Chapter 1

Basic Algebraic Operations

1.1 Numbers

1.

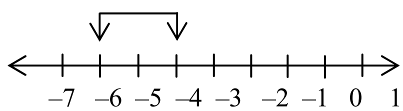
The numbers –3 and 14 are integers. They are also rational numbers since they can be written as $\frac{-3}{1}$ and $\frac{14}{1}$.

2.

The absolute value of –6 is 6 because –6 is six units to the left of the origin at zero.

3.

$-6 < -4$ should be read "–6 is less than –4" (–6 is to the left of –4 on the number line).



4.

The reciprocal of $\frac{3}{2}$ is $\frac{1}{\frac{3}{2}} = 1 \times \frac{2}{3} = \frac{2}{3}$.

5.

3 is an integer, rational $\left(\frac{3}{1}\right)$, and real.

$\sqrt{-4}$ is imaginary.

6.

$\frac{\sqrt{7}}{3}$ is irrational (because $\sqrt{7}$ is an irrational number) and real.

-6 is an integer, rational $\left(\frac{-6}{1}\right)$, and real

7.

$-\frac{\pi}{6}$ is irrational (because π is an irrational number) and real.

$\frac{1}{8}$ is rational and real

8.

$-\sqrt{-6}$ is imaginary.

$-2.33 = \frac{-233}{100}$ is rational and real

9.

$|3| = 3$

$|-4| = 4$

$\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$

10.

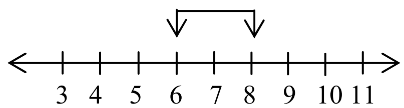
$|-0.857| = 0.857$

$|\sqrt{2}| = \sqrt{2}$

$\left|-\frac{19}{4}\right| = \frac{19}{4}$

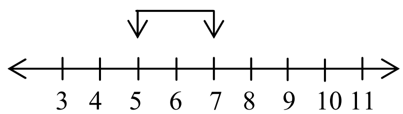
11.

$6 < 8$; 6 is to the left of 8.



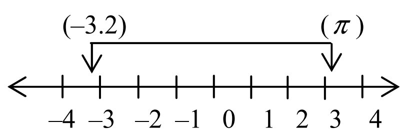
12.

$7 > 5$; 7 is to the right of 5.



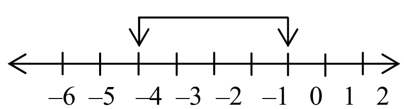
13.

$\pi > -3.2$; π (3.14159…) is to the right of –3.2.



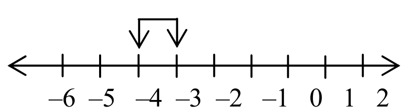
14.

$-4 < 0$; –4 is to the left of 0.



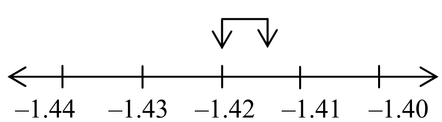
15.

$-4 < -|-3|$; –4 is to the left of $-|-3|$, $\left(-|-3| = -(3) = -3\right)$.



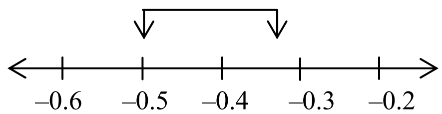
16.

$-\sqrt{2} > -1.42$; $(-\sqrt{2} = -(1.414....) = -1.414...)$, $-\sqrt{2}$ is to the right of –1.42.



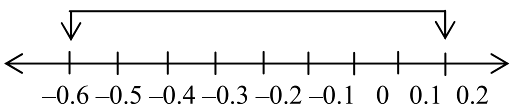
17.

$-\frac{1}{3} > -\frac{1}{2}; -\frac{1}{3} = -0.33\ldots$ is to the right of $-\frac{1}{2} = -0.5$.



18.

$-0.6 < 0.2$; –0.6 is to the left of 0.2.



19.

The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $-\frac{4}{\sqrt{3}}$ is $-\frac{1}{\frac{4}{\sqrt{3}}} = -\frac{\sqrt{3}}{4}$.

The reciprocal of $\frac{y}{b}$ is $\frac{1}{\frac{y}{b}} = \frac{b}{y}$.

20.

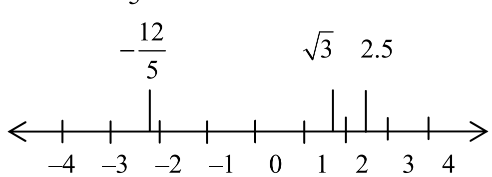
The reciprocal of $-\frac{1}{3}$ is $-\frac{1}{\frac{1}{3}} = -\frac{3}{1} = -3$.

The reciprocal of $-0.25 = -\frac{1}{4}$ is $-\frac{1}{\frac{1}{4}} = -\frac{4}{1} = -4$.

The reciprocal of x is $\frac{1}{x}$.

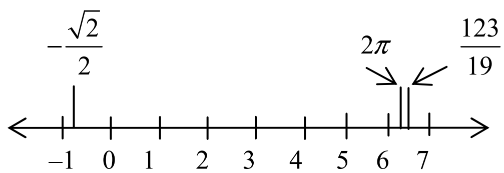
21.

Find 2.5, $-\frac{12}{5} = -2.4$; $\sqrt{3} = 1.732...$



22.

Find $-\frac{\sqrt{2}}{2} = -\frac{1.414\ldots}{2} = -0.707$; $2\pi = 2 \times 3.14\ldots = 6.28$; $\frac{123}{19} = 6.47$.



23.
An absolute value is not always positive, $|0| = 0$ which is not positive.

24.

Since $2.17 = \frac{217}{100}$, it is rational.

25.
The reciprocal of the reciprocal of any positive or negative number is the number itself.

The reciprocal of n is $\frac{1}{n}$; the reciprocal of $\frac{1}{n}$ is $\frac{1}{\frac{1}{n}} = 1 \cdot \frac{n}{1} = n$.

26.
A rational number can be expressed as a fraction of integers. So if the denominator in the fraction must be 11, then find the integer *x* so that $-1.0 < \frac{x}{11} < -0.9$. If $x = -11$, it would equal the lower limit $\frac{-11}{11} = -1.0$. So it must be an integer larger than –11. If $x = -10$, then $\frac{-10}{11} = -0.90909\ldots$, which is rational.

27.
A rational number can be expressed as a fraction of integers. So, if the numerator in the fraction must be 3, then find the integer *x*, so that $0.13 < \frac{3}{x} < 0.14$. So $0.13 = \frac{13}{100}$, and we can find the equivalent fraction of $\frac{13}{100}$ that has 3 as a numerator by rearranging and solving the following equation:

$$\frac{13}{100} = \frac{3}{x}$$
$$13(x) = 100 \times 3$$
$$x = \frac{300}{13}$$
$$x = 23.077$$
$$0.13 = \frac{3}{23.077}$$

However, since *x* must be an integer and less than the answer above to make our fraction with the numerator of 3 greater than 0.13, we assign 23 to *x*, making $\frac{3}{23} = 0.1304$, which is rational. $0.13 < 0.1304 < 0.14$.

28.
No, $|b-a| = |b|-|a|$, as shown below.

If $a>0$, then $|a|=a$.

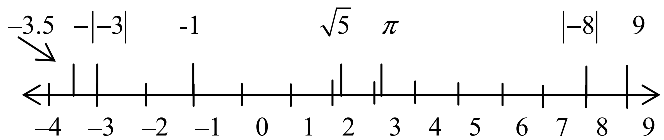
If $b>a$ and $a>0$, then $|b|=b$.

If $b>a$ then $b-a>0$, then $|b-a|=b-a$.

Therefore, $|b-a|=b-a=|b|-|a|$.

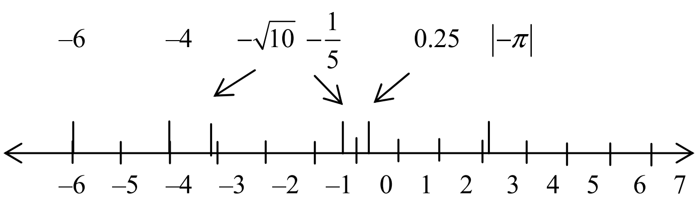
The two sides of the expression are equivalent, one side is not less than the other.

29.
List these numbers from smallest to largest: $-1,\ 9,\ \pi=3.14,\ \sqrt{5}=2.236,\ |-8|=8,\ -|-3|=-3,\ -3.5$.



So, from smallest to largest, they are $-3.5,\ -|-3|,\ -1,\ \sqrt{5},\ \pi,\ |-8|,\ 9$.

