Chapter 2

Mission Analysis Fundamentals

2.8-4 Homework Problems

2.8-1 Assume that you are a deer hunter sitting around a mountain campfire recalling the days events. Your companion starts in with one of those "big fish stories" and claims that she once shot a deer on a ridge over 4000 feet above her location in the the valley. Knowing the muzzle velocity of her gun is 500 f/s, either confirm or refute this story.

FROM PHYSICS 152 . . . THE MAX height is found from energy.

$\frac{1}{2} mV^2 = mgh$ find h and see if the bullet will go that high.

$$\frac{V^2}{2g} = h \qquad \frac{(500)^2}{2(32.174)} \doteq \boxed{3885.12 \text{ ft}}$$

$\therefore$ Even with $\varnothing$ drag. the bullet can not reach the deer.

2.8-2 The comic on the last page shows a ballistic missile which has been modified to "deliver" garbage to a remote site (say New Jersey) 600 miles away. Determine the burnout flight path angle and burnout velocity for this mission. Plot your results for burnout heights ranging from 5 to 15 miles.

$$\text{Range} = r_e \Psi = 600 \text{ miles.}$$

$r_e = 3963$ miles $\Rightarrow \Psi = .1514$ rad.

$$Q_{bo} = \left(\frac{V_{bo}}{V_c}\right)^2 \quad \text{where} \quad V_c = \sqrt{\frac{\mu}{r_e + h_{bo}}}$$

from max range:

$$Q_{bo} = \frac{2\sin(\Psi/2)}{1+\sin(\Psi/2)} = \boxed{2,639\times10^{-3}}$$

flight path angle equation gives

$$\sin(2\phi_{bo} + \Psi/2) = \frac{2-Q_{bo}}{Q_{bo}} \sin(\Psi/2)$$

subing in the max range eq^n.

$$\sin(2\phi_{bo} + \Psi/2) = \frac{\left[2 - \dfrac{2\sin(\Psi/2)}{1+\sin(\Psi/2)}\right]}{\left[\dfrac{2\sin(\Psi/2)}{1+\sin(\Psi/2)}\right]} \sin(\Psi/2)$$

$$\sin(2\phi_{bo} + \Psi/2) = \frac{[2(1+\sin(\Psi/2)) - 2\sin(\Psi/2)]}{2\sin(\Psi/2)} \sin(\Psi/2)$$

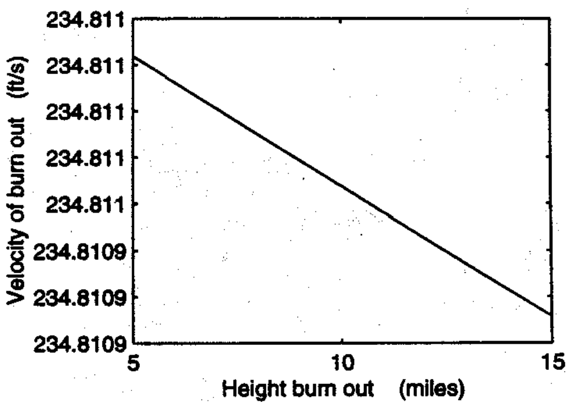
$$\sin(2\phi_{bo} + \Psi/2) = 1$$

$$\Rightarrow \quad 2\phi_{bo} + \Psi/2 = \pi/2 \Rightarrow \boxed{\phi_{bo} = .74175 \text{ rad}}$$

$$Q_{bo} = 2.639\times10^{-3} = \frac{V_c^2}{\left(\frac{\mu}{r_e + h_{bo}}\right)}$$

$$\Rightarrow \quad V_{bo} = \sqrt{\frac{(2.639\times10^{-3})\mu}{(r_e + h_{bo})}}$$

See plot for Results



2.8-3 Some interceptors make use of a two pulse design (with a coast phase in between) in order to optimize performance. Let's assume our interceptor is launched horizontally from an aircraft flying 200 m/s and that each pulse provides 500 m/s ΔV. Furthermore, assume that both pulses are 20 seconds in duration and that the velocity history during the pulse if given by:

$$V = V_o\left(1 + a\left(t - t_{ig}\right)^2\right) \quad a = \text{const}$$

Where V_o is the velocity at the start of the pulse and t_{ig} is the time the pulse is initiated. During the coast phase, drag acts to reduce the missile velocity according to:

$$V = \frac{V_o}{1 + 0.01\left(t - t_c\right)^2}$$

Where t_c is the time of initiation of the coasting phase. Finally, our mission requires that we cover a range of 30 km in the one minute flight time of the device.

a) Assuming the first pulse is fired as the missile is launched from the aircraft, determine the coast time required to accomplish the mission.

b) Sketch the velocity history for this slight, noting velocity values at the start and end of each flight segment.

c) Sketch range as a function of time for the flight, noting velocity values at the end and start of each flight segment.

Four regions for the flight

1) 1st burn $\quad 0 \le t \le 20\,sec \qquad V = V_0 (1 + g_1 (t - t_0)^2)$

V_0 = velocity @ time zero = 200 m/sec

$t_0 = 0$

2) 1st coast $\quad 20 \le t \le t_2 \qquad V = \dfrac{V_1}{1 + b(t - t_c)^2}$

V_1 = velocity @ t = 20sec = 700 m/sec

since $\Delta V = 500$ m/sec

$t_c = 20\,sec \qquad b = 0.01\ sec^{-2}$

3) 2nd burn $\quad t_2 \le t \le t_2 + 20 \qquad V = V_2 (1 + g_2 (t - t_2)^2)$

V_2 = velocity @ end of 1st coast

4) 2nd coast $\quad t_2 + 20 \le t \le 60\,sec \qquad V = \dfrac{V_3}{1 + b(t - (t_2 + 20))^2}$

$V_3 = V_2 + \Delta V = V_2 + 500$ m/sec

total range is integral of velocity in all 4 regions

$$30\,km = \int_0^{20} V_{①}\,dt + \int_{20}^{t_2} V_{②}\,dt + \int_{t_2}^{t_2+20} V_{③}\,dt + \int_{t_2+20}^{60} V_{④}\,dt$$

2 types of integrals

During burns: $$Range = \int_{t_{ij}}^{t_{ij}+20} V_0 \left[1 + a(t - t_{ij})^2\right] dt$$

$$= \left[V_0 t + \frac{V_0 a}{3}(t - t_{ij})^3\right]_{t_{ij}}^{t_{ij}+20}$$

$$= 20 V_0 + \frac{(20)^3}{3} V_0 a$$

During coast periods: $\quad \text{Range} = \int_{t_c}^{t_2} \frac{V_1}{1+b(t-t_c)^2}\,dt$

$$= V_1 \int_{t_c}^{t_2} \frac{dt}{bt^2 - 2bt_c t + (bt_c^2+1)}$$

From integral tables

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)$$

so $\quad \text{Range} = V_1 \frac{2}{\sqrt{4b(bt_c^2-1) - 4b^2t_c^2}} \tan^{-1}\left(\frac{2bt - 2bt_c}{\sqrt{4b(bt_c^2-1) - 4b^2t_c^2}}\right)$

$$= \frac{V_1}{\sqrt{b}} \tan^{-1}\left(\sqrt{b}\,(t-t_c)\right)$$

Find range in each region

region 1 $\quad \Delta V = 500 = V_1 - V_0 = V_0\left[1 + a_1(20-0)^2\right] - V_0$

so $\quad a_1 = \frac{500}{200(400)} = .00625$

range ① $= 20V_0 + \frac{20^3}{3} V_0 a_1 = 200\left[20 + \frac{(20)^3}{3} 0.00625\right] = \frac{22000}{3}\ m$

$= 7.33\ km$

Region 2 $\quad \text{Range } 2 = \frac{V_1}{\sqrt{b}} \tan^{-1}\left[\sqrt{b}\,(t_2-20)\right] \quad V_1 = 700\ \frac{m}{sec}$

Region 3

$$\text{Range } 3 = V_2\left[20 + \frac{(20)^3}{3} a_2\right]$$

V_2 is velocity at end of 1st coast period

$\Delta V = 500\ \frac{m}{sec} = V_3 - V_2 = V_2\left[1 + a_2(20)^2\right] - V_2$

$a_2 = \frac{500}{400(V_2)}$

$\text{Range } 3 = 20V_2 + \frac{(20)^3}{3} V_2\left(\frac{500}{(20)^2 V_2}\right) = 20V_2 + \frac{10000}{3}$

$V_2 = \frac{V_1}{1+b(t_2-20)^2} = \frac{700}{1+b(t_2-20)^2} \Rightarrow \text{Range ③} = \frac{14000}{1+b(t_2-20)^2} + \frac{10000}{3}$

Region 4

$$\text{Range④} = \frac{V_3}{\sqrt{b}} \tan^{-1}\left(\sqrt{b}\,(60-(t_2+20))\right)$$

$$V_3 = \text{velocity after 2nd burn} = 500 + V_2 = \left[500 + \frac{700}{1+b(t_2-20)^2}\right]$$

$$\text{Range④} = \frac{1}{\sqrt{b}}\left[500 + \frac{700}{1+b(t_2-20)^2}\right]\tan^{-1}\left[\sqrt{b}\,(40-t_2)\right]$$

sum ranges for all 4 regions and set $= 30$ km

$$30000 = \frac{22000}{3} + \frac{700}{\sqrt{.01}}\tan^{-1}\left[\sqrt{.01}\,(t_2-20)\right] + \frac{10000}{3} + \frac{14000}{1+0.01(t_2-20)^2}$$

$$\left(\frac{1}{.01}\right)^{\frac{1}{2}}\left[500 + \frac{700}{1+.01(t_2-20)^2}\right]\tan^{-1}\left[\sqrt{.01}\,(40-t_2)\right]$$

$$\frac{58000}{3} = \frac{700}{\sqrt{0.01}}\tan^{-1}\left[\sqrt{.01}\,(t_2-20)\right] + \frac{14000}{1+.01(t_2-20)^2} +$$

$$\left(\frac{1}{.01}\right)^{\frac{1}{2}}\left[500 + \frac{700}{1+.01(t_2-20)^2}\right]\tan^{-1}\left[\sqrt{.01}\,(40-t_2)\right]$$

itterate on $t_2 \Rightarrow t_2 = 29.84 \Rightarrow 9.84$ sec. coast

ii) $V_1 = 700$ m/sec @ $t = 20$ sec

after first coast period, $t = 29.84$ sec $\qquad V_2 = \dfrac{700}{1+.01(9.84)^2} = 355.6 \frac{m}{sec}$

