A Short Course in Intermediate Microeconomics with Calculus

2nd edition

Solutions to Exercises[1]

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Chapter 2 Solutions

1. (a) Our consumer prefers a cup of coffee with one teaspoon (6 g) of sugar to a cup of coffee with two teaspoons (12 g) of sugar; that is, $6 \succ 12$. However, she is indifferent between a cup of coffee with n grams of sugar and a cup of coffee with $n+1$ grams; that is, $n \sim n+1$. By transitivity, if $n \sim n+1$ and $n+1 \sim n+2$, then $n \sim n+2$. With repeated applications of transitivity, if $6 \sim 7$, $7 \sim 8$, ..., and $11 \sim 12$, then $6 \sim 12$. However, $6 \sim 12$ contradicts our first statement, $6 \succ 12$.

   (b) Vote 1: x versus y.
   Person 1 votes for x, Person 2 votes for y, and Person 3 votes for x. Therefore, $x \succ y$.
   Vote 2: y versus z.
   Person 1 votes for y, Person 2 votes for y, and Person 3 votes for z. Therefore, $y \succ z$.
   Vote 3: x versus z.
   Person 1 votes for x, Person 2 votes for z, and Person 3 votes for z. Therefore, $z \succ x$.

   Now if $x \succ y$ and $y \succ z$, then by transitivity we should have $x \succ z$. But in Vote 3, we see $z \succ x$, which means majority voting violates transitivity.

2. (a) The indifference curve corresponding to $u = 1$ passes through the points $(0.5, 2)$, $(1, 1)$, and $(2, 0.5)$. The indifference curve corresponding to $u = 2$ passes through the points $(0.5, 4)$, $(1, 2)$, $(2, 1)$, and $(4, 0.5)$.

   (b) The MRS equals 1 along the ray from the origin $x_2 = x_1$, and it equals 2 along the ray from the origin $x_2 = 2x_1$.

3. (a) The indifference curves are downward-sloping parallel lines with a slope of -1 and the arrow pointing northeast.

   (b) The indifference curves are upward sloping with the arrow pointing northwest.

(c) The indifference curves are vertical with the arrow pointing to the right.

(d) The indifference curves are downward sloping and convex with the arrow pointing northeast.

4. (a) The indifference curves are horizontal; the consumer is neutral about x_1 and likes x_2.

(b) The indifference curves are downward-sloping parallel lines with a slope of -1; the consumer considers x_1 and x_2 to be perfect substitutes.

(c) The indifference curves are L-shaped, with kinks along the ray from the origin $x_2 = \frac{1}{2}x_1$; the consumer considers x_1 and x_2 to be perfect complements.

(d) The indifference curves are upward sloping and convex (shaped like the right side of a U); the consumer likes x_2, but dislikes x_1; that is, good 1 is a bad for the consumer.

5. (a) $MU_1 = \frac{\partial u(x_1,x_2)}{\partial x_1} = \frac{\partial(3x_1^2x_2^4)}{\partial x_1} = 6x_1x_2^4$.

(b) $MU_2 = \frac{\partial u(x_1,x_2)}{\partial x_2} = \frac{\partial(3x_1^2x_2^4)}{\partial x_2} = 12x_1^2x_2^3$.

(c) $MRS = \frac{MU_1}{MU_2} = \frac{6x_1x_2^4}{12x_1^2x_2^3} = \frac{x_2}{2x_1}$.

(d) $MRS = \frac{x_2}{2x_1} = \frac{4}{2\cdot 2} = 1$.

(e) $MRS = \frac{x_2}{2x_1} = \frac{2}{2\cdot 8} = \frac{1}{8}$.

His MRS has decreased. As he spends more and more time fishing, he is increasingly loath to give up hammock time for an extra hour of fishing.

(f) Last week, $u(x_1, x_2) = 3x_1^2x_2^4 = 3 \cdot 2^2 \cdot 4^4 = 3 \cdot 2^2 \cdot 2^8 = 3 \cdot 2^{10}$.

This week, $u(x_1, x_2) = 3x_1^2x_2^4 = 3 \cdot 8^2 \cdot 2^4 = 3 \cdot 2^6 \cdot 2^4 = 3 \cdot 2^{10}$.

Thus, he is as happy this week as he was last week.

6. (a) The MRS is the amount of money I am willing to give up in exchange for working an hour.

   (b) Since work is a bad, I would need to receive a positive amount of money for every hour I work. Therefore, my indifference curves are upward sloping, and the slope of an upward-sloping curve is positive. Given that $MRS = -$ Indifference Curve Slope, the MRS should be negative.

   (c) If I'm working only an hour a day, a low hourly wage, say, \$10, would be sufficient to induce me to work another hour. If I'm already working 12 hours a day, I would need a very high hourly wage, say, \$100, to induce me to work another hour. Thus, the slope of the indifference curve is increasing as the hours of work increase. Given that $MRS = -$ Indifference Curve Slope, the MRS is decreasing as the hours of work increase.

7. (a) The *MRS* is the negative of the slope of an indifference curve at a point. Between a bad and a good, the curves must be upward sloping, and the slope of an upward-sloping curve is positive.

   (b) In this case, the absolute value of the *MRS* is the additional number of units of good 2 that the consumer would offer in exchange for an increase in one unit of good 1. That is, it tells us her willingness to work (extra hours) as a function of increases in pay.

   (c) The absolute value of the *MRS* (which is what we really care about) is decreasing. That is, she would offer less and less additional work per additional dollars of pay increase. The indifference curves are convex.

8. (a) They are L-shaped: the case of perfect complementts.

   (b) While we don't know much about their shape, what is striking about these preferences is that they are monotonically decreasing, with a global satiation point at zero.

   (c) The curves are horizontal because microeconomics is a neutral for this consumer.

(d) They are downward sloping and concave.

9. (a) The indifference curve through the point (1,1) consists exclusively of that point. That is, no other bundle is indifferent to it.

   (b) These preferences are complete (any two bundles can be compared to one another).

   They are transitive: if $x \succ y$, it must be that either $x_1 > y_1$ or $x_1 = y_1, x_2 > y_2$. If $y \succ z$, it must be that either $y_1 > z_1$ or $y_1 = z_1, y_2 > z_2$. Then, it follows that either $x_1 > z_1$ or $x_1 = z_1, x_2 > z_2$, i.e., $x \succ Z$.

   And they also satisfy monotonicity: more is preferred to less. That is, if $x_1 \geq y_1$ and $x_2 \geq y_2$ with at least one strict inequality, we have that $x \succ y$.

10. (a) They are concentric circles around the point $(1, 2)$, which is the consumer's bliss point.

    (b) These preferences are complete, transitive, and also convex (averages are preferred to extremes). However, they violate monotonicity: for example, the point $(2, 3)$ is less preferred than the point $(1, 2)$.