30.
List these numbers from smallest to largest: $-\frac{1}{5}=-0.20,\ -\sqrt{10}=-3.16,\ -|-6|=-6,\ -4,\ 0.25,\ |-\pi|=3.14$.



So, from smallest to largest, they are $-|-6|,\ -4,\ -\sqrt{10},\ -\frac{1}{5},\ 0.25,\ |-\pi|$.

31.
If a and b are positive integers and $b>a$, then

(a) $b-a$ is a positive integer.
(b) $a-b$ is a negative integer.
(c) $\frac{b-a}{b+a}$, the numerator and denominator are both positive, but the numerator is less than the denominator, so the answer is a positive rational number than is less than 1.

32.
If a and b are positive integers, then

(a) $a+b$ is a positive integer
(b) a/b is a positive rational number
(c) $a\times b$ is a positive integer

33.

(a) Is the absolute value of a positive or a negative integer always an integer? Yes:
$|x| = x$, so the absolute value of a positive integer is an integer.
$|-x| = x$, so the absolute value of a negative integer is an integer.
(b) Is the reciprocal of a positive or negative integer always a rational number? Yes:
If x is a positive or negative integer, then the reciprocal of x is $\frac{1}{x}$. Since both 1 and x are integers, the reciprocal is a rational number.

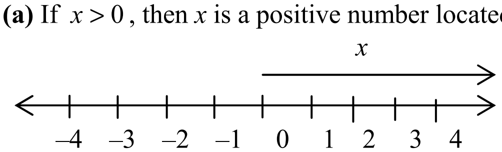
34.

(a) Is the absolute value of a positive or negative rational number rational?
$|x| = x$, so if x is a positive or negative rational number, the absolute value of it is also a rational number.
(b) Is the reciprocal of a positive or negative rational number a rational number?

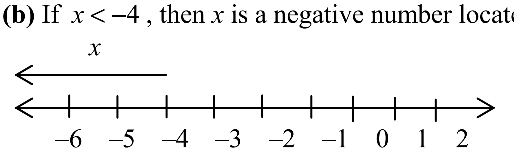
A rational number is a number that can be expressed as a fraction where both the numerator and denominator are integers and the denominator is not zero. So a rational number $\frac{\text{integer } a}{\text{integer } b}$ has a reciprocal of $\frac{1}{\frac{\text{integer } a}{\text{integer } b}} = \frac{\text{integer } b}{\text{integer } a}$, which is also a rational number if integer a is not zero.

35.

(a) If $x > 0$, then x is a positive number located to the right of zero on the number line.

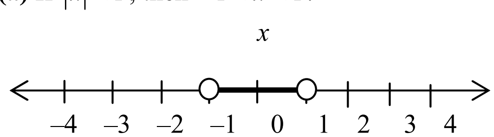


(b) If $x < -4$, then x is a negative number located to the left of –4 on the number line.

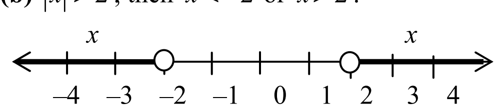


36.

(a) If $|x| < 1$, then $-1 < x < 1$.

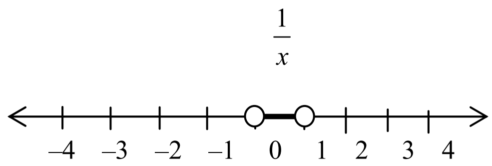


(b) $|x| > 2$, then $x < -2$ or $x > 2$.



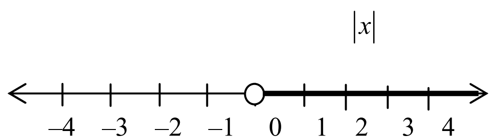
37.

If $x > 1$, then $\frac{1}{x}$ is a positive number less than 1. Or $0 < \frac{1}{x} < 1$.



38.

If $x < 0$, then $|x|$ is a positive number greater than zero.



39.

$a + bj = a + b\sqrt{-1}$ is a real number when $\sqrt{-1}$ is eliminated, which is when $b = 0$. So $a + bj$ is a real number for all real values of a and $b = 0$.

40.

The variables are *w* and *t*.
The constants are *c*, 0.1, and 1.

41.

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$. Find C_T, where $C_1 = 0.0040\text{ F}$ and $C_2 = 0.0010\text{ F}$.

$$\frac{1}{C_T} = \frac{1}{0.0040} + \frac{1}{0.0010}$$

$$\frac{1}{C_T} = \frac{1(0.0040) + 1(0.0010)}{0.0040 \times 0.0010}$$

$$C_T = \frac{0.0040 \times 0.0010}{0.0040 + 0.0010}$$

$$C_T = 0.00080\text{ F}$$

42.

$$|100V| = 100V$$

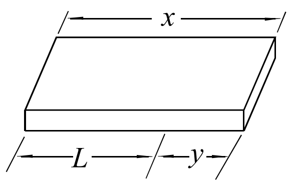
$$|-200V| = 200V$$

$$|-200V| > |100V|$$

43.

$$N = \frac{a\text{ bits}}{\text{bytes}} \times \frac{1000\text{ bytes}}{1\text{ kilobyte}} \times n\text{ kilobytes} = 1000an\text{ bits}$$

44.



$x =$ length of base in m

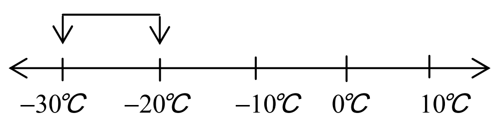
$y =$ the shortened length in cm.

$100x =$ length of base in cm

$y + L = 100x,$ all dimensions in cm

$L = 100x - y$

45.
Yes, $-20°C > -30°C$ because $-30°C$ is found to the left of $-20°C$ on the number line.



46.
For $I < 4$ A, $V > 12$ V.

47. In words, we would state: "The mass of the Tesla car is less than the mass of the perogy statue." It would be equally correct to state: "The mass of the perogy statue is greater than the mass of the Tesla car." In symbols, if m_T denotes the mass of the Tesla car and m_P denotes the mass of the perogy statue, both of the following statements are equally appropriate: $m_P > m_T$ or $m_T < m_P$.

1.2 Fundamental Operations of Algebra

1.

$16-2\times(-3)=16-(-6)=16+6=22$

2.

$\frac{-18}{-6}+5-(-2)(3)=3+5-(-6)=8+6=14$

3.

$\frac{-12}{8-2}+\frac{5-1}{2(-1)}=\frac{-12}{6}+\frac{4}{-2}=-2+(-2)=-4$

4.

$$\begin{aligned}\frac{1500(40)+1400(-20)}{1500+1400}&=\frac{60\ 000+(-28\ 000)}{2900}\\&=\frac{60\ 000-28\ 000}{2900}\\&=\frac{32\ 000}{2900}\\&=11.034\ 482\ 76\text{ km/h}\\&=11\text{ km/h if rounded to two significant digits}\end{aligned}$$

5.

$8+(-4)=8-4=4$

6.

$-4+(-7)=-4-7=-11$

7.

$-3+9=6$ or alternatively

$-3+9=+(9-3)=+(6)=6$

8.

$18-21=-3$ or alternatively

$18-21=-(21-18)=-(3)=-3$

9.

$-19-(-16)=-19+16=-3$

10.

$8-(-4)=8+4=12$

11.

$8(-3)=-(8\times3)=-24$

12.