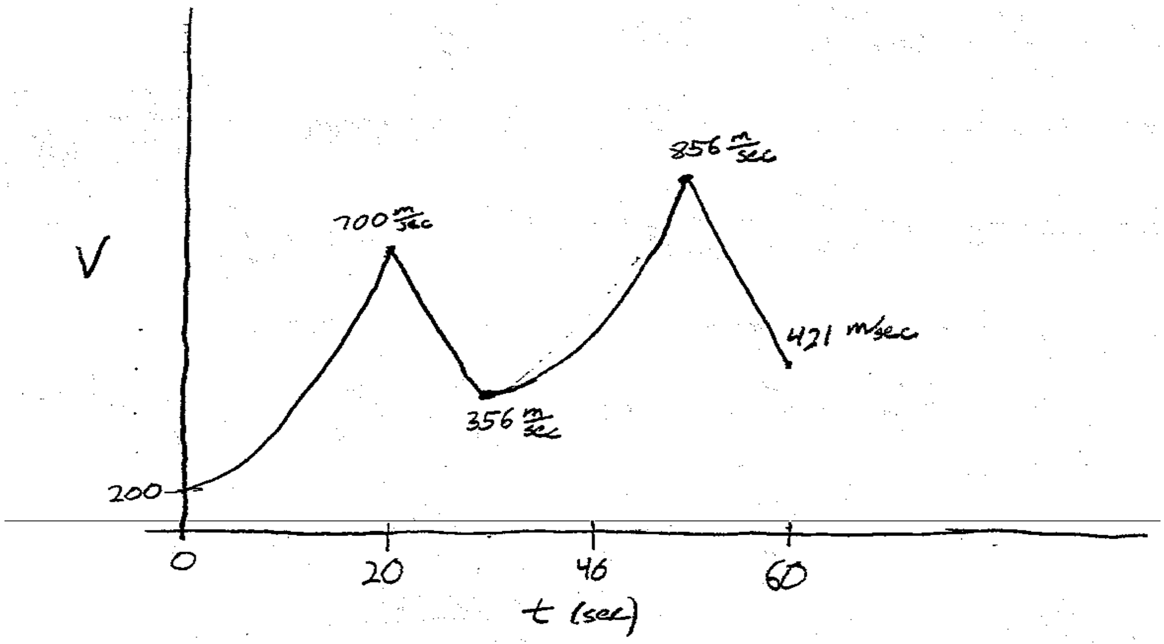
second burn adds $500 \frac{m}{s}$ $\Delta V \Rightarrow V_3 = 855.6 \frac{m}{sec}$ @ $t = 49.84$ sec

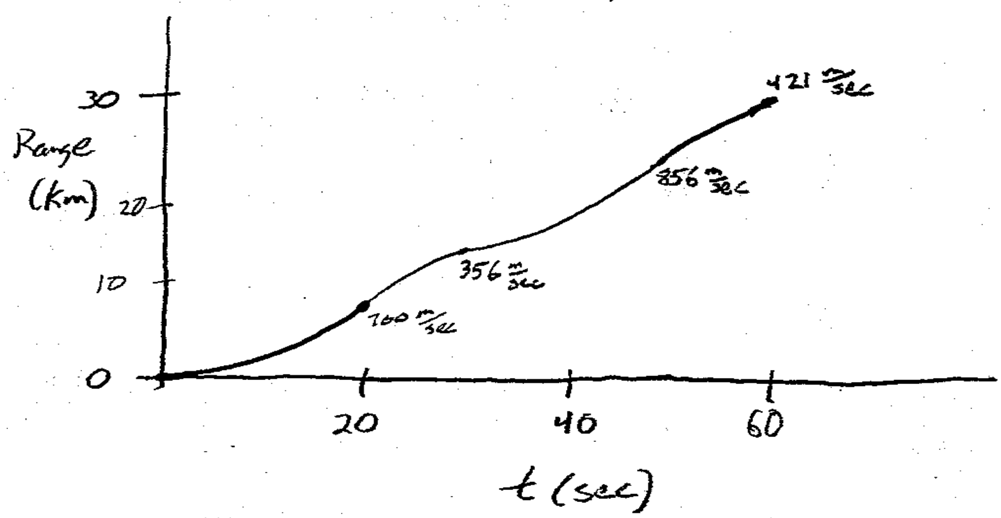
Final coast of $(60-49.84) = 10.16$ sec

$$V_4 = \frac{855.6}{1+.01(10.16)^2} = 421.0 \frac{m}{sec}$$

iii) range ① $= 7.33$ km $\qquad$ range ② $= \dfrac{700}{\sqrt{.01}}\tan^{-1}\left(\sqrt{.01}\,(9.84)\right) = 5.44$ km

range ③ $= 20V_2 + \dfrac{10000}{3} = 10.45$ km $\qquad$ range ④ $= \dfrac{855.6}{\sqrt{.01}}\tan^{-1}\left(\sqrt{.01}\,(10.16)\right) = 6.79$ km





2.8-4 In 1997, the Mars Global Surveyor was captured into a highly elliptic orbit about the panet. Over the next few months, the spacecraft will use the Martian atmosphere as an aerobrake to eventually arrive at a low-altitude circular orbit about the planet. Assuming that the initial ellipic orbit had an apogee of 30,000 Km and a perigee of 50 Km, estimate the Δv saved by using the aerobrake maneuver. i.e. What Δv would have been required if Mars didn't have an atmosphere?

If Mars did not have an atmosphere, the spacecraft would have to burn retrograde at perigee to lower its apogee. Assume that the circular orbit is 50 km in altitude (lacking further information).

mass of mars : 6.417×10^{23} kg

$G = 6.674 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

$\mu_{mars} = GM_{mars} = (6.674 \times 10^{-11})(6.417 \times 10^{23})$

$= 4.283 \times 10^{13}\ m^3 s^{-2}$

$v_{c1} = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{4.283 \times 10^{13}}{(3,390+50) \times 10^3}}$

$v_{c1} = 3,529$ m/s

$\Delta v_1 = v_{c1}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$ $r_1 = 3,440$ km , $r_2 = 33,390$ km

$= (3,529)\left(\sqrt{\frac{2 \times 33,390 \times \cancel{10^3}}{(33,390+3,440) \times \cancel{10^3}}} - 1\right)$

$\boxed{\Delta v_1 = 1,223 \text{ m/s}}$ Δv saved from having to circularize.

2.8-5 Suppose we wish to resupply the new space station using Boeing's Sea Launch Vehicle. Using a Sea Launch, we don't have any inclination changes and we can launch from the equator to minimize Δv.

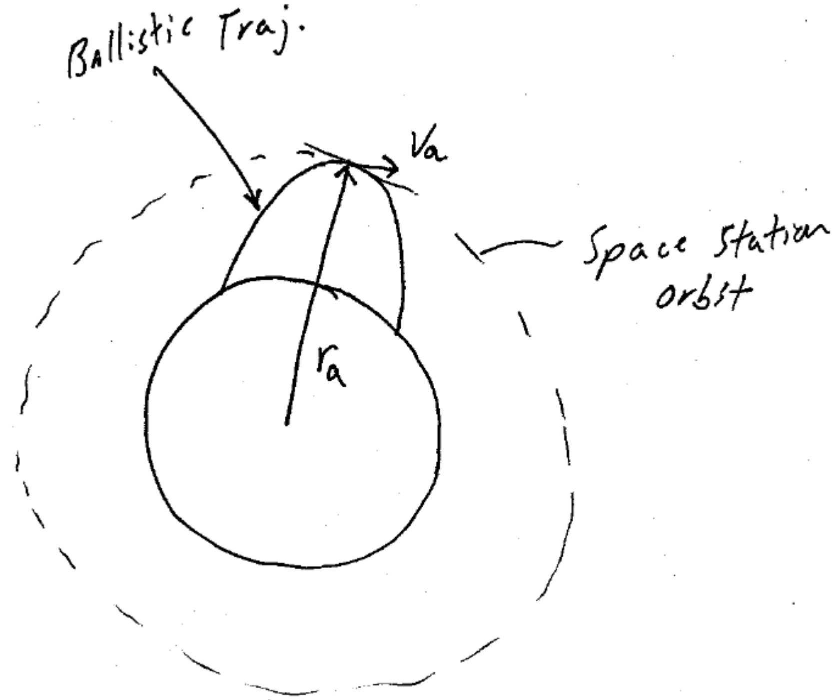
a) Assuming the space station is in a 200 km circular orbit, determine the total ΔV required using a conventional launch vehicle approach.

b) An engineer has an idea as outlined below to resupply the station using a ballistic trajectory in which the payload is deposited on the station as the missile reaches apogee. Assuming a burnout flight path angle of 30° and a burnout altitude of 10 km, determine the required burnout velocity of the missile in this case.

c) How much Δv is saved by using option b)? Is this a good idea? You may wish to caluclate the horizontal velocity at apogee

$$v = \sqrt{2\left(\frac{\mu}{r_a} - \frac{\mu}{r_{bo}}\right) + v_{bo}^2}$$

Is there an optimal flight path angle for the ballistic missile option?



a) $r_c = 6378 + 200 = 6578$ km.

