsibly and exercise your professional discretion when choosing how you share this material with your students. Other instructors may wish to use the exercises in our book for assessment purposes and their usefulness will be undermined if they are shared online or via social networks. These solutions and the other resources that accompany our book are subject to copyright (© Samuel S. P. Shen and Richard C. J. Somerville 2019). Instructors are permitted to view, print or download these resources for use in their teaching, but may not change them or use them for commercial gain.

While every effort has been made to check the accuracy of these solutions, we would be extremely grateful to receive prompt notification of any errors, and will post corrected versions of this manual. Feedback on the Climate Mathematics textbook and the other digital resources that accompany it, is also welcomed and will help inform future plans for a second edition. Errata and feedback can be submitted online at

`https://docs.google.com/forms/d/1w22xDKhukR1o-Kv5RlaCyxr55feYJwlUSyvKc10C6w4/viewform?edit_requested=true`

or through the URL link "Readers feedback" on the book website `www.climatemathematics.org`.

Samuel S. P. Shen and Richard C. J. Somerville

La Jolla, California, USA

20 September 2019

1 Dimensional analysis for Climate Science

Exercises

1.1 Taking dimension for both sides of the equation

$$v = \alpha m^a g^b t^c \tag{1.1}$$

yields

$$[v] = [\alpha m^a g^b t^c] = [\alpha][m]^a[g]^b[t]^c, \tag{1.2}$$

where α is a dimensionless constant.

According to Table 1.2, the above equation becomes

$$LT^{-1} = 1 \times M^a(LT^{-2})^b T^c. \tag{1.3}$$

This can be re-written as

$$M^0L^1T^{-1} = M^aL^bT^{-2b+c}. \tag{1.4}$$

Comparing the exponents of both sides of the equation, we obtain the following three linear equations for the three unknowns a, b and c:

$$a = 0, \tag{1.5}$$

$$b = 1, \tag{1.6}$$

$$-2b + c = -1. \tag{1.7}$$

The solution of the above three equations is

$$a = 0, b = 1, c = 1. \tag{1.8}$$

Thus,

$$v = \alpha g t. \tag{1.9}$$

The velocity is independent of mass m, and depends only on the gravitational acceleration and time.

1.2 Taking dimension for both sides of the equation

$$h = \beta m^a g^b t^c \tag{1.10}$$

yields

$$[h] = [\beta m^a g^b t^c] = [\beta][m]^a[g]^b[t]^c, \tag{1.11}$$

where β is a dimensionless constant.

According to Table 1.2, the above equation becomes

$$L = 1 \times M^a (LT^{-2})^b T^c. \tag{1.12}$$

This can be re-written as

$$M^0 L^1 T^0 = M^a L^b T^{-2b+c}. \tag{1.13}$$

Comparing the exponents of both sides of the equation, we obtain the following three linear equations for the three unknowns a, b and c:

$$a = 0, \tag{1.14}$$

$$b = 1, \tag{1.15}$$

$$-2b + c = 0. \tag{1.16}$$

The solution of the above three equations is

$$a = 0, b = 1, c = 2. \tag{1.17}$$

Thus,

$$h = \beta g t^2. \tag{1.18}$$

The falling distance is independent of mass m, and depends only on the gravitational acceleration and time.

1.3 (a) The solution of this problem depends on one's experimental data. The steps are outlined as follows.
(i) Do the experiments.
(ii) Record the data of h and t.
(iii) Compute β by

$$\beta = \frac{h}{gt^2}. \tag{1.19}$$

In (b), the gravitational acceleration in the direction of motion is reduced to

$$g' = g \sin \phi. \tag{1.20}$$

Then,

$$\beta = \frac{h}{g't^2} \tag{1.21}$$

where h is the distance the ball has travelled on the included plane.

1.4 In the book, Page 371.

$$h = \frac{1}{2}gt^2. \tag{1.22}$$

Comments: This formula is given in many textbooks of physics. It is derived traditionally using calculus by solving a differential equation. Our derivation using the dimensional analysis approach has the advantage of simplicity in mathematical derivations, and clarity of physical concepts. Determining an unknown constant by using a constraint, which can be an experimental observation, or an independent physical law, is a universal principle, because the constraint establishes an equation. The solution of the equation determines the unknown quantity.

1.5 In the book, Page 372.

1.6 In the book, Page 372.

1.7 In the book, Page 373.

1.8 The dimensional analysis for the universal gravitation law is below

$$[F_g] = [G]\frac{[m_1][m_2]}{[r^2]} \tag{1.23}$$

Assume that the dimension of force is given MLT^{-2}. The only unknown dimension is $[G]$. We will find $[G]$:

$$[G] = \frac{[r^2][F_g]}{[m_1][m_2]} = \frac{L^2MLT^{-2}}{M^2} = L^3M^{-1}T^{-2}. \tag{1.24}$$

1.9 Taking the dimension for both sides of the equation

$$F = ma, \tag{1.25}$$

we have

$$[F] = [ma] = [m][a] = M(LT^{-2}) = MLT^{-2}. \tag{1.26}$$

1.10 The solution depends on your experimental data.

Comments: Force is a derived quantity and may be defined as mass times acceleration. One thus can use an observed acceleration to quantify force. On the other hand, if the data of force and acceleration are both known, one can use regression to determine the linear relationship between a *and* F *with* $a = (1/m)F + \epsilon$, *where* ϵ *is a random error with its mean equal to zero.*

1.11 In the book, Page 374.