$-9(3)=-27$

13.
$-7(-5)=+(7\times5)=35$

14.
$\frac{-9}{3}=-3$

15.
$\frac{-6(20-10)}{-3}=\frac{-6(10)}{-3}=\frac{-60}{-3}=20$

16.
$\frac{28}{-7(6-5)}=\frac{28}{-7(1)}=\frac{28}{-7}=-4$

17.
$-2(4)(-5)=-8(-5)=40$

18.
$3(-4)(6)=-12(6)=-72$

19.
$2(2-7)\div10=2(-5)\div10=-10\div10=-1$

20.
$\frac{-64}{-2|4-8|}=\frac{-64}{-2|-4|}=\frac{-64}{-2(4)}=\frac{-64}{-8}=8$

21.
$16\div2(-4)=8(-4)=-32$

22.
$-20\div5(-4)=-4(-4)=16$

23.
$-9-|2-10|=-9-|-8|=-9-8=-17$

24.
$(7-7)\div(5-7)=0\div(-2)=0$

25.
$\frac{17-7}{7-7}=\frac{10}{0}=\text{is undefined}$

26.
$\frac{7-7}{7-7}=\frac{0}{0}=\text{is indeterminate}$

27.
$8-3(-4)=8+12=20$

28.
$-20+8\div 4=-20+2=-18$

29.
$-2(-6)+\left|\frac{8}{-2}\right|=12+|-4|=12+4=16$

30.
$-10-(-6)(-8)=-10-(48)=-58$

31.
$$\begin{aligned}10(-8)(-3)\div(10-50) &= 10(-8)(-3)\div(-40)\\ &= -80(-3)\div(-40)\\ &= 240\div(-40)\\ &= -6\end{aligned}$$

32.
$\frac{7-|-5|}{-1(-2)}=\frac{7-5}{2}=\frac{2}{2}=1$

33.
$\frac{24}{3+(-5)}-4(-9)=\frac{24}{-2}+(4\times 9)=-12+36=24$

34.
$\frac{-18}{3}-\frac{4-|-6|}{-1}=\frac{-18}{3}-\frac{4-6}{-1}=-6-\frac{-2}{-1}=-6-2=-8$

35.
$$\begin{aligned}-7-\frac{|-14|}{2(2-3)}-3|6-8| &= -7-\frac{14}{2(-1)}-3|-2|\\ &= -7-\frac{14}{-2}-3(2)\\ &= -7-(-7)-6\\ &= -7+7-6\\ &= -6\end{aligned}$$

36.
$$\begin{aligned}-7(-3)+\frac{6}{-3}-(-9) &= +(7\times 3)+(-2)+9\\ &= 21-2+9\\ &= 28\end{aligned}$$

37.

$$\begin{aligned}\frac{3(-9)-2(-3)}{3-10} &= \frac{-(3\times 9)+(2\times 3)}{-7}\\ &= \frac{-27+6}{-7}\\ &= \frac{-21}{-7}\\ &= 3\end{aligned}$$

38.

$$\frac{20(-12)-40(-15)}{98-|-98|} = \frac{-240+600}{98-98} = \frac{360}{0} = \text{is undefined}$$

39.

$6(7)=(7)6$ demonstrates the commutative law of multiplication.

40.

$6+8=8+6$ demonstrates the commutative law of addition.

41.

$6(3+1)=6(3)+6(1)$ demonstrates the distributive law.

42.

$4(5\times\pi)=(4\times 5)\pi$ demonstrates the associative law of multiplication.

43.

$3+(5+9)=(3+5)+9$ demonstrates the associative law of addition.

44.

$8(3-2)=8(3)-8(2)$ demonstrates the distributive law.

45.

$(\sqrt{5}\times 3)\times 9=\sqrt{5}\times(3\times 9)$ demonstrates the associative law of multiplication.

46.

$(3\times 6)\times 7=7\times(3\times 6)$ demonstrates the commutative law of multiplication.

47.

$-a+(-b)=-a-b$, which is expression (d).

48.

$b-(-a)=b+a=a+b$, which is expression (a).

49.

$-b-(-a)=-b+a=a-b$, which is expression (b).

50.

$-a-(-b)=-a+b=b-a$, which is expression (c).

51.

Since $|5-(-2)|=|5+2|=|7|=7$ and $|-5-|-2||=|-5-2|=|-7|=7$,
$|5-(-2)| = |-5-|-2||$.

52.

Since $|-3-|-7|| = |-3-7| = |-10|=10$ and $||-3|-7|=|3-7|=|-4|=4$,
$|-3-|-7|| > ||-3|-7|$.

53.
(a) The sign of a product of an even number of negative numbers is positive. Example: $-3(-6)=18$

(b) The sign of a product of an odd number of negative numbers is negative. Example: $-5(-4)(-2)=-40$

54.
Subtraction is not commutative because $x-y \neq y-x$. Example: $7-5=2$ does not equal $5-7=-2$

55.
Correct. From the definition in Section 1.1, the absolute value of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number. So for values of *x* where $x>0$ (positive) or $x=0$ (neutral) then $|x|=x$. Example: $|4|=4$.

The claim that absolute values of negative numbers $|x|=-x$ is also true.

Example: if x is -6, then $|-6|=-(-6)=6$.

56.
The incorrect answer was achieved by subtracting before multiplying or dividing which violates the order of operations.
$24-6\div 2\times 3 \neq 18\div 2\times 3=9\times 3=27$
The correct value is:
$24-6\div 2\times 3=24-3\times 3=24-9=15$

57.

(a) $-xy=1$ is true for values of *x* and *y* that are negative reciprocals of each other or $y=-\dfrac{1}{x}$, provided that the number *x* in the denominator is not zero. So if $x=12$, then $y=-\dfrac{1}{12}$ and $-xy=-(12)\left(-\dfrac{1}{12}\right)=1$.

(b) $\dfrac{x-y}{x-y}=1$ is true for all values of *x* and *y*, provided that $x \neq y$ to prevent division by zero.

58.
(a)
$|x+y|=|x|+|y|$ is true for values where both *x* and *y* are positive or either are zero:

$|x+y|=|x|+|y|$, when $x \geq 0$ and $y \geq 0$.
Example:
$|6+3|=6+3=9$ and

$|6|+|3|=6+3=9$

$|x+y|=|x|+|y|$ is also true for values where <u>both</u> x and y are negative

$|x+y|=|x|+|y|$, when $x<0$ and $y<0$.

Example:

$|-11+(-7)|=|-18|=18$

$|-11|+|-7|=11+7=18$

$|x+y|=|x|+|y|$ is not true however, when x and y have opposite signs

$|x+y|\neq|x|+|y|$, when $x>0$ and $y<0$; or $x<0$ and $y>0$.

Example:

$|-21+6|=|-15|=15,$

$|-21|+|6|=21+6=27\neq15$

$|4+(-5)|=|-1|=1,$

$|4|+|-5|=4+5=9\neq1$

(b)

The same argument as above holds true for $|x-y|=|x|-|y|$

$|x-y|=|x|-|y|$ is true for values where both x and y are positive or either are zero:

$|x-y|=|x|-|y|$, when $x\geq0$ and $y\geq0$.

Example:

$|6-3|=6=3=3$ and

$|6|-|3|=6-3=3$

$|x-y|=|x|-|y|$ is also true for values where both x and y are negative:

$|x-y|=|x|-|y|$, when $x<0$ and $y<0$.

Example:

$|-11-(-7)|=|-11+7|=|-4|=4$

$|-11|-|-7|=11-7=4$

$|x-y|=|x|-|y|$ is not true however, when x and y have opposite signs

$|x-y|\neq|x|-|y|$, when $x>0$ and $y<0$; or $x<0$ and $y>0$.

Example:

$|21-(-6)|=|21+6|=|27|=27,$

$|21|-|-6|=21-6=15\neq27$

59.

The total change in the price of the stock is $-0.68+0.42+0.06+(-0.11)+0.02=-0.29$ dollars.

60.

The difference in altitude is $-86-(-1396)=-86+1396=1310$ m.

61. The change in the meter energy reading *E* would be:

$E_{change} = E_{used} - E_{generated}$

$E_{change} = 2.1\text{ kW}\cdot\text{h} - 1.5\text{ kW}(3.0\text{ h})$

$E_{change} = 2.1\text{ kW}\cdot\text{h} - 4.5\text{ kW}\cdot\text{h}$

$E_{change} = -2.4\text{ kW}\cdot\text{h}$

62.
Assuming that this batting average is for the current season only which is just starting, the number of hits is zero and the total number of at-bats is also zero giving us a $\text{batting average} = \dfrac{\text{number of hits}}{\text{at} - \text{bats}} = \dfrac{0}{0}$ which is indeterminate, not 0.000.

63.
The average temperature for the week is:

$$T_{avg} = \frac{-7 + (-3) + 2 + 3 + 1 + (-4) + (-6)}{7}\ ^\circ\text{C}$$

$$T_{avg} = \frac{-7 - 3 + 2 + 3 + 1 - 4 - 6}{7}\ ^\circ\text{C}$$

$$T_{avg} = \frac{-14}{7}\ ^\circ\text{C} = -2.0\ ^\circ\text{C}$$

64.
The vertical distance from the flare gun is

$d = (20)(5) + (-5)(25)$

$d = 100 + (-125)$

$d = 100 - 125$

$d = -25\text{ m}$

The flare is 25 m below the flare gun.