$$V_{surf} = r_e \omega = \left(6378 \times 10^3\right)\left(\frac{2\pi}{24 \times 60 \times 60}\right)$$

$$= 463.8 \text{ m/s}$$

$$V_c = \sqrt{\frac{\mu_e}{r_c}} = \sqrt{\frac{3.986 \times 10^{14}}{6578 \times 10^3}}$$

$$V_c = 7,784 \text{ m/s}$$

$$\Delta V = 7,784 - 463.8 = \boxed{7,320 \text{ m/s}}$$

b) $\phi_{bo} = 30°$

$r_{bo} = 6388$ km

$$Alt_{max} = 200 \text{ km} = \frac{r_{bo}}{2 - Q_{bo}}\left[1 + \sqrt{\left(1 + Q_{bo}\left(Q_{bo} - 2\right)\cos^2 \phi_{bo}\right)}\right] - r_e$$

$$Q_{bo} = \left(\frac{V_{bo}}{V_c}\right)^2$$

$$AH^*_{max} = (r_e + AH_{max}) = \frac{r_{bo}}{2 - \frac{V_{bo}^2}{V_c^2}}\left[1 + \sqrt{1 + \cos^2 30^\circ\left(\frac{V_{bo}^4}{V_c^4} - \frac{2V_{bo}^2}{V_c^2}\right)}\right] - r_c$$

$$= \frac{r_{bo} V_c^2}{2V_c^2 - V_{bo}^2}\left[1 + \sqrt{1 + \frac{0.75}{V_c^4}V_{bo}^4 - \frac{1.5}{V_c^2}V_{bo}^2}\right]$$

$$2AH^*_{max}V_c^2 - AH^*_{max}V_{bo}^2 = r_{bo}V_c^2 + r_{bo}V_c^2\sqrt{1 + \frac{0.75}{V_c^4}V_{bo}^4 - \frac{1.5}{V_c^2}V_{bo}^2}$$

$$\left[\left(2AH^*_{max}V_c^2 - r_{bo}V_c^2\right) - AH^*_{max}V_{bo}^2\right]^2 = r_{bo}^2 V_c^4\left(1 + \frac{0.75}{V_c^4}V_{bo}^4 - \frac{1.5}{V_c^2}V_{bo}^2\right)$$

$$AH^{*2}_{max}V_{bo}^4 - 2\left(2AH^*_{max}V_c^2 - r_{bo}V_c^2\right)AH^*_{max}V_{bo}^2 + \left(2AH^*_{max}V_c^2 - r_{bo}V_c^2\right)^2 = r_{bo}^2V_c^4 + 0.75\,r_{bo}^2V_{bo}^4 - 1.5\,r_{bo}^2V_c^2V_{bo}^2$$

$$\left(0.75\,r_{bo}^2 - AH^{*2}_{max}\right)V_{bo}^4 + \left(4AH^{*2}_{max}V_c^2 - 2AH^*_{max}r_{bo}V_c^2 - 1.5\,r_{bo}^2V_c^2\right)V_{bo}^2 + \left[r_{bo}^2V_c^4 - \left(2AH^*_{max}V_c^2 - r_{bo}V_c^2\right)^2\right] = 0$$

solve quadratic equation for V_{bo} : will get 4 solutions.

let $x = V_{bo}^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

close to escape velocity, too high.

$V_{bo} = \sqrt{x} = \pm 11{,}010$ or $\pm 3{,}458$ m/s

solution we're interested in.

$$\boxed{V_{bo} = 3{,}458 \text{ m/s}}$$

c.) amt. of ΔV saved $= 7{,}320 - 3{,}458$

$$= \boxed{3{,}862 \text{ m/s}}$$

$$V_{apogee} = \sqrt{2\left(\frac{\mu}{r_a} - \frac{\mu}{r_{bo}}\right) + V_{bo}^2}\quad , \quad r_a = r_c = 6{,}578 \text{ km}.$$

$$= \sqrt{2\left(\frac{3.986\times10^{14}}{6{,}578\times10^3} - \frac{3.986\times10^{14}}{6{,}388\times10^3}\right) + 3{,}458^2}$$

$$= \boxed{2{,}890 \text{ m/s}}$$

The horizontal velocity of the ballistic vehicle at apogee is much too low for a rendezvous with the ISS.

For optimal flight path angle, assume that we want the ballistic system to work, i.e. max altitude = 200 km

$V_{apogee} = 7{,}784$ m/s

Additionally, assume we want the same burnout altitude, so $r_{bo} = 6388$ km. With these constraints, we can solve for V_{bo}, then Q_{bo}, and finally ϕ_{bo}.

$$V_{apogee}^2 = 2\left(\frac{\mu}{r_a} - \frac{\mu}{r_{bo}}\right) + V_{bo}^2$$

$$V_{bo} = \sqrt{V_{apogee}^2 - 2\left(\frac{\mu}{r_a} - \frac{\mu}{r_{bo}}\right)}$$

$$= 8{,}012 \text{ m/s}$$

$$Q_{bo} = \left(\frac{V_{bo}}{V_c}\right)^2 = 1.06 > 1$$

$$AH^*_{max} = \frac{r_{bo}}{2-Q_{bo}}\left[1 + \sqrt{\left(1 + Q_{bo}(Q_{bo}-2)\cos^2\phi_{bo}\right)}\right]$$

$$\left[\frac{AH^*_{max}(2-Q_{bo})}{r_{bo}} - 1\right]^2 = 1 + Q_{bo}(Q_{bo}-2)\cos^2\phi_{bo}$$

$$\phi_{bo} = \cos^{-1} \pm\sqrt{\frac{\left[\frac{AH_{max}(2-Q_{bo})}{r_{bo}} - 1\right]^2 - 1}{Q_{bo}(Q_{bo}-2)}} = \text{imaginary numbers.}$$

$\rightarrow$ $\boxed{\text{no solution exits for the constraints.}}$

2.8-6 Typical ballistic missiles have a burnout height of 100 miles with a maximum range of 8000 miles. How much extra Δv capability would be required to use the missile as a launch vehicle to orbit payloads in polar orbits (90° inclination) at the 100 mile altitude?

2.8-7 A small interceptor is launched horizontally from an aircraft flying at $M = 0.8$ at an altitude of 40,000 ft. The rocket motor operates over a one second duration after release from the aircraft. The missile velocity history during this time is given by

$$V = V_o\left(1 + 2\sin\left(\pi\, t/2\right)\right) \qquad 0 \le t \le 1$$

where V_o is the aircraft velocity at the time of release. Guidance experts indicate that the missile will have adequate agilty to intercept its moving target as long as its velocity is at least 1000 f/s. Assume that we can neglect drag during the brief boost phase. During the coast pahse, assume the following:

Missile Mass = 300 lb.

Avg. Drag Coefficient = 0.2 (i.e., drag *not* negligible during coast)
Reference Area = 50 in^2

Determine:

a) The range at the end of the boost phase.

b) The total range of the missile.

i) for alt = 40,000 ft, speed of sound, a = 968 ft/s ⇒ $V_0 = aM = 774.4\ \text{ft/s}$

$$\text{range})_{boost} = \int_0^{1\,sec} V(t)\,dt = \int_0^1 V_0\left(1 + 2\sin(\pi t/2)\right)dt$$

$$= V_0\left[\int_0^1 dt + \int_0^1 2\sin(\pi \tfrac{t}{2})\,dt\right]$$

$$= V_0\left\{1\,sec + \left[2\left(\tfrac{-2}{\pi}\right)(\cos(1) - \cos(0))\right]sec\right\}$$

$$= (2.273\,sec)V_0 \Rightarrow \boxed{\text{range}_{bo} = 1760.3\ ft}$$

ii) know range from boost, need range of coast

$V_{bo} = 2{,}323.2$ ft/s

want the "acceleration" of rocket $\frac{dV_r}{dt} = \frac{-1}{2} C_D A_r \frac{\rho_g}{m_r} V_r^2$ where $\rho_g = 5.876\times10^{-4}$ slug/ft³

$F = ma \Rightarrow$ but $\frac{dV_r}{dt} = \frac{dV_r}{dx}\frac{dx}{dt} = \frac{dV_r}{dx}\cancel{V_r} = \frac{-1}{2}C_D A_r \frac{\rho_g}{m_r} V_r^{\cancel{2}}$

rearrange: $\frac{-dV_r}{V_r}\frac{2m_r}{C_D A_r \rho_g} = dx \Rightarrow x = \frac{-2m_r}{C_D A_r \rho_g}\ln\left(\frac{V_f}{V_{bo}}\right)$

$\Rightarrow x = 73$ miles

$$\Rightarrow \boxed{\text{range}_{tot} = 73.24\ \text{miles}}$$

2.8-8 In this problem, consider an orbital transfer vehicle raising an orbit of radius r_1, to a higher orbit of radius r_2 with no plane change. We would like to determine the ΔV penalty (or benefit) in using a spiral trajectory versus a standard Hohmann transfer for this mission. Let ΔV_S and ΔV_H represent the *total* ΔV required for the spiral and Hohmann trnasfers, respectively.

a) Show that the ratio $\Delta V_S/\Delta V_H$ depends only on the radius ratio r_1/r_2 and determine the form of that dependence.

b) If $r_1/r_2 = 0.2$, how much ΔV penalty (benefit) do we incur by employing the spiral trasfer (as compared to Hohmann transfer)

8 a)

$$\Delta V_H = \Delta V_1 + \Delta V_2$$

$$= V_{c1}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right) + V_{c1}\left(\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2\left(1+\frac{r_2}{r_1}\right)}}\right)$$

$$\Delta V_S = \sqrt{V_{c1}^2 - 2V_{c1}V_{c2} + V_{c2}^2}$$

$$= \sqrt{(V_{c1} - V_{c2})^2}$$

$$= V_{c1} - V_{c2}$$

$$= V_{c1}\left(1 - \frac{V_{c2}}{V_{c1}}\right)$$

$$= V_{c1}\left(1 - \sqrt{\frac{r_1}{r_2}}\right)$$

$$\frac{\Delta V_S}{\Delta V_H} = \frac{\cancel{V_{c1}}\left(1 - \sqrt{\frac{r_1}{r_2}}\right)}{\cancel{V_{c1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right) + \cancel{V_{c1}}\left(\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2\left(1+\frac{r_2}{r_1}\right)}}\right)}$$

$$= \frac{1 - \sqrt{\frac{r_1}{r_2}}}{\sqrt{\frac{2r_2}{r_1\left(1+\frac{r_2}{r_1}\right)}} - 1 + \sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2\left(1+\frac{r_2}{r_1}\right)}}}$$

$$\boxed{\frac{\Delta V_S}{\Delta V_H} = \frac{1 - \sqrt{\frac{r_1}{r_2}}}{\sqrt{\frac{2}{1+\frac{r_2}{r_1}}}\left(\sqrt{\frac{r_2}{r_1}} - \sqrt{\frac{r_1}{r_2}}\right) + \sqrt{\frac{r_1}{r_2}} - 1}}$$

All in terms of $\frac{r_1}{r_2}$ or $\frac{r_2}{r_1}$

b. let $x = \frac{r_1}{r_2}$

$$\frac{\Delta V_S}{\Delta V_H} = \frac{1 - \sqrt{x}}{\sqrt{\frac{2}{1+\frac{1}{x}}}\left(\sqrt{\frac{1}{x}} - \sqrt{x}\right) + \sqrt{x} - 1}$$

when $x = 0.2$,

$$\boxed{\frac{\Delta V_S}{\Delta V_H} = 1.15}$$

ΔV penalty of 15%

2.8-9 In the Apollo lunar missions, the lunar lander had to rendezvous with the command module which was orbiting the moon at an altitude of 80 nmi (1 nmi = 6076 ft). Estimate the Δv required to accomplish this mission assuming that no plane change is required and that gravity losses amount to 20% of the ideal velocity increment.

1. $V_c = \sqrt{\mu/r}$ $\quad r = 938 + 80$ nmi $\quad \mu = 1.73 \times 10^{14}\ f^3/s^2$

$= 6,185,368$ ft

$\therefore V_c = 5288$ f/s

for moon, $V_{surf} = 0$ (Moon doesn't Rotate WRT earth)

So

$\Delta V_{id} = V_c - V_{surf} = 5288$ f/s 15

$\Delta V_{g-t} = 0.2\ 5288 = 1058$ f/s 5

$\Delta V_{tot} = \Delta V_{id} + \Delta V_{g-t} = 6346$ f/s

2.8-10 A ballistic missile has a maximum range of 3000 km when launched on the earth. If we use an identical missile on Jupiter, what will it's maximum range be? You may assume that the burning time of the missile is so short that it effectively burns out at the surface of the planet (i.e., zero altitude).

2 $\psi_e = 3000/r_e = \frac{3000}{6378} = 0.47$ rad.

$$Q_{bo_e} = \frac{2\sin\psi_e/2}{1+\sin\psi_e/2} = \boxed{0.378}$$

$$V_{bo_e} = \sqrt{\mu_e Q_{bo_e}/r_{bo_e}} \approx \sqrt{\frac{\mu Q_{bo_e}}{r_e}} = \sqrt{\frac{3.986\times10^5\ 0.378}{6378}} = \boxed{4.86\ \text{Km/s}}$$

Since t_s is small - Neglect g-t loss so $V_{bo_e} = V_{bo_j}$

then

$$Q_{bo_j} = \frac{V_{bo_j}^2 r_j}{\mu_j} = \frac{4.86^2\ 71380}{1265\times10^5} = \boxed{0.01333}$$

max range eq gives

$$Q_{bo_j}(1+\underbrace{\sin\frac{\psi_j}{2}}_{x}) = 2x \qquad x(2-Q) = Q$$

$$\sin\frac{\psi_j}{2} = \frac{Q_{bo_j}}{2-Q_{bo_j}} \qquad \psi_j = 2\sin^{-1}\frac{Q_{bo_j}}{2-Q_{bo_j}} = 0.0134$$

$$R_j = r_j\psi_j = 71380\ 0.0134 = \underline{958}\ \text{Km}$$

2.8-11 A ballistic missile is launched so as to maintain a 70°angle with respect to the local horizon during the burn. The acceleration history for the vehicle is given by

$$a = 10 + 0.8t \qquad 0 \le t \le 50\ \text{sec}$$

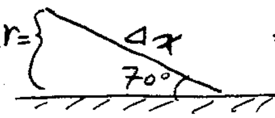
where a is in m/s^2. Determine the range of this missile.

$\phi_{bo} = 70°$ $\quad a = \frac{dV}{dt} = \frac{d^2x}{dt^2}$

$\Rightarrow \Delta V = \int_0^{50} (10 + 0.8t)\,dt = 1500 \frac{m}{s} = V_{bo}$

$\Delta x = \iint a\, dt^2 = \int_0^{50} (10t + 0.4t^2)\,dt = 29{,}166.7\ m$

$r_{bo} = r_e + \Delta r$ where $\Delta r =$



$\Rightarrow \Delta r = \Delta x \sin 70°$

$r_e = 6378\ Km$

$\Rightarrow r_{bo} = 6405407.7\ m \Rightarrow Q_{bo} = \frac{u_{bo}^2 r_{bo}}{\mu} = 0.036157$

$$\cos\left(\frac{\psi}{2}\right) = \frac{1 - Q_{bo}\cos^2\phi_{bo}}{\sqrt{1 + Q_{bo}(Q_{bo} - 2)\cos^2\phi_{bo}}}$$

solving for ψ we get $\psi = 1.337°$

$\text{Range} = r_e \psi^{\text{in radians}} = (6378 km)(0.023339) \Rightarrow \boxed{\text{Range} = 148.86\ Km}$

2.8-12 The U.S. launches the bulk of its satellites from Cape Kennedy (inclination 28.5°) while the French launch the Ariane from New Guinea (inclination 3°).

a) Determine the advantage the French have in terms of lower ideal velocity requirements if an 85 nmi equatorial orbit is desired.

b) How do the two launch sites compare for a 90 nmi orbit which passes over the center of the continental U.S. (inclination 35°).

Find surface velocities: $V_{surf} = \frac{2\pi r_e \cos\theta}{1\,day}$

$\Rightarrow V_{surf}$ @ New Guinea $= 0.463 \frac{Km}{s} = 1520 \frac{ft}{s}$

@ Cape Kennedy $= 0.408 \frac{Km}{s} = 1340 \frac{ft}{s}$

85_{nmi} Equatorial orbit $V_c = \sqrt{\frac{\mu}{r_e + 85_{nmi}}} = 25{,}620 \frac{ft}{s}$

$\Delta V = V_c - V_{surf} + \Delta V_{plane\ change}$ where $\Delta V_{pc} = 2V_c \sin\left(\frac{\phi}{2}\right)$ where here ϕ = latitude of launch site

$\Rightarrow$ for New Guinea $\Delta V = 25{,}620 - 1520 + 2(25{,}620)\sin(1.5°)$

$= 25{,}440\ ft/s$

for Kennedy $\Delta V = 25{,}620 - 1340 + 2(25{,}620)\sin(14.25°) = 36{,}890\ ft/s$

$\Rightarrow$ French have a $\underline{\Delta V = 11{,}450\ ft/s}$ advantage!

b) for 90 nmi orbit $V_c = 25{,}597\ \frac{ft}{s}$

now $\phi = 35^\circ$ – initial launch latitude

$\Rightarrow$ for New Guinea $\Delta V = 25{,}597\ \frac{ft}{s} - 1520\ \frac{ft}{s} + (25{,}597)(2)\sin(16^\circ)$

$= 38{,}188\ \frac{ft}{s}$

for Kennedy $\Delta V = 25{,}597\ \frac{ft}{s} - 1340\ \frac{ft}{s} + (25{,}597)(2)\sin(3.25^\circ)$

$= 27{,}159\ \frac{ft}{s}$

2.8-13 Compare the ideal ΔV between a Hohmann and spirla transfer from LEO to GEO. Assume the LEO orbit is circular at 150 km altitude for your calculation.

$LEO = 150\ Km = z_1$ $\quad GEO = 35{,}855\ Km = z_2$ $\quad \Rightarrow r_1 = r_e + z_1 = 6528\ Km$

$r_2 = 42{,}233\ Km$

Hohmann
$$\Delta V = V_{c_1}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1 + \sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2\left(1+\frac{r_2}{r_1}\right)}}\right]$$

@ $z_1 = 150\ Km$ $\quad V_{c_1} = 7.814\ \frac{Km}{s}$ $\quad \Rightarrow \boxed{\Delta V = 3953\ \frac{m}{s}}$

Spiral transfer (w/ no plane change)

$$\Delta V = \sqrt{V_{c_1}^2 - 2V_{c_1}V_{c_2} + V_{c_2}^2} = V_{c_1} - V_{c_2}$$ where for $z_2 = 35{,}855\ Km$

$V_{c_2} = 3.072\ \frac{Km}{s}$

$\Rightarrow \boxed{\Delta V = 4{,}742\ \frac{m}{s}}$ $= 20\%$ more than Hohmann

2.8-14 An interceptor has a velocity history:

$$V(t) = at - bt^2 \qquad a, b \text{ constants} \qquad t \le t_b$$

a) If the total mission time is t_m, determine the range of the interceptor, z, in terms of a, b, t_b, and t_m.

b) If $a < 2bt_m$, determine the burn time which maximizes the range.

c) What is the mazimized range, Z_{max}, corresponding to the optimized burn time derived in Part (b) above.

a) $z = \text{range} = \int_0^{t_b} V(t)\,dt + \Delta V (t_m - t_b)$ where $\Delta V = V(t_b)$ by definition

$$= \int_0^{t_b} (at - bt^2)\,dt + (at_b - bt_b^2)(t_m - t_b)$$

$$= \tfrac{1}{2}at_b^2 - \tfrac{1}{3}bt_b^3 + at_m t_b - at_b^2 - bt_m t_b^2 + bt_b^3$$

$$\Rightarrow \boxed{z = \tfrac{2}{3}b\,t_b^3 - \left(\tfrac{1}{2}a + bt_m\right)t_b^2 + at_m t_b}$$

T[cont d]

b) t_b for max range at $\frac{dz}{dt_b} = 0$

$$\frac{dz}{dt_b} = 2b\,t_b^2 - (a + 2bt_m)t_b + at_m = 0$$

$$t_b = \frac{(a+2bt_m) \pm \left[(a+2bt_m)^2 - (4)(2b)(at_m)\right]^{\frac{1}{2}}}{4b}$$

$$= \frac{(a+2bt_m) \pm (a - 2bt_m)}{4b} \Rightarrow t_b = \frac{a}{2b},\ t_m$$

one is max range, one is min. range

$$\frac{d^2z}{dt_b^2} = 4bt_b - (a + 2bt_m);\ \text{for } t_b = \frac{a}{2b} \quad \frac{d^2z}{dt_b^2} = (a - 2bt_m) \ \text{negative for } a < 2bt_m$$

$$\text{for } t_b = t_m \quad \frac{d^2z}{dt_b^2} = (2bt_m - a) \ \text{positive for } a < 2bt_m$$

(−) indicates max t_b $\Rightarrow \boxed{t_b)_{max} = \frac{a}{2b}}$

c) $$z_{max} = z\left(\frac{a}{2b}\right) = \frac{2}{3}b\left(\frac{a}{2b}\right)^3 - \left(\frac{1}{2}a + bt_m\right)\left(\frac{a}{2b}\right)^2 + at_m\left(\frac{a}{2b}\right)$$

$$= \frac{a^3}{12b^2} - \frac{a^3}{8b^2} - \frac{a^2 t_m}{4b} + \frac{a^2 t_m}{2b}$$

$$= \frac{-a^3}{24b^2} + \frac{a^2 t_m}{4b} = \underline{\underline{\left(\frac{a}{2b}\right)^2\left[bt_m - \frac{a}{6}\right]}}$$

2.8-15 As many of you know, airbreathing engines (scramjets, etc.) have been proposed as a means to develop a singe stage to orbit (SSTO) vehicle. The airbreather operates over a portion of the trajectory at which point the vehicle transitions to a rocket-propelled mode. The other option is a pure rocket vehicle. Assume a Scramjet engine can accelerate the vehicle to Mach 15 at 150,000 feet, and that a 90 nmi circular orbit is desired (no plane change). What fraction of the ideal energy is constant? Suppose the mass varies with time (as it really does). How would this fraction be effected?