65.
The sum of the voltages is

$V_{sum} = 6\text{V} + (-2\text{V}) + 8\text{V} + (-5\text{V}) + 3\text{V}$

$V_{sum} = 6\text{V} - 2\text{V} + 8\text{V} - 5\text{V} + 3\text{V}$

$V_{sum} = 10\text{V}$

66.
(a)
The change in the current for the first interval is the second reading – the first reading
$\text{Change}_1 = -0.2\text{ mA} - 0.7\text{ mA} = -0.9\text{ mA}.$

(b)
The change in the current for the middle intervals is the third reading – the second reading
$\text{Change}_2 = -0.9\text{ mA} - (-0.2\text{ mA}) = -0.9\text{ mA} + 0.2\text{ mA} = -0.7\text{ mA}.$

(c)
The change in the current for the last interval is the last reading – the third reading
$\text{Change}_3 = -0.6\text{ mA} - (-0.9\text{ mA}) = -0.6\text{ mA} + 0.9\text{ mA} = 0.3\text{ mA}.$

67.
The oil drilled by the first well is $100\text{ m} + 200\text{ m} = 300\text{ m}$ which equals the depth drilled by the second well:
$200\text{ m} + 100\text{ m} = 300\text{ m}.$
$100\text{ m} + 200\text{ m} = 200\text{ m} + 100\text{ m}$ demonstrates the commutative law of addition.

68.

The first tank leaks $12\frac{\text{L}}{\text{h}}(7\text{ h}) = 84\text{ L}$ .The second tank leaks $7\frac{\text{L}}{h}(12\text{ h}) = 84\text{ L}$.

$12\times 7 = 7\times 12$ demonstrates the commutative law of multiplication.

69.

The total time spent browsing these websites is the total time spent browsing the first site on each day + the total time spent browsing the second site on each day, for all 7 days.

$t = 7\text{ days}\times 25\frac{\text{min}}{\text{day}} + 7\text{ days}\times 15\frac{\text{min}}{\text{day}}$

$t = 175\text{ min} + 105\text{ min}$

$t = 280\text{ min}$

OR

$t = 7\text{ days}\times(25+15)\frac{\text{min}}{\text{day}}$

$t = 7\text{ days}\times 40\frac{\text{min}}{\text{day}}$

$t = 280\text{ min}$

This illustrates the distributive law.

70.

Distance = rate × time

$d = \left(600\frac{\text{km}}{\text{h}} + 50\frac{\text{km}}{\text{h}}\right)3\text{ h}$

$d = 600\frac{\text{km}}{\text{h}}(3\text{ h}) + 50\frac{\text{km}}{\text{h}}(3\text{ h})$

$d = 1800\text{ km} + 150\text{ km} = 1950\text{ km}$

OR

$d = \left(600\frac{\text{km}}{\text{h}} + 50\frac{\text{km}}{\text{h}}\right)3\text{ h}$

$d = \left(650\frac{\text{km}}{\text{h}}\right)3\text{ h}$

$d = 1950\text{ km}$

This illustrates the distributive law.

1.3 Measurement, Calculation, and Approximate Numbers

1.
0.3900 has four significant digits since the trailing zeros are after the decimal. Trailing zeros after the decimal are not necessary as placeholders and should not be written unless they are significant.

2.
35.303 rounded off to four significant digits is 35.30.

3.
In finding the product of the approximate numbers, $2.483 \times 30.5 = 75.7315$, but since 30.5 has three significant digits, the answer is 75.7.

4.
$38.3 - 21.9(-3.58) = 116.702$ using exact numbers; if we estimate the result, $40 - 20(-4) = 120$.

5.
1 megahertz = 1 MHz = 1 000 000 Hz

6.
1 kilowatt = 1 kW = 1000 W

7.
1 millimetre = 1 mm = 0.001 m

8.
1 picosecond = 1 ps = 1×10^{-12} s

9.
1 kV = 1 kilovolt = 1000 volts

10.
1 GΩ = 1 gigaohm = 1×10^{9} ohms

11.
1 mA = 1 milliampere = 0.001 amperes

12.
1 pF = 1 picofarad = 1×10^{-12} farads

13.
$$1\text{ km}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\cdot\left(\frac{100\text{ cm}}{1\text{ m}}\right) = 100\ 000\text{ cm}$$

14.
$$1\text{ kg}\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\cdot\left(\frac{1000\text{ mg}}{1\text{ g}}\right) = 1\ 000\ 000\text{ mg}$$

15.

$$20\text{ s}\left(\frac{1\text{ Ms}}{1\ 000\ 000\text{ s}}\right) = 0.\ 000\ 02\text{ Ms}$$

16.

$$800\text{ Pa}\left(\frac{1\text{ kPa}}{1000\text{ Pa}}\right) = 0.8\text{ kPa}$$

17.

$$250\text{ mm}^2\left(\frac{1\text{ m}}{1000\text{ mm}}\right)^2 = 0.000\ 25\text{ m}^2$$

18.

$$1.75\text{ m}^2\left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 = 17\ 500\text{ cm}^2$$

19.

$$80.0\text{ m}^3\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) = 80\ 000\text{ L (with three significant digits)}$$

20.

$$0.125\text{ L}\left(\frac{1000\text{ mL}}{1\text{ L}}\right) = 125\text{ mL}$$

21.

$$45.0\text{ m/s}\left(\frac{100\text{ cm}}{1\text{ m}}\right) = 4500\text{ cm/s (with three significant digits)}$$

22.

$$1.32\ \frac{\text{km}}{\text{h}}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\cdot\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.367\text{ m/s}$$

23.

$$9.80\ \frac{\text{m}}{\text{s}^2}\left(\frac{100\text{ cm}}{1\text{ m}}\right)\cdot\left(\frac{60\text{ s}}{1\text{ min}}\right)^2 = 3\ 530\ 000\text{ cm/min}^2$$

24.

$$5.10\ \frac{\text{g}}{\text{cm}^3}\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\cdot\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 = 5100\text{ kg/m}^3\ \text{(with three significant digits)}$$

25.

$$25\text{ h}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\cdot\left(\frac{1000\text{ ms}}{1\text{ s}}\right) = 90\ 000\ 000\text{ ms (with two significant digits)}$$

26.

$$5.25\text{ mV}\left(\frac{1\text{ V}}{1000\text{ mV}}\right)\cdot\left(\frac{1\text{ W/A}}{1\text{ V}}\right) = 0.005\ 25\text{ W/A}$$

27.

$$15.0\ \mu\text{F}\left(\frac{1\ \text{F}}{1\ 000\ 000\ \mu\text{F}}\right)\cdot\left(\frac{1\ \text{C/V}}{1\ \text{F}}\right)\cdot\left(\frac{1000\ \text{mC}}{1\ \text{C}}\right)=0.0150\ \text{mC/V}$$

28. The metric equivalence is given as: $1\ \text{lb} = 0.4536\ \text{kg}$. Dividing each side: $\frac{1\ \text{lb}}{1\ \text{lb}}=\frac{0.4536\ \text{kg}}{1\ \text{lb}}$, which reduces to $1=\frac{0.4536\ \text{kg}}{1\ \text{lb}}$. Using this value as a multiple: $\frac{42\ \cancel{\text{lb}}}{1}\cdot\frac{0.4536\ \text{kg}}{1\ \cancel{\text{lb}}}=19.05\ \text{kg}.$

29. The statement of equivalence is given as: $1\ \text{km} = 0.6214\ \text{mi}.$ Dividing each side: $\frac{1\ \text{km}}{1\ \text{km}}=\frac{0.6214\ \text{mi}}{1\ \text{km}}$, which reduces to $1=\frac{0.6214\ \text{mi}}{1\ \text{km}}$. Using this value as a multiple: $\frac{27\ \cancel{\text{km}}}{1}\cdot\frac{0.6214\ \text{mi}}{1\ \cancel{\text{km}}}=16.78\ \text{mi}.$

30. The statement of equivalence between feet and metres is: $1\ \text{ft} = 0.304\ 80\ \text{m}.$ Squaring each side: $1\ \text{ft}^2 = 0.09290\ \text{m}^2.$ Dividing we get: $1=\frac{0.09290\ \text{m}^2}{1\ \text{ft}^2}.$ Using this value as a multiple:

$$\frac{10\ \cancel{\text{ft}^2}}{1}\cdot\frac{0.092\ 90\ \text{m}^2}{1\ \cancel{\text{ft}^2}}=0.929\ 0\ \text{m}^2.$$

31. In this case, we need two statements of equivalence. First, $1\ \text{kg} = 2.204\ \text{lb}.$. Dividing: $1=\frac{2.204\ \text{lb}}{1\text{kg}}$. The second statement of equivalence relates metres to feet: $1\ \text{m} = 3.281\ \text{ft}.$ Squaring each side: $1\ \text{m}^2 = 10.76\ \text{ft}^2.$ Dividing: $\frac{1\ \text{m}^2}{10.76\ \text{ft}^2}=1.$ Now, using dimensional analysis multiply by each fraction equal to 1 to cancel units we wish to eliminate: $\frac{200\ \cancel{\text{kg}}}{\cancel{\text{m}^2}}\cdot\frac{2.204\ \text{lb}}{1\ \cancel{\text{kg}}}\cdot\frac{1\ \cancel{\text{m}^2}}{10.76\ \text{ft}^2}=40.97\ \text{lb}/\text{ft}^2.$

32. To travel 1 m, light takes 1/299 792 458 s.

$$1\ \text{year}\left(\frac{365.25\ \text{days}}{1\ \text{year}}\right)\cdot\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\cdot\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)=31\ 557\ 600\ \text{s}$$

$$\frac{1\ \text{m}}{1/299\ 792\ 458\ \text{s}}\left(\frac{31\ 557\ 600\ \text{s}}{1\ \text{year}}\right)=9\ 460\ 730\ 472\ 580\ 800\ \text{m}=9.461\times10^{15}\ \text{m/year}$$

33.