@ $z = 90\,nmi$ $\quad E = KE + PE = \frac{1}{2} m V_c^2 + mgh$ $\quad$ where $V_c = 25{,}597 \frac{ft}{s}$ (from 2b)

$\Rightarrow E = \frac{1}{2} m (25{,}597 \frac{ft}{s})^2 + m (32.174 \frac{ft}{s^2})(546{,}850\,ft)$ $\quad$ (note: $g \cong$ constant is an ok assumption)

$= 3.452 m \times 10^8 \frac{ft^2}{s^2}$

@ $h = 150\,Kft$ $\quad M = 15$ $\quad a = 1072.97 \frac{ft}{s}$ (tables) $\Rightarrow V = 16{,}094.6 \frac{ft}{s}$

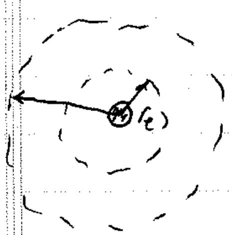
$\Rightarrow E = \frac{1}{2} m (16{,}094.6 \frac{ft}{s})^2 + m (32.174 \frac{ft}{s^2})(150{,}000\,ft)$

$= 1.343 m \times 10^8 \frac{ft^2}{s^2}$

$\Rightarrow$ Energy input by scramjet $= \underline{38.9\%\ E_{total}}$

for variable mass this fraction or percent would be higher.

2.8-16 What Δv is required to travel from Earth's orbit (around the sun) to Mars' orbit? The radii from the su to earth and mars are 1.5×10^8 km and 2.28×10^8 km, respectively.



$1.325\times10^{11}\ km^3/s^2$

$\mu = 47{,}900\ f^3/s^2$ $\quad r_1 = r_e = 1.496\times10^8\ km$

$r_2 = r_m = 2.279\times10^8\ km$

$v_{c_1} = \sqrt{\frac{\mu}{r_1}} = 30\ km/s$

$v_{c_2} = \sqrt{\mu/r_2}$

$\Delta v_1 = v_{c_1} \left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1 \right] = 30\ [0.098] = 2.95\ km/s$

$\Delta v_2 = v_{c_1} \left[\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2(1+r_2/r_1)}} \right] = 30 \left[0.8111 - 0.722 \right] = 2.65\ km/s$

$\Delta v = \Delta v_1 + \Delta v_2 = 5.6\ km/s$

2.8-17 Determine the burnout flight path angle and burnout velocity for a ballistic missile with a maximum range of 5000 miles. Plot your results as a function of the height at burnout (h_{bo}) for 10 miles < h_{bo} < 100 miles.

$$\psi = \frac{range}{r_e} = \frac{5000 mi}{(6378/1.6) mi} = 1.25 \text{ rad}$$

$$Q_{bo} = \left(\frac{V_{bo}}{V_c}\right)^2 , \quad V_c = \sqrt{\frac{\mu}{r_e + h_{bo}}}$$

$$Q_{bo} = \frac{2 \sin(\psi/2)}{1 + \sin(\psi/2)} = 0.7396$$ max. range condition

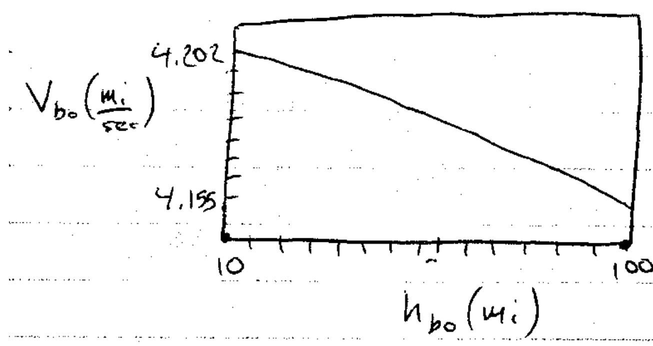
$$\sin\left(2\phi_{bo} + \psi/2\right) = \frac{2 - Q_{bo}}{Q_{bo}} \sin(\psi/2) \Rightarrow$$

$$\sin\left(2\phi_{bo} + \frac{\psi}{2}\right) = 1 \Rightarrow$$

$$\boxed{\phi_{bo} = 0.472 \text{ rad}}$$

since $Q_{bo} = 0.7396 = \dfrac{V_{bo}^2}{\frac{\mu}{r_e + h_{bo}}} \Rightarrow$

$$\boxed{V_{bo} = \left(\frac{0.7396}{r_e + h_{bo}}\right)^{1/2}}$$ which looks like



2.8-18 Estimate the Δv required to reach a 100 nmi polar orbit assuming southerly launch from Vanderberg AFB.

Southerly launch, polar orbit

$$\boxed{\Delta V = V_c = 25562 \text{ ft/s}}$$

2.8-19 A very important orbit utilized by many satellites has a period of 24 hours. This orbit, called a Geostationary Earth Orbit (GEO) is such that a satellite remains fixed with respect to a point on the ground (obviously desirable for communications).

a) Find the altitude of this orbit in feet, miles, and kilometers.
b) What ideal ΔV would be required for a launch vehicle to deliver a payload to this altitude.

) Period of orbit = 24 hrs = 86,400 sec.

r_{earth} = 3961 miles

i) $\tau = \frac{2\pi (r_e + z)^{3/2}}{\mu^{1/2}} \Rightarrow z = 1.18 \times 10^8$ ft, or $\boxed{= 22,300 \text{ miles} \quad 36,200 \text{ Km.}}$

ii) $U_{\substack{surface \\ @equator}} = \frac{2\pi (20.9 \times 10^6 \text{ft})}{86400 \text{ sec}} = 1520$ ft/s

$U_{GEO} = \left(\frac{\mu_{earth}}{R_{GEO}}\right)^{1/2} = 10,100$ ft/sec

So ideal $\Delta V \big|_{\substack{sea \\ level @ equator}} = U_{GEO} - U_{surf} = 8590$ ft/sec

2.8-20 Prove that the ideal ΔV required to travel from earth to the moon and return to earth is 88,900 f/s. What is the ideal value assuming we can use the earth's atmosphere as an "aerobrake" during the descent pahse in the return to earth?

a) ΔV_i = 88900 ft/s $\qquad \frac{1}{2} mV^2 = mgh$

$r_e = 20.9 \times 10^6$ ft $\qquad r_{moon} = 5.7 \times 10^6$ ft $\qquad \frac{1}{2} mV^2 = mgh$

so $g_{moon} = \frac{\mu_{moon}}{r^2_{moon}} = 5.32$ ft/sec² $\qquad = \frac{V^2}{2} = gr \Rightarrow V = \sqrt{2gr}$

$U_{\infty earth} = [2 g_e r_e]^{1/2} = 36672.51$ ft/sec

$U_{\infty moon} = [2 g_{moon} r_{moon}]^{1/2} = 7787.68$ ft/sec

$\Delta V_{Ideal} = 2 [U_{\infty earth} + U_{\infty moon}] = \boxed{88920.39 \text{ ft/sec}}$

b) $\Delta V_{Ideal}\big|_{\substack{aero \\ brake}} = U_{\infty earth} + U_{\infty moon} + U_{\infty moon} = \boxed{52,247.87 \text{ ft/s}}$

The excess energy is used up in creating heat!

2.8-21 The Titan Launch vehicle boosts on IUS and payload to a 85×90 nmi equatorial parking orbit. Assuming the first stage of the IUS fires from the apogee of this orbit determine:

a) The Δv required by the IUS first and second stage burns required to place the satellite in GEO orbit.

b) The ideal Δv input by Titan to attain the parking orbit including contributions for the plane change to equatorial orbit.

$\mu = 1.407 \times 10^{16}$ ft³/sec²

$r_e = 3444$ nmi $= 20.9 \times 10^6$ ft.

$$\Delta v_1 = V_{c_1}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right] \qquad V_{c_1} = \sqrt{\frac{\mu}{r_1}} = 25600 \ \frac{ft}{sec}.$$

$r_1 = r_e + 90$ nmi $= 21.46 \times 10^6$ ft.

$r_2 = r_{GEO} + r_e$

Since $\tau = 24$ hours for GEO, & $\tau = \frac{2\pi (r_e + r_{GEO})^{\frac{3}{2}}}{\mu^{1/2}}$

we have $r_{GEO} = 1.18 \times 10^8$ ft. $\therefore r_2 = 1.389 \times 10^8$ ft.

So,
$$\Delta v_1 = \left[\sqrt{\frac{2(1.389\times10^8)}{(21.46\times10^6)+(1.389\times10^8)}} - 1\right] 25600 \ \frac{ft}{sec}$$
$$= 8095 \text{ ft/sec.}$$

$$\Delta V_2 = V_{c_1}\left[\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2\left(1+\frac{r_2}{r_1}\right)}}\right]$$

So,
$$\Delta V_2 = 25600 \frac{ft}{sec}\left[\sqrt{\frac{21.45\times10^6}{1.389\times10^8}} - \sqrt{\frac{2(21.45\times10^6)}{(1.389\times10^8)\left(1+\frac{1.389\times10^8}{21.45\times10^6}\right)}}\right]$$
$$= 4857 \text{ ft sec.}$$

Assume launch from Cape Kennedy, 28.5° LATITUDE, and plane change over the equator.

then:
$$\Delta V = V_C - V_{SURF} + \Delta V_{\substack{PLANE\\CHANGE}}$$

where $\Delta V_{\substack{PLANE\\CHANGE}} = 2V_C \sin\left(\frac{\Phi}{2}\right)$ where $\frac{\Phi}{2} = 14.25$ for this case.

V_{SURF} at cape $= 1320$ ft/sec.

So,
$$\Delta V = \left\{25600 - 1320 + 2(25600)\sin(14.25^\circ)\right\} \frac{ft}{sec}$$
$$= 36883 \text{ ft/sec}$$

2.8-22 Suppose we wish to replace the IUS with an electric OTV. What Δv would be required by this vehicle assuming the same initial and final orbits?

FOR AN ELECTRIC OTV, WE ASSUME A SPIRAL TRAJECTORY AND A ΔV OF THE FORM:

$$\Delta V = \left[V_{C_1}^2 - 2V_{C_1}V_{C_2} + V_{C_2}^2 \right]^{1/2}$$

$V_{C_1} = 25600$ ft/sec

$$V_{C_2} = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{1.407\times10^{16}\ \text{ft}^3/\text{sec}^2}{1.389\times10^8\ \text{ft}}} = 10{,}065\ \frac{\text{ft}}{\text{sec}}.$$

$\therefore$

$$\Delta V = \left[(25600)^2 - 2(25600)(10065) + (10065)^2 \right]$$
$$= 15540\ \text{ft/sec}.$$

2.8-23 What ideal ΔV is required to travel from a LEO of 200 km to Mars and back? Include ΔV budget to actually land on the Martian surface.