The distance around the orbit should be $d=2\pi r=2\pi(150\ 000\ 000\ \text{km})=942\ 477\ 796.1\ \text{km}$.

$$1\ \text{year}\left(\frac{365.25\ \text{days}}{1\ \text{year}}\right)\cdot\left(\frac{24\ \text{h}}{1\ \text{day}}\right)=8766\ \text{h}$$

Orbital speed is the ratio of distance travelled to time elapsed:

$$v=\frac{d}{t}=\frac{942\ 477\ 796.1\ \text{km}}{8766\ \text{h}}=107\ 515\ \text{km/h}=110\ 000\ \text{km/h}$$

34.

$$101\ 300\ \text{Pa}\left(\frac{1\ \text{kPa}}{1\ 000\ \text{Pa}}\right)=101.3\ \text{kPa}$$

35. $$56\ \text{L}\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3=56\ 000\ \text{cm}^3$$

36.

$$0.160\ \text{kg}\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right)\cdot\left(\frac{1000\ \text{mg}}{1\ \text{g}}\right)=160\ 000\ \text{mg}$$

37.

$$6800\ \frac{\text{m}}{\text{s}}\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\cdot\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)=24\ 480\ \text{km/h}=24\ 000\ \text{km/h}$$

38.

Convert 1.5 TB to kB:

$$1.5\ \text{TB}\left(\frac{1\ 000\ 000\ 000\ 000\ \text{B}}{1\ \text{TB}}\right)\cdot\left(\frac{1\ \text{kB}}{1000\ \text{B}}\right)=1\ 500\ 000\ 000\ \text{kB}$$

Compare the memories:

$$\left(\frac{1\ 500\ 000\ 000\ \text{kB}}{64\ \text{kB}}\right)=23\ 437\ 500\ \text{times greater}$$

39.

$$112\ \text{cm}^2\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2=0.0112\ \text{m}^2$$

40.

$$0.024\ \text{MW}\cdot\text{h}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)\cdot\left(\frac{1\ 000\ 000\ \text{W}}{1\ \text{MW}}\right)\cdot\left(\frac{1\ \text{J/s}}{1\ \text{W}}\right)=86\ 400\ 000\ \text{J}=86\ 000\ 000\ \text{J}$$

41.

$$1000\ \frac{\text{kg}}{\text{m}^3}\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right)\cdot\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)=1000\ \text{g/L}$$

42.

$$8500\ \frac{\text{mL}}{\text{min}}\left(\frac{1\ \text{L}}{1000\ \text{mL}}\right)\cdot\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\cdot\left(\frac{1\ \text{min}}{60\ \text{s}}\right)=0.000\ 14\ \text{m}^3/\text{s}$$

43.

$$332\ \frac{\text{m}}{\text{s}}\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\cdot\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)=1195.2\ \text{km/h}=1200\ \text{km/h (with three significant digits)}$$

44.

$$\frac{15.0\ \text{g}}{0.060\ \text{L}}\left(\frac{1000\ \text{mg}}{1\ \text{g}}\right)\cdot\left(\frac{1\ \text{L}}{10\ \text{dL}}\right)=25\ 000\ \text{mg/dL}$$

45.
$$1.35\ \frac{\text{kW}}{\text{m}^2}\left(\frac{1000\ \text{W}}{1\ \text{kW}}\right)\cdot\left(\frac{1\ \text{J/s}}{1\ \text{W}}\right)\cdot\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 0.135\ \text{J/(s}\cdot\text{cm}^2)$$

46.
Orbital speed is the ratio of distance travelled to time elapsed:
$$v = \frac{d}{t} = \frac{2\ 400\ 000\ \text{km}}{28\ \text{days}}\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\cdot\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\cdot\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 992.06\ \text{m/s} = 990\ \text{m/s}$$

47.
$$1.2\times10^6\ \frac{\text{A}}{\text{m}^2}\left(\frac{1000\ \text{mA}}{1\ \text{A}}\right)\cdot\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 120\ 000\ \text{mA/cm}^2$$

48.
$$\frac{2.0\ \text{L}}{24\ \text{km}}\left(\frac{100\ \text{km}}{100\ \text{km}}\right) = 8.3\ \text{L/(100 km)}$$

49. 8 cylinders is exact because they can be counted. 55 km/h is approximate since it is measured.

50. 0.002 mm thick is a measurement and is therefore an approximation. $7.50 is an exact price.

51. 24 h and 1440 min (60 min/h × 24 h =1140 min) are both exact numbers.

52. 50 keys is exact because you can count them; 50 h of use is approximate since it is a measurement of time.

53. 107 has three significant digits; 3004 has four significant digits; 1040 has three significant digits.

54. 3600 has two significant digits; 730 has two significant digits; 2055 has four significant digits.

55. 6.80 has three significant digits since the zero indicates precision; 6.08 has three significant digits; 0.068 has two significant digits.

56. 0.8735 has four significant digits; 0.0075 has two significant digits; 0.0305 has three significant digits.

57. 3000 has one significant digit; 3000.1 has five significant digits; 3000.10 has six significant digits.

58. 1.00 has three significant digits since the zeros indicate precision; 0.01 has one significant digit since leading zeros are not significant; 0.0100 has three significant digits.

59.
(a) 0.01 has more decimal places (2) and is more precise.
(b) 30.8 has more significant digits (3) and is more accurate.

60.
(a) Both 0.041 and 7.673 have the same precision as they have the same number of decimal places (3).
(b) 7.673 is more accurate because it has more significant digits (4) than 0.041, which has two significant digits.

61.
(a) Both 0.1 and 78.0 have the same precision as they have the same number of decimal places.
(b) 78.0 is more accurate because it has more significant digits (3) than 0.1, which has one significant digit.

62.
(a) 0.004 is more precise because it has more decimal places (3).
(b) 7040 is more accurate because it has more significant digits (3) than 0.004, which has only one significant digit.

63.
(a) 0.004 is more precise because it has more decimal places (3).
(b) Both have the same accuracy as they both have one significant digit.

64.
(a) Both 50.060 and 8.914 have the same precision as they have the same number of decimal places (3).
(b) 50.060 is more accurate because it has more significant digits (5) than 8.914, which has four significant digits.

65.
(a) 4.936 rounded to three significant digits is 4.94.
(b) 4.936 rounded to two significant digits is 4.9.

66.
(a) 80.53 rounded to three significant digits is 80.5.
(b) 80.53 rounded to two significant digits is 81.

67.
(a) −50.893 rounded to three significant digits is −50.9.
(b) −50.893 rounded to two significant digits is −51.

68.
(a) 7.005 rounded to three significant digits is 7.01.
(b) 7.005 rounded to two significant digits is 7.0.

69.
(a) 9549 rounded to three significant digits is 9550.
(b) 9549 rounded to two significant digits is 9500.

70.
(a) 30.96 rounded to three significant digits is 31.0.
(b) 30.96 rounded to two significant digits is 31.

71.
(a) 0.9449 rounded to three significant digits is 0.945.
(b) 0.9449 rounded to two significant digits is 0.94.

72.
(a) 0.9999 rounded to three significant digits is 1.00.
(b) 0.9999 rounded to two significant digits is 1.0.

73.
(a) Estimate: $13+1-2=12$
(b) Calculator: $12.78+1.0495-1.633=12.1965$, which is 12.20 to 0.01 precision

74.
(a) Estimate: $4\times 17=68$
(b) Calculator: $3.64(17.06)=62.0984$, which is 62.1 to three significant digits

75.
(a) Estimate $9+(1)(4)=9+4=13$
(b) Calculator: $8.75+(1.2)(3.84)=13.358$, which is 13 to two significant digits

76.

(a) Estimate $30-\frac{20}{2}=30-10=20$

(b) Calculator: $28-\frac{20.955}{2.2}=18.475$, which is 18 to two significant digits

77.

(a) Estimate $\frac{9(15)}{9+15}=\frac{135}{24}=6,$ to one significant digit

(b) Calculator: $\frac{8.75(15.32)}{8.75+15.32}=5.569173$, which is 5.57 to three significant digits

78.

(a) Estimate: $\frac{0.7+0.05}{300\times 3}=0.0083...$

(b) Calculator: $\frac{0.69378+0.04997}{257.4\times 3.216}=0.000\ 898\ 467\ 549\ 6$, which is 0.000 898 5 to four significant digits

79.

(a) Estimate $4.5-\frac{2(300)}{400}=3.0,$ to two significant digits

(b) Calculator: $4.52-\frac{2.056(309.6)}{395.2}=2.9093279$, which is 2.91 to three significant digits

80.

(a) Estimate: $\frac{1}{0.6}+\frac{4}{3-1}=3.66....$

(b) Calculator: $\frac{1.00}{0.5926}+\frac{3.6957}{2.935-1.054}=3.652\ 231\ 698$, which is 3.65 to three significant digits

81. $0.9788+14.9=15.8788$ since the least precise number in the question has four decimal places.

82. $17.311-22.98=-5.669$ since the least precise number in the question has three decimal places.

83. $-3.142\times 65=-204.23$, which is –204.2 because the least accurate number has four significant digits.

84. $8.62\div 1728=0.004988$, which is 0.00499 because the least accurate number has three significant digits.

85. With a frequency listed as 2.75 MHz, the least possible frequency is 2.745 MHz, and the greatest possible frequency is 2.755 MHz. Any measurements between those limits would round to 2.75 MHz.

86. For an engine displacement stated at 2400 cm^3, the least possible displacement is 2350 cm^3, and the greatest possible displacement is 2450 cm^3. Any measurements between those limits would round to 2400 cm^3.

87. The speed of sound is $5.23\text{ km}\div 15\text{ s} = 0.3486...\text{ km/s} = 348.6...\text{ m/s}$. However, the least accurate measurement was time since it has only two significant digits. The correct answer is 350 m/s.

88. $4.4\text{ s}-2.72\text{ s}=1.68\text{ s}$, but the answer must be given according to precision of the least precise measurement in the question, so the correct answer is 1.7 s.

89.
(a) $2.2+3.8\times4.5=2.2+(3.8\times4.5)=19.3$
(b) $(2.2+3.8)\times4.5=6.0\times4.5=27$

90.
(a) $6.03\div2.25+1.77=(6.03\div2.25)+1.77=4.45$
(b) $6.03\div(2.25+1.77)=6.03\div4.02=1.5$

91.
(a) $2+0=2$
(b) $2-0=2$
(c) $0-2=-2$
(d) $2\times0=0$
(e) $2\div0=$ error; from Section 1.2, an equation that has 0 in the denominator is undefined when the numerator is not also 0.

92.
(a) $2\div0.0001=20\ 000$; $2\div0=$ error
(b) $0.0001\div0.0001=1$; $0\div0=$ error
(c) Any number divided by zero is undefined. Zero divided by zero is indeterminate.

93.
Pick any six digit integer for x = 231 465 and rearrange those digits for y = 164 352.
$(x-y)\div9=(231\ 465-164\ 352)\div9=7457$. A smaller *integer* number results.

94. $9\times8+7+6+5+4+3+2+1+0=100$

95. $\pi=3.14159265...$
(a) $\pi<3.1416$
(b) $22\div7=3.1428$
$\pi<(22\div7)$

96.
(a) $8\div33=0.2424...=0.\overline{24}$
(b) $\pi=3.14159265...$

97.
(a) $1\div3=0.333...$ It is a rational number since it is a repeating decimal.
(b) $5\div11=0.454545...$ It is a rational number since it is a repeating decimal.
(c) $2\div5=0.400...$ It is a rational number since it is a repeating decimal (0 is the repeating part).

98. $124\div990=0.12525....$ the calculator may show the answer as 0.1252525253 because it has rounded up the next 2 because of the next 5 that doesn't fit on the screen.

99. $32.4\text{ MJ}+26.704\text{ MJ}+36.23\text{ MJ}=95.334\text{ MJ}$. The answer must be to the same precision as the least precise measurement. The answer is 95.3 MJ.

100. The difference in speed of the two jets is the speed of the second jet (1450 km/h) minus the speed of the first jet (938 km/h): $1450\text{ km/h}-938\text{ km/h}=512\text{ km/h}$. But since 1450 is rounded to the tens precision, the answer must be as well. The answer is 510 km/h.

101.
1 K = 1024 bytes

$$256\text{ K}\cdot\left(\frac{1024\text{ bytes}}{1\text{ K}}\right) = 262\ 144\text{ bytes}$$

102.

$$V = (15.2\ \Omega + 5.64\ \Omega + 101.23\ \Omega)\times 3.55\text{ A}$$

$$V = 122.07\ \Omega\times 3.55\text{ A}$$

$$V = 433.3485\text{ V}$$

$V = 433$ V to three significant digits

103.

$$\frac{100(40.63+52.96)}{105.30+52.96} = 59.1386\ \% = 59.14\ \%$$ to four signficiant digits

104.

$$T = \frac{50.45(9.80)}{1+100.9\div 23} = 91.779\text{ N} = 92\text{ N}$$ to two significant digits

1.4 Exponents

1.
$(3k)^2 = 3^2 k^2 = 9k^2$

2.
$2x^0 = 2(1) = 2$

3.
$$\left(\frac{b^2t}{a^3x}\right)^{-2} = \frac{\left(b^2t\right)^{-2}}{\left(a^3x\right)^{-2}} = \frac{\left(a^3x\right)^2}{\left(b^2t\right)^2} = \frac{\left(a^3\right)^2\left(x^2\right)}{\left(b^2\right)^2\left(t^2\right)} = \frac{a^6x^2}{b^4t^2}$$

4.
$8-(-1)^3-2(-3)^2 = 8-(-1)-2(9) = 8+1-18 = -9$

5.
$x^3x^4 = x^{3+4} = x^7$

6.
$y^2y^7 = y^{2+7} = y^9$

7.
$2b^4b^2 = 2b^{4+2} = 2b^6$

8.
$3k^5k = 3k^{5+1} = 3k^6$

9.
$$\frac{m^5}{m^3} = m^{5-3} = m^2$$

10.
$$\frac{2x^6}{x} = 2x^{6-1} = 2x^5$$

11.
$$\frac{-n^5}{7n^9} = -\frac{n^{5-9}}{7} = -\frac{n^{-4}}{7} = \frac{-1}{7n^4}$$

12.
$$\frac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \frac{3}{s^3}$$

13.
$\left(P^2\right)^4 = P^{2(4)} = P^8$

14.
$\left(x^8\right)^3 = x^{8(3)} = x^{24}$

15.
$\left(2\pi\right)^3 = (2)^3(\pi^3) = 8\pi^3$

16.
$\left(ax\right)^5 = a^5x^5$

17.
$\left(aT^2\right)^{30} = a^{30}T^{2(30)} = a^{30}T^{60}$

18.
$\left(3r^2\right)^3 = (3)^3r^{2(3)} = 27r^6$

19.
$$\left(\frac{2}{b}\right)^3 = \frac{(2)^3}{b^3} = \frac{8}{b^3}$$

20.
$$\left(\frac{F}{t}\right)^{20} = \frac{F^{20}}{t^{20}}$$

21.
$$\left(\frac{x^2}{2}\right)^4 = \frac{x^{2(4)}}{(2)^4} = \frac{x^8}{16}$$

22.
$$\left(\frac{3}{n^3}\right)^3 = \frac{(3)^3}{n^{3(3)}} = \frac{27}{n^9}$$

23.
$\left(8a\right)^0 = 1$

24.
$-v^0 = (-1)(1) = -1$

25.
$-3x^0 = -3(1) = -3$

26.
$-(-2)^0 = -1(1) = -1$

27.

$6^{-1} = \frac{1}{6^1} = \frac{1}{6}$

28.

$-w^{-5} = -\frac{1}{w^5}$

29.