$\mu_{mars} = 0.438\times10^5\ \text{km}^3/\text{sec}^2$ $\qquad g_{mars} = \frac{\mu_M}{r_M^2} = 3.81\ \frac{\text{m}}{\text{sec}^2}$

$r_{mars} = 3390$ km.

$$V_{\infty_{earth}} = \left[2 g_e r_e\right]^{\frac{1}{2}} = 11\,180\ \text{m/sec}$$

$$V_{\infty_{mars}} = \left[2 g_e r_e\right]^{\frac{1}{2}} = 5080\ \text{m/sec}.$$

So,

$$\Delta V_{TOTAL} = 2\left\{\left[V_{\infty_{earth}} - V_{C_{LEO}}\right] + V_{\infty_{mars}}\right\}$$
$$= 2\left[(11180 - 7802) + 5080\right]$$
$$= 16900\ \text{m/sec}.$$

2.8-24 An interceptor is required to travel a distance Z in a mission time, t_m. Because the rocket motor on the interceptor has a thrust which decreases with time, the velocity of the missile during rocket operation can be expressed:

$$v(t) = K_1 t^{0.5} + K_2 \qquad 0 < t < t_b$$

where K_1 and K_2 are constants and t_b is the motor burn time.

a) Derive and expression for Z in terms of K_1, K_2, t_m, and t_b.

b) If the missile is intially at rest, determine vales for K_1 and K_2 in terms of Z, t_m, and t_b.

c) Suppose $K_1 = 30\ \text{f/s}^{0.5}$, $K_2 = 100\ \text{f/s}$, $t_m = 30\ \text{sec}$, and $t_b = 9\ \text{sec}$. Determine the range Z at $t = t_m$ and the portion of this range attained during the coasting period.

$v(t) = K_1 t^{1/2} + K_2 \qquad 0 < t < t_b$

a) $Z = \int_0^{t_b} v(t)\,dt + \Delta V (t_m - t_b) \qquad \Delta V = v(t_b) = K_1 t_b^{1/2} + K_2$

→ sub in + integrate

$$\boxed{Z = \frac{2}{3} K_1 t_b^{3/2} + K_2 t_b + (K_1 t_b^{1/2} + K_2)(t_m - t_b)}$$

b) $v(0) = 0 \qquad 0 = \cancel{K_1}(0)^0 + K_2 \quad$ so $\quad K_2 = 0$

so: $Z = \frac{2}{3} K_1 t_b^{3/2} + K_1 t_b^{1/2} (t_m - t_b)$

$$\boxed{K_1 = \frac{Z}{\frac{2}{3} t_b^{3/2} + t_b^{1/2} (t_m - t_b)} \quad + \quad K_2 = 0}$$

c) $Z = \frac{2}{3} K_1 t_b^{3/2} + K_2 t_b + (K_1 t_b^{1/2} + K_2)(t_m - t_b)$

$K_1 = 30 \qquad K_2 = 100 \qquad t_m = 30 \qquad t_b = 9$

$$\boxed{Z(t_m) = 5430 \text{ ft}}$$

$Z_c = \Delta V (t_m - t_b)$

$$\boxed{Z_c = 3990 \text{ ft}}$$

coasting is 73% of the flight distance

2.8-25 An ambitious student wishes to launch a rocket into low Earth orbit from his/her backyard. Nearby power lines inhibit an easterly launch, so the student elects to launch in a westerly direction. The Isp of the vehicle is 300 sec. Assuming the launch site is at a latitude of 35 degrees, and a 200 km circular orbit is desired, what ideal velocity increment would be required to complete this mission.

$V_{surf} = \dfrac{2\pi r_e \cos 35°}{1\,\text{day}} = 380 \text{ m/s}$

$V_c = \sqrt{\dfrac{\mu}{r_e + z}} = 7784 \text{ m/s}$

10

$\Delta V_{id} = V_c + V_{surf} = 8164 \text{ m/s}$ 10

westerly

2.8-26 We would like to modify our analysis of mission requirements for Earth escapes to take into account dissipation due to atmospheric drag. Assume we can write the average drag during the ascent as

$$\overline{D} = c_D \frac{\overline{\rho}}{2}\left(\frac{u_z}{2}\right)^2 A$$

$\overline{D}$ = Average drag force

c_D = Drag coefficient

$\overline{\rho}$ = Average atmospheric density during ascent

$\frac{u_z}{2}$ = Average velocity during ascent

A = Vehicle cross-sectional area

a) Using this expression, derive a relation (analogous to 4.17) for the impulsive velocity to attain a height $Z(u_z)$ in terms of the parameters above, the body mass (m) and known characteristics of planet Earth.

b) How much additional impulsive velocity (above the case with no drag) will be required if $C_D \overline{\rho} A/m = 1\times10^{-5}\text{m}^{-1}$ and z = 100 km?

Solve through energy balance: PE + KE = constant

However, PE needs drag term. Can turn drag force into energy loss by multiplying by distance (height) $E = Fd$

a)
$$\frac{m g_e r_e Z}{r_e + Z} + \underbrace{c_D \frac{\overline{\rho}}{2}\left(\frac{u_z}{2}\right)^2 A}_{\overline{D}} Z = \frac{m}{2} V_z^2$$

$$\frac{m g_e r_e Z}{r_e + Z} = V_z^2\left(\frac{m}{2} - C_D \frac{\overline{\rho}}{8} A Z\right)$$

$$V_z = \sqrt{\frac{m g_e r_e Z}{\left(\frac{m}{2} - \frac{C_D \overline{\rho} A Z}{8}\right)(r_e + Z)}} = \boxed{\sqrt{\frac{g_e r_e Z}{\left(\frac{1}{2} - \frac{C_D \overline{\rho} A Z}{8m}\right)(r_e + Z)}}}$$

b) $\frac{C_D \overline{\rho} A}{m} = 1\cdot10^{-5}$ $Z = 100$ km

$$(V_z)_{\text{NO DRAG}} = \sqrt{\frac{2(9.81\ \frac{m}{s^2})(6378\text{ km})(100\text{ km})}{6378\text{ km} + 100\text{ km}}} = \underline{1{,}389.86\ \frac{m}{s}}$$

$$(V_z)_{\text{DRAG}} = \sqrt{\frac{(9.81\ \frac{m}{s^2})(6378\text{ km})(100\text{ km})}{\left(\frac{1}{2} - \frac{(1\cdot10^{-5})(100\text{ km})}{8}\right)(6378\text{ km} + 100\text{ km})}} = \underline{1{,}604.87\ \frac{m}{s}}$$

$\underline{\Delta V_z = 215\ \frac{m}{s}}$ 15.5% increase

2.8-27 What ΔV is required to escape Jupiter's gravitational field assuming one starts on the surface of the planet and drag is neglected?

$\mu_j = 447 \times 10^{14}\ f^3/s^2 = G\, m_j \qquad r_j = 71380\ Km\ \frac{3281\ f}{Km} = 2.34 \times 10^8\ f$

$= 1265 \times 10^5\ Km^3/s^2$

$V_\infty = \sqrt{2\, g_j\, r_j} \qquad But\ \ g_j\, r_j = \mu_j / r_j$

$V_\infty = \sqrt{2\mu_j / r_j} = \sqrt{2\ 1265 \times 10^5 / 0.7138 \times 10^5} = \underline{59.5\ Km/s}$

2.8-28 The initial portion of the return trajectory for a future Mars probe would involve a launch from the surface of Mars to a temporary circular orbit 150 mi above the planet's surface. Assuming the probe is departing from the equator, estimate the ΔV required to accomplish this portion of the mission. Hint: The length of a Martian day is 97.5% that of a day on Earth.

$V_c = \sqrt{\mu)_{mars} / r)_{mars}} = \left[\frac{0.1515 \times 10^{14}}{(1830\,(1.15) + 150)\ 5280}\right]^{1/2} = 11{,}280\ f/s$

~~130~~ Km/s

3.5

$V_{surf} = \frac{2\pi\ 1830\ 1.15\ 5280}{24\ 3600\ 0.975} = 830\ f/s$

$0.258\ Km/s$

$\Delta V = V_c - V_{surf} = 10{,}450\ f/s$

2.8-29 The "g-t" loss for a typical launch from Earth is roughly 5000 f/s. Can you estimate the loss for a launch from the surface of Venus? Carefully state all assumptions required to make this estimate.

$g_{venus} / g_{earth} = \mu_{venus} / \mu_{earth} = \frac{1.145}{1.407} = 0.81$

$\therefore\ g\text{-}t\ loss)_{venus} = 0.81\ (5000) = 4070\ f/s$

Assumes similar traj., t_b values

2.8-30 What altitude would be attained by a projectile launched vertically at half of earth's escape velocity?

$$\text{ESCAPE VELOCITY} = u_\infty = \sqrt{2 g_e r_e}$$

$$u_z = \sqrt{\frac{2 g_e r_e z}{r_e + z}} \qquad \text{if } u_z = \tfrac{1}{2} u_\infty,$$

$$\tfrac{1}{2}\sqrt{2 g_e r_e} = \sqrt{\frac{2 g_e r_e z}{r_e + z}}$$

$$\left(\frac{1}{2}\right)^2 = \frac{z}{r_e + z}$$

$$r_e + z = 4z$$

$$r_e = 3z \quad \Rightarrow \quad z = \tfrac{1}{3} r_e$$

2.8-31 We have had considerable discussion on the Δv estimates for launch vehicles. The highly simplified result that $\Delta v = v_c - v_{surf}$ (with v_c evaluated at earth's surface) works okay for orbits very near the surface of the earth (neglecting gravity and drag losses, of course). For Kennedy Space Center, this technique gives Δv of 7.5 km/s.

a) How does this estimate compare with a Hohmann transfer from Kennedy Space Center to a 150 km orbit inclined at $28.5°$?

b) Which approach do you prefer and why? Do you have a better suggestion?