$\frac{1}{R^{-2}} = R^2$

30.

$\frac{1}{-t^{-48}} = -t^{48}$

31.

$(-t^2)^7 = \left[(-1)(t^2)\right]^7 = (-1)^7 t^{2(7)} = (-1)t^{14} = -t^{14}$

32.

$(-y^3)^5 = \left[(-1)(y^3)\right]^5 = (-1)^5 y^{3(5)} = (-1)y^{15} = -y^{15}$

33.

$-\frac{L^{-3}}{L^{-5}} = -L^{-3-(-5)} = -L^2$

34.

$2i^{40}i^{-70} = 2i^{40+(-70)} = 2i^{-30} = \frac{2}{i^{30}}$

35.

$\frac{2v^4}{(2v)^4} = \frac{2v^4}{(2)^4(v^4)} = \frac{2v^4}{16v^4} = \frac{1}{8}$

36.

$\frac{x^2x^3}{(x^2)^3} = \frac{x^{2+3}}{x^{2(3)}} = \frac{x^5}{x^6} = \frac{1}{x}$

37.

$\frac{(n^2)^4}{(n^4)^2} = \frac{n^{2(4)}}{n^{4(2)}} = \frac{n^8}{n^8} = 1$

38.

$\frac{(3t)^{-1}}{3t^{-1}} = \frac{(3)^{-1}t^{-1}}{3t^{-1}} = \frac{t}{3(3)t} = \frac{1}{9}$

39.

$(\pi^0 x^2 a^{-1})^{-1} = \pi^{0(-1)}x^{2(-1)}a^{-1(-1)} = \pi^0 x^{-2} a^1 = \frac{a}{x^2}$

40.

$$(3m^{-2}n^{4})^{-2} = (3)^{-2}m^{-2(-2)}n^{4(-2)} = \frac{m^4}{9n^8}$$

41.

$$(-8g^{-1}s^{3})^{2} = (-8)^{2}g^{-1(2)}s^{3(2)} = \frac{64s^6}{g^2}$$

42.

$$ax^{-2}(-a^2x)^3 = ax^{-2}(-1)^3(a^{2(3)})x^3 = -\frac{a(a^6)x^3}{x^2} = -a^{1+6}x^{3-2} = -a^7x$$

43.

$$\left(\frac{4x^{-1}}{a^{-1}}\right)^{-3} = \frac{(4)^{-3}x^{-1(-3)}}{a^{-1(-3)}} = \frac{x^3}{64a^3}$$

44.

$$\left(\frac{2b^2}{y^5}\right)^{-2} = \frac{(2)^{-2}b^{2(-2)}}{y^{5(-2)}} = \frac{b^{-4}}{4y^{-10}} = \frac{y^{10}}{4b^4}$$

45.

$$\frac{15n^2T^5}{3n^{-1}T^6} = \frac{5n^{2-(-1)}}{T} = \frac{5n^3}{T}$$

46.

$$\frac{(nRT^{-2})^{32}}{R^{-2}T^{32}} = \frac{n^{32}R^{32-(-2)}T^{-2(32)}}{T^{32}} = \frac{n^{32}R^{34}T^{-64}}{T^{32}} = \frac{n^{32}R^{34}}{T^{32-(-64)}} = \frac{n^{32}R^{34}}{T^{96}}$$

47.

Using the quotient rule, we can simplify the values inside of the bracket:

$$\left(\frac{16x^2y^{-2}}{xy^{-1}}\right)^{-4} = \left(\frac{16}{1}\cdot\frac{x^2}{x^1}\cdot\frac{y^{-2}}{y^{-1}}\right)^{-4} = \left(16\cdot x^{2-1}\cdot y^{-2-(-1)}\right)^{-4} = \left(16xy^{-1}\right)^{-4}$$

Next, we apply the power outside of the bracket to the coefficient multiple, and each algebraic multiple: $\left(16xy^{-1}\right)^{-4} = 16^{-4}\cdot x^{1(-4)}\cdot y^{(-1)(-4)} = (2^4)^{-4}\cdot x^{-4}\cdot y^4 = 2^{-16}x^{-4}y^4$. Finally, we use the negative exponents rule to eliminate negative powers: $2^{-16}x^{-4}y^4 = \dfrac{y^4}{2^{16}x^4}$ or if the coefficient is written numerically instead: $\dfrac{y^4}{2^{16}x^4} = \dfrac{y^4}{65536x^4}$.

48.

$$\left(\frac{7x^{-3}y^5}{x^2y^{-2}}\right)^{-2} = \left(7x^{-3-2}y^{5-(-2)}\right)^{-2} = \left(7x^{-5}y^7\right)^{-2} = 7^{-2}x^{(-5)(-2)}y^{(7)(-2)} = \frac{x^{10}}{49y^{14}}$$

49.

$$7(-4)-(-5)^2 = -28-25 = -53$$

50.
$6+(-2)^5-(-2)(8)=6+(-32)-(-16)=6-32+16=-10$

51.
$-(-26.5)^2-(-9.85)^3=-(702.25)-(-955.671625)=253.421625$
which gets rounded to 253 because 702.25 and –955.671625 are both accurate to only three significant digits due to the original numbers having only three significant digits.

52.
$-0.711^2-(-0.809)^6=(-1)(0.711)^2-(-0.809)^6=(-1)(0.505521)-(0.2803439122)=-0.7858649122$
which gets rounded to three significant digits: –0.786.

53.
$$\frac{3.07(-1.86)}{(-1.86)^4+1.596}=\frac{-5.7102}{11.96883216+1.596}=\frac{-5.7102}{13.56483216}=-0.420956185$$
which gets rounded to three significant digits: –0.421.

54.
$$\frac{15.66^2-(-4.017)^4}{1.044(-3.68)}=\frac{245.2356-260.379822692}{-3.84192}=\frac{-15.144222692}{-3.84192}=3.941837074$$
which gets rounded to three significant digits: 3.94.

55.

$$\begin{aligned}2.38(-10.7)^2-\frac{254}{1.17^3}&=2.38(114.49)-\frac{254}{1.601613}\\&=272.4862-158.5901213\\&=113.8960787\end{aligned}$$
which gets rounded to three significant digits: 114

56.
$$\begin{aligned}&0.513(-2.778)-(-3.67)^3+\frac{0.889^4}{1.89-1.09^2}\\&=-1.425114-(-49.430863)+\frac{0.624607283}{1.89-1.1881}\\&=48.005749+\frac{0.624607283}{0.7019}\\&=48.005749+0.889880728\\&=48.895629728\end{aligned}$$

which gets rounded to three significant digits: 48.9.

57. Yes: $\left(\frac{1}{x^{-1}}\right)^{-1}=\frac{1^{-1}}{x^{-1(-1)}}=\frac{1}{x}$, which is the reciprocal of x.

58.

$$\left(\frac{0.2-5^{-1}}{10^{-2}}\right)^0=\left(\frac{0.2-\frac{1}{5}}{\frac{1}{100}}\right)^0=\left(\frac{0}{0.01}\right)^0=0^0\neq 1$$, since $a^0=1$ requires that $a\neq 0$.

59.

If $a^3=5$, then

$a^{12}=a^{3(4)}$

$a^{12}=\left(a^3\right)^4$

$a^{12}=(5)^4$

$a^{12}=625$

60.

If $\frac{1}{a^2}<\frac{1}{a}$, then for any negative value of a, a will be negative, and a^2 will be positive, making all values of $\frac{1}{a^2}$ greater than $\frac{1}{a}$.

61.

$(x^a\cdot x^{-a})^5=(x^{a-a})^5=(x^o)^5=x^{0(5)}=x^0=1$, provided that $x\neq 0$.

62.

$(y^{a-b}\cdot y^{a+b})^2=(y^{a-b+a+b})^2=(y^{2a})^2=y^{2a(2)}=y^{4a}$.