4.8-31) Hohmann Transfer

$r_1 = r_e = 6378$ km

$r_2 = r_e + 150$ km $= 6528$ km

$$V_{c1} = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.98 \cdot 10^5}{6378}} = 7.9 \text{ km/s}$$

$$\Delta V_1 = V_{c1}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right] = 7.9\left[\sqrt{\frac{2(6528)}{6378+6528}} - 1\right] = 0.046 \ \frac{\text{km}}{\text{s}}$$

$$\Delta V_2 = V_{c1}\left[\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2(1+\frac{r_2}{r_1})}}\right]$$

$$= 7.9\left[\sqrt{\frac{6378}{6528}} - \sqrt{\frac{2(6378)}{6528(1+\frac{6528}{6378})}}\right] = 0.046 \ \frac{\text{km}}{\text{s}}$$

$$\Delta V = \underbrace{V_{c1}}_{\text{Needed to get to orbital velocity. This term dominates } \Delta V} + \Delta V_1 + \Delta V_2 = 7.9 + .046 + .046 = \boxed{7.99 \ \frac{\text{km}}{\text{s}}}$$

The two figures are very similar.

2.8-32 Typical ballistic missiles have a burnout height of 100 miles with a maximum range of 8000 miles. How much extra Δv capability would be required to use the missile as a launch vehicle to orbit payloads in polar orbits (90° inclination) at the 100 mile altitude?

$$Q_{bo} = \frac{2\sin(\psi/2)}{1+\sin(\psi/2)} \qquad \psi = \frac{\text{Range}}{R_E} = \frac{8000}{4000} = 2 \text{ radians}$$
$$= 114.6^\circ$$

$\therefore Q_{bo} = 0.914$

$$V_c = \sqrt{\mu/r} = \sqrt{\frac{1.407\times10^{16}\ f^3/s^2}{4060\ \text{miles}\ 5280\ f/mi}} = 25,600\ f/s$$

This is also ΔV for a polar orbit since $V_{surf} = V_{ROT}\cos 90^\circ = 0$

now

$$V_{bo} = V_c\sqrt{Q_{bo}} = 25,600\sqrt{0.914} = 24,500\ f/s$$

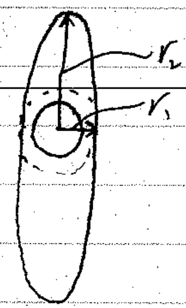
we are only 1100 f/s short of being able to orbit something.... If we consider the launch veh. requirement as:

$$\Delta V = V_c - V_{surf} = V_c - \underbrace{\frac{2\pi r_e}{1\,\text{Day}}\cancel{\cos 90^\circ}}_{\text{Get no help from earth's Rotation for Polar orbit}}$$

now, this isn't quite right since earth rotates while we are flying - could also compute ΔV assuming a plane change to 90° inclination from some init. orbit.

2.8-33 Recently, the Mars Global Surveryor was captured into a highly elliptic orbit about the planet. Over the next few months, the spacecraft will use the Martian atmosphere as an aerobrake to eventually arrive at a low-altitude circular orbit about the planet. Assuming that the initial elliptic orbit had an apogee of 30,000 Km and a perigee of 50 Km, estimate the Δv saved by using the aerobrake maeuver. i.e. What Δv would have been required if Mars didn't have an atmosphere?

For Mars $\mu = 0.438 \times 10^5 \frac{km^3}{s^2}$, $r_m = 3390$ km



If we started at the lower orbit in a circ. traj., How much ΔV would we need to form this elliptic transfer orbit? i.e. $\Delta V = \Delta V_1 = V_{c1}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right]$

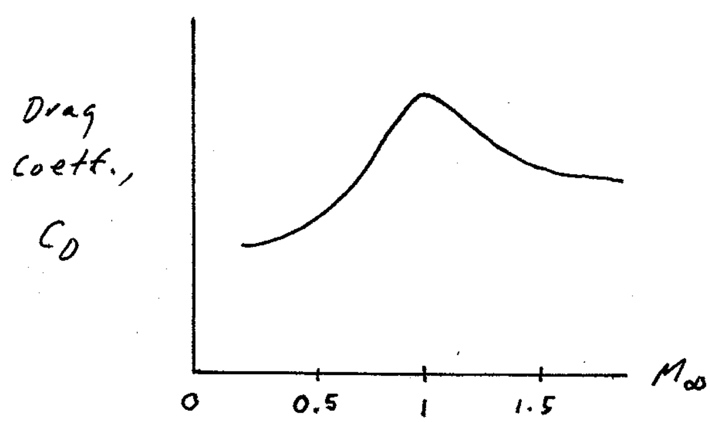
$r_1 = 3390 + 50 = 3440$ km

$r_2 = 30{,}000 + 3390 = 33{,}390$ km

$V_{c1} = \sqrt{\mu/r_1} = 3.57$ km/s

$$\Delta V = 3.57\left[\sqrt{\frac{2(33390)}{33390+3440}} - 1\right] = 0.347\, V_{c1} = 1.24 \text{ km/s}$$

2.8-34 A solid rocket motor is to be utilized for propulsion on an air-launched missile whose drag characteristics are shown in the plot below. Assuming the missile is launched at $M_\infty = 0.9$, discuss the implications of the SRM grain design on the missiles performance. Which grain design should be used (regressive, neutral, or progressive) assuming that each one provides the same total impulse?



Additional Problems

1.)

Determine the burnout flight path angle and burnout velocity for a ballistic missile with a maximum range of 7000 km. Plot your results as a function of the height at burnout (h_{bo}) for 100 km < h_{bo} < 500 km.

3. BALLISTIC MISSILE: RANGE $= r_e \psi = 7000$ km.

since $r_e = 6378$ km, $\psi = 1.0975$ RAD.

$$Q_{bo} = \left(\frac{V_{bo}}{V_c}\right)^2 \quad \text{where} \quad V_c = \sqrt{\frac{\mu}{r_e + h_{bo}}}$$

Maximum Range Condition: $Q_{bo} = \dfrac{2\sin(\psi/2)}{1+\sin(\psi/2)} = 0.686$

flight path angle equation:

$$\sin(2\phi_{bo} + \psi/2) = \frac{2 - Q_{bo}}{Q_{bo}} \sin(\psi/2)$$

Substituting in the maximum range condition, $\sin(2\phi_{bo} + \psi/2) = 1$

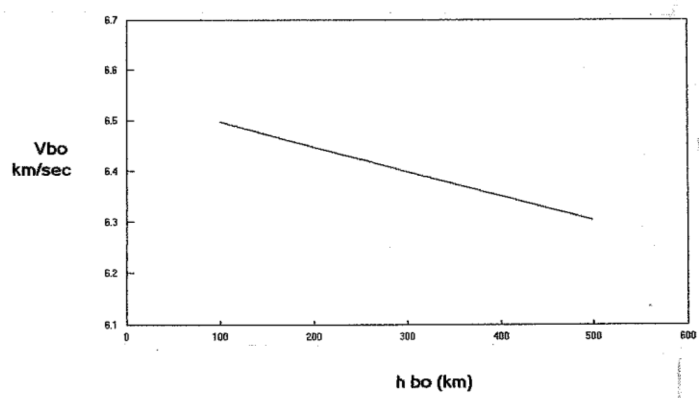
so, $2\phi_{bo} + \frac{\psi}{2} = \frac{\pi}{2} \Rightarrow \phi_{bo} = \left(\frac{\pi}{2} - \frac{\psi}{2}\right)\frac{1}{2} = 0.511$ RAD

since $Q_{bo} = 0.686 = \dfrac{V_{bo}^2}{\left(\frac{\mu}{r_e + h_{bo}}\right)}$

$$V_{bo} = \left(\frac{0.686\,\mu}{r_e + h_{bo}}\right)^{1/2}$$

$\rightarrow$ CHOOSE h_{bo}, find V_{bo}, PLOT RESULTS FOR RANGE 100 km < h_{bo} < 500 km.

h bo (km)	Vbo (km/sec)
100	6.496961
150	6.472032
200	6.447387
250	6.423023
300	6.398932
350	6.37511
400	6.351553
450	6.328255
500	6.305211



2.)

1) You have been directed to decide on Propulsion System Alternatives for a vertically launched sounding rocket with a payload of 150 kg and an empty weight of 1500 kg (excluding the payload). The rocket employs twin liquid engines with a storable propellant combination which provides an average Isp of 280 sec. The payload contains sensitive instruments which cannot withstand acceleration levels in excess of 4 g's.

i) Assuming negligible drag and that the engines are not throttleable, determine the propellant load, engine burn time, and engine thrust level which will maximize the performance (Δv) of this vehicle. What is the Δv achieved under these conditions?

Since the engines are not trottleable, $F = const.$

$F = \dot{m} I_{sp} g \qquad m_p = \dot{m} t_b \qquad m_o = m_p + m_f$

Neglecting drag, we have

$$\Delta V = g I_{sp} \ln\left(\frac{m_o}{m_f}\right) - g t_b$$

$$= g I_{sp} \ln\left(\frac{\dot{m} t_b + m_f}{m_f}\right) - g t_b$$

To find t_b for max ΔV, Set $\frac{d\Delta V}{dt_b} = 0$

$$\frac{d\Delta V}{dt_b} = \frac{g I_{sp} m_f}{\dot{m} t_b + m_f}\left(\frac{\dot{m}}{m_f}\right) - g = 0$$

$$t_b = I_{sp} - \frac{m_f}{\dot{m}}$$

Since $F = \dot{m} I_{sp} g$, $\quad t_b = I_{sp} - \left(\frac{I_{sp} g}{F}\right) m_f$

$$\dot{m} t_b = \dot{m} I_{sp} - m_f$$
$$= \frac{F}{g} - m_f$$

Substituting back into the ΔV equation:

$$\Delta V = g I_{sp} \ln\left[\frac{\left(\frac{F}{g}\right) - m_f + m_f}{m_f}\right] - g\left[I_{sp} - \frac{I_{sp} g}{F} m_f\right]$$

$$\Delta V = g I_{sp}\left[\ln\left(\frac{F}{m_f g}\right) - 1 + \left(\frac{m_f g}{F}\right)\right]$$

We now know ΔV for a given F.

$$a = \underset{\text{in } g\text{'s}}{\text{acceleration}} = \frac{F}{mg} + \overbrace{1}^{\text{with no thrust, 1 gravity.}} \leq 4$$

So, $F \leq 3mg$.

Since F is constant, max a at minimum m.

ie, $F_{max} \leq 3 m_f g = 48510$ N. (total thrust)

A high thrust gives a short t_b for some required ΔV. So we want F as high as possible.

$$\therefore \Delta V = (9.8 \tfrac{m}{sec^2})(280 sec)\left[\ln\left(\frac{48510}{9.8(1650)}\right) - 1 + \frac{(1650)9.8}{48510}\right]$$

$$\frac{kg\,m}{sec^2}\,\frac{sec^2}{m}\,\frac{1}{kg} = \text{non dim.}$$

$\Delta V = 1185.3$ m/sec.

$$t_b = I_{sp} - \frac{I_{sp} g}{F} m_f = 280 - 280\frac{(9.8)(1650)}{48510}\ \frac{sec^2\,m\,kg}{kg\,m\,sec^2}$$

(sec, sec)

$t_b = 186.7$ sec.

$$m_p = \dot{m} t_b = \frac{F t_b}{I_{sp} g} = \frac{(48510)(186.7)}{280\ (9.8)}\ \frac{kg\,m}{sec^2}\,\frac{sec^2\,sec}{sec\,m}$$

$m_p = 3300$ kg.

ii) Being the bright young engineer that you are (must be a Purdue thing), you suggest that the rocket may have additional performance if we are able to shut one engine down at some appropriate point in the flight. Determine the burning times, propellant consumption, thrust level, and Δv achieved during both phases of the flight.

Since we shut down one engine during the flight each engine can have a thrust of

$$F = 48510 \text{ N}.$$

So, at launch, $F = 97020$ N.

Now, one engine must be shut down during the flight when the acceleration reaches the maximum allowable value.

we had $a = \frac{F}{mg} + 1 \leq 4.$

So $m \geq \frac{F}{3g}$ at the end of the first burn.

$$m_{f_1} = \text{mass at the end of burn 1} = \frac{2(48510)\text{N}}{3\ (9.8)\frac{m}{sec^2}} = 3300 \text{ kg}.$$

$\therefore$ during the second phase when only one engine is firing,

$$m_p = 3300 - 1650 = 1650 \text{ kg}.$$

Since the constant massflow equations from part (i) still hold,

$$t_b = I_{sp} - \frac{I_{sp}\, g}{F} m_f$$

$$= 280\text{sec} - \frac{280\text{sec}\ 9.8\frac{m}{sec^2}\ 3300\text{ kg}}{97020\text{ N}}.$$

$$= 186.7 \text{ sec}.$$

$$\Delta V_1 = g I_{sp}\left[\ln\left(\frac{F_1}{m_{f_1} g}\right) - 1 + \left(\frac{m_{f_1} g}{F_1}\right)\right]$$

$$= (9.8)(280)\left[\ln\left(\frac{2(48510)}{2(1650)9.8}\right) - 1 + \left(\frac{2(1650)9.8}{2(48510)}\right)\right]$$

$$= 1185.3 \text{ m/sec}.$$

$$m_{p_1} = \dot{m} t_b = \frac{F}{g} - m_f = \frac{2(48510)}{9.8} - 3300$$

$$= 6600 \text{ kg}.$$

$$\dot{m} = 35.35 \frac{\text{kg}}{\text{sec}} \text{ (2 engines)}$$

$$m_{p2} = 1650 \text{ kg.}, \quad \dot{m} = 17.675 \frac{kg}{sec} \Big\} \quad t_{b2} = 93.35 \text{ sec.}$$

$$\Delta V_2 = g I_{sp} \ln\left(\frac{m_0}{m_f}\right) - g t_b$$
$$= (9.8)(280) \ln\left(\frac{3300}{1650}\right) - 9.8(93.35)$$
$$= 987.2 \text{ m/sec.}$$

In summary:

$t_{b1} = 186.7$ sec $\quad m_{P1} = 6600$ kg
$F_1 = 97020$ N $\quad \Delta V_1 = 1185.3$ m/sec
$t_{b2} = 93.4$ sec $\quad m_{P2} = 1650$ kg
$F_2 = 48510$ N $\quad \Delta V_2 = 987.2$ m/sec.

3.) Refers to problem above

2) **Determine the burnout and apogee heights the sounding rocket in Problem 1 attains for both conditions stated in that problem.**

$$h_{bo} = g I_{sp} t_b \left[\frac{m_f}{m_0 - m_f} \ln\left(\frac{m_f}{m_0}\right) + 1\right] - \frac{g}{2} t_b^2$$

neglect changes in g, constant mass flow.

(i) $m_0 = m_p + m_f = 3300 + 1650 = 4950$ kg.
$t_b = 186.7$ sec

So, $[m/sec^2 \;\; sec \;\; sec]$. $\quad [\frac{m}{sec^2} \;\; sec^2]$

$$h_{bo} = (9.8)(280)(186.7)\left[\frac{1650}{3300} \ln\left(\frac{1650}{4950}\right) + 1\right] - \frac{9.8}{2}(186.7)^2$$

$h_{bo} = 60.1$ km.

$h_c = \frac{1}{2}\frac{u_e^2}{g} = \frac{1}{2}\frac{(1185.3 \text{ m/sec})^2}{(9.8)} = 71.7$ km. $\Big\} h_{TOT} = 131.8$ km

(ii) 2 ENGINES:
$$h_{bo_1} = (9.8)(280)(186.7)\left[\frac{3300}{6600} \ln\left(\frac{3300}{9900}\right) + 1\right] - \frac{9.8}{2}(186.7)^2$$

$h_{bo_1} = 60.1$ km.

note initial velocity $\neq 0$

$$h_{bo_2} = g I_{sp} t_b \left[\frac{m_f}{m_0 - m_f} \ln\left(\frac{m_f}{m_0}\right) + 1\right] - \frac{g}{2} t_b^2 + u_0 t_b$$
$$= (9.8)(280)(93.4)\left[\frac{1650}{1650} \ln\left(\frac{1650}{3300}\right) + 1\right] - \frac{9.8}{2}(93.4)^2 + (1185.3)(93.4)$$
$$= 146.6 \text{ km.}$$

$$h_c = \frac{1}{2}\frac{u_e^2}{g} = \frac{1}{2}\frac{(1185.3 + 987.2)^2}{9.8} = 240.8 \text{ km.}$$

$$h_{TOT} = h_{bo_1} + h_{bo_2} + h_c = 447.5 \text{ km}$$

4.)

On OTV orbits the earth at an altitude of 200 Km (circular orbit). The vehicle utilizes a LRE with an Isp for 450 sec. and has an initial gross mass of 15,000 Kg. The engine is ignited and burns for 300 sec. at a thrust level of 75 KN to initiate the first burn in a Hohmann transfer. Determine:

a) The ΔV imparted by the first burn.

b) The apogee height attained as a result of the first burn.

The engine is reignited to circularize the elliptic orbit formed as a result of the first burn. Assuming another constant thrust firing at 75 KN, determine:

c) The ΔV required to circularize the orbit.

d) The firing duration required to attain this ΔV.

1)

$$V_{c_1} = \sqrt{\mu / r_1} = \sqrt{\frac{3.986 \times 10^5}{6578}} = 7.78 \text{ km/s} = 7780 \text{ m/s}$$

$$M_p = F t_b / (g I_{sp}) = 75{,}000 (300) / ((9.81 (450)) = 5097 \text{ Kg}$$

5 a) $\Delta V_1 = \Delta V = g I_{sp} \ln MR = 9.81 \; 450 \ln \frac{15000}{15000-5097} = \underline{\underline{1830 \text{ m/s}}}$

Hohmann xfer $\quad \frac{\Delta V_1}{V_{c_1}} = \sqrt{\frac{2 r_2}{r_1 + r_2}} - 1$

$$2 r_2 = (r_1 + r_2) \left[\frac{\Delta V_1}{V_{c_1}} + 1 \right]^2 \qquad r_2 = r_1 \frac{\left(\frac{\Delta V_1}{V_{c_1}} + 1\right)^2}{2 - \left(\frac{\Delta V_1}{V_{c_1}} + 1\right)^2}$$

5 b) so $\quad r_2 = 6578 \frac{1.524}{2 - 1.524} = 21{,}170 \text{ Km} \qquad h_2 = r_2 - r_e$

$$= 21{,}170 - 6378 = \underline{\underline{14{,}800 \text{ Km}}}$$

5 c) $\Delta V_2 = V_{c_1} \left[\sqrt{\frac{1}{x}} - \sqrt{\frac{2}{x(1+x)}} \right]$ where $x = \frac{r_2}{r_1} = \frac{21170}{6578} = 3.22$

$$= 7780 \left[0.557 - 0.384 \right] = \underline{\underline{1350 \text{ m/s}}}$$

10 d) $\Delta V = 1350 = g I_{sp} \ln MR \quad \therefore \; MR = e^{\Delta V / g I_{sp}} = 1.356$

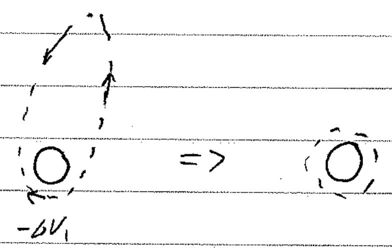
$MR = \frac{M_0}{M_0 - M_p} \qquad$ Here $M_0 = 15000 - 5097 = 9903$

$9903 = (9903 - M_p) \, 1.356 \qquad M_p = 2600 \text{ Kg}$

$t_b = g I_{sp} M_p / F = 9.81 \; 450 \; 2000 / 75{,}000 = \underline{\cancel{108} \; 153 \text{ sec}}$

5.)

An expedition sent out to mine near-earth asteroids is exploring economical alternatives to bringing the minerals back into low earth orbit. In this connection, they propose to capture the payload in a highly elliptic orbit about earth with apogee at 50,000 km and perigee at 100 km. They intend to use aerobraking to ultimately arrive at a 100 km circular orbit which could then be a starting point for re-entry to earth. How much Δv is saved by the aerobraking process assuming we would need to use retro-rockets if this option weren't employed?



Basically, aerobraking provides the equiv. of ΔV_1 in a Hohmann Xfer from 100 Km to 50,000 Km

Data

$r_1 = r_e + 100\,Km = 6478 \qquad r_2 = 56378 \qquad V_{c1} = \sqrt{\frac{\mu}{r_1}} = \left(\frac{3.986 \times 10^5}{6478}\right)^{1/2}$

$= 7.84\ Km/s$

$\Delta V_1 = V_{c1}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right] = 7.84\left[\sqrt{\frac{112756}{62856}} - 1\right] = \underline{2.66\ Km/s}$

2. $\beta = \frac{m}{C_D A_p} = \frac{\rho_p \frac{\pi}{6} D_p^3}{C_D \frac{\pi}{4} D_p^2} = \frac{2}{3}\frac{\rho_p D_p}{C_D}$ Particle w/ lowest β will have lowest 2ϕ Flow Loss

Particle	$\rho * D$	
1	100*200 = 20,000	← 2nd Lowest loss
2	300*100 = 30,000	← 3rd Lowest Loss
3	50*300 = 15,000	← Lowest Loss