63.

$$\begin{aligned}\left(\frac{kT}{hc}\right)^3(GkThc)^2c&=\frac{k^3T^3}{h^3c^3}\bullet(G^2k^2T^2h^2c^2)c\\&=\frac{k^3T^3}{h^3c^3}\cdot(G^2k^2T^2h^2c^3)\\&=\frac{(G^2k^{2+3}T^{2+3}c^{3-3})}{h^1}\\&=\frac{G^2k^5T^5}{h}\end{aligned}$$

64.

$$GmM(mr)^{-1}(r^{-2})=\frac{GmM}{mr^{1+2}}=\frac{GM}{r^3}$$

65.

$$\pi\left(\frac{r}{2}\right)^3\left(\frac{4}{3\pi r^2}\right)=\pi\left(\frac{r^3}{8}\right)\left(\frac{4}{3\pi r^2}\right)=\frac{4r}{24}=\frac{r}{6}$$

66.

$$\frac{gM}{2\pi fC(2\pi fM)^2} = \frac{gM}{2\pi fC(4\pi^2 f^2 M^2)}$$
$$= \frac{gM}{8\pi^3 f^3 CM^2}$$
$$= \frac{g}{8\pi^3 f^3 CM}$$

67.

$$2500\left(1+\frac{0.042}{4}\right)^{24} = \$2500(1.0105)^{24}$$
$$= \$2500(1.28490602753)$$
$$= \$3212.26700688$$
$$= \$3212.27$$

68.

$$\frac{6.85(1000-20(6.85)^2+(6.85)^3}{1850} = \frac{6.85(1000-20(46.9225)+(321.419125)}{1850}$$
$$= \frac{6.85(1321.419125-938.45)}{1850}$$
$$= \frac{6.85(382.969125)}{1850}$$
$$= \frac{2623.33850625}{1850}$$
$$= 1.418020814$$
$$= 1.42 \text{ cm}$$

1.5 Scientific Notation

1.
$8.06 \times 10^3 = 8060$

2.
$$\begin{aligned} 750\ 000\ 000\ 000^{-1} &= (7.5 \times 10^{11})^{-1} \\ &= 7.5^{-1} \times 10^{-11} \\ &= 0.1333... \times 10^{-11} \\ &= 1.33 \times 10^{-12} \end{aligned}$$
rounded to three significant digits.

3.
$4.5 \times 10^4 = 45\ 000$

4.
$6.8 \times 10^7 = 68\ 000\ 000$

5.
$2.01 \times 10^{-3} = 0.002\ 01$

6.
$9.61 \times 10^{-5} = 0.000\ 096\ 1$

7.
$3.23 \times 10^0 = 3.23 \times 1 = 3.23$

8.
$8 \times 10^0 = 8 \times 1 = 8$

9.
$1.86 \times 10 = 18.6$

10.
$1 \times 10^{-1} = 0.1$

11.
$4000 = 4 \times 10^3$

12.
$56\ 000 = 5.6 \times 10^4$

13.
$0.0087 = 8.7 \times 10^{-3}$

14.
$0.7 = 7 \times 10^{-1}$

15.
$609\ 000\ 000 = 6.09 \times 10^8$

16.
$100 = 1\times10^{2}$

17.
$0.063 = 6.3\times10^{-2}$

18.
$0.000\ 090\ 8 = 9.08\times10^{-5}$

19.
$1 = 1\times10^{0}$

20.
$10 = 1\times10^{1}$

21.
$28\ 000(2\ 000\ 000\ 000) = 2.8\times10^{4}(2\times10^{9}) = 5.6\times10^{13}$

22.
$50\ 000(0.006) = 5\times10^{4}(6\times10^{-3}) = 300 = 3\times10^{2}$

23.
$$\frac{88\ 000}{0.0004} = \frac{8.8\times10^{4}}{4\times10^{-4}} = 2.2\times10^{8}$$

24.
$$\frac{0.000\ 03}{6\ 000\ 000} = \frac{3\times10^{-5}}{6\times10^{6}} = 5\times10^{-12}$$

25.
$2\times10^{-35} + 3\times10^{-34} = 0.2\times10^{-34} + 3\times10^{-34} = 3.2\times10^{-34}$

26.
$5.3\times10^{12} - 3.7\times10^{10} = 530\times10^{10} - 3.7\times10^{10} = 526.3\times10^{10} = 5.263\times10^{12}$

27.
$(1.2\times10^{29})^{3} = 1.2^{3}\times10^{29(3)} = 1.728\times10^{87}$

28
$(2\times10^{-16})^{-5} = 2^{-5}\times10^{-16(-5)} = 0.031\ 25\times10^{80} = 3.125\times10^{78}$

29.
$1280(865\ 000)(43.8) = 4.849\ 536\times10^{10}$
which gets rounded to 4.85×10^{10}.

30.
$0.0000569(3\ 190\ 000) = 181.511$
which in scientific notation and rounded is 1.82×10^{2}.

31.
$$\frac{0.0732(6710)}{0.00134(0.0231)}=\frac{491.172}{0.000\,030\,954}=1.586\,780\,3\times10^{7}$$
which gets rounded to 1.59×10^{7}.

32.
$$\frac{0.00452}{2430(97100)}=\frac{0.00452}{235\,953\,000}=1.915\,635\,741\times10^{-11}$$
which gets rounded to 1.92×10^{-11}.

33.
$$(3.642\times10^{-8})(2.736\times10^{5})=9.964\,512\times10^{-3}$$
which gets rounded to 9.965×10^{-3}.

34.
$$\frac{(7.309\times10^{-1})^{2}}{5.9843(2.5036\times10^{-20})}=\frac{0.534\,214\,81}{1.497\,870\,29\times10^{-19}}=3.566\,567\,233\,94\times10^{18}$$
which gets rounded to 3.567×10^{18}.

35.
$$\frac{(3.69\times10^{-7})(4.61\times10^{21})}{0.0504}=\frac{1.701\,09\times10^{15}}{0.0504}=3.375\,178\,571\,42\times10^{16}$$
which gets rounded to 3.38×10^{16}.

36.
$$\frac{(9.907\times10^{7})(1.08\times10^{12})^{2}}{(3.603\times10^{-5})(2054)}=\frac{(9.907\times10^{7})(1.1664\times10^{24})}{0.074\,005\,62}=\frac{1.015\,552\,48\times10^{32}}{0.074\,005\,62}=1.561\,438\,820\,45\times10^{33}$$ which gets rounded to 1.56×10^{33}.

37.
$2\,000\,000$ kW $=2\times10^{6}$ kW

38.
$17\,200\,000\,000$ bytes $=1.72\times10^{10}$ bytes

39.
$0.000\,003$ W $=3\times10^{-6}$ W

40.
0.0075 mm $=7.5\times10^{-3}$ mm

41.
$1\,200\,000\,000$ Hz $=1.2\times10^{9}$ Hz

42. $1.84\times10^{12}=1\,840\,000\,000\,000$

43. $12\,000\,000\,000\ \text{m}^2=1.2\times10^{10}\ \text{m}^2$

44.
3.086×10^{16} m = 30 860 000 000 000 000 m

45.
1.6×10^{-12} W = 0.000 000 000 001 6 W

46.
2.4×10^{-43} = 0.000 000 000 000 000 000 000 000 000 000 000 000 000 000 24

47.
$$\begin{aligned} 2\ 000\ 000 \text{ kW} &= 2\times10^{6} \text{ kW} \\ &= 2\times10^{6}\times10^{3} \text{ W} \\ &= 2\times10^{9} \text{ W} \\ &= 2 \text{ GW} \end{aligned}$$

48.
$$\begin{aligned} 17\ 200\ 000\ 000 \text{ bytes} &= 17.2\times10^{9} \text{ bytes} \\ &= 17.2 \text{ gigabytes} \end{aligned}$$

49.
$$\begin{aligned} 0.000\ 003 \text{ W} &= 3\times10^{-6} \text{ W} \\ &= 3\ \mu\text{W} \end{aligned}$$

50.
$$\begin{aligned} 0.0075 \text{ mm} &= 7.5\times10^{-3} \text{ mm} \\ &= 7.5\times10^{-3}\times10^{-3} \text{ m} \\ &= 7.5\times10^{-6} \text{ m} \\ &= 7.5\ \mu\text{m} \end{aligned}$$

51.
$$\begin{aligned} 1\ 200\ 000\ 000 \text{ Hz} &= 1.2\times10^{9} \text{ Hz} \\ &= 1.2 \text{ GHz} \end{aligned}$$

52.

(a) $2300 = 2.3\times10^{3}$
(b) $0.23 = 230\times10^{-3}$
(c) $23 = 23\times10^{0}$

53.
(a) $8\ 090\ 000 = 8.09\times10^{6}$
(b) $809\ 000 = 809\times10^{3}$
(c) $0.0809 = 80.9\times10^{-3}$

54.

(a) $\text{googol} = 1\times10^{100} = 10^{100}$

(b) $\text{googolplex} = 10^{\text{googol}} = 10^{10^{100}}$

55.

$\text{googol} = 10^{100}$, so to find the ratio $\dfrac{10^{100}}{10^{79}} = 10^{100-79} = 10^{21}$

A googol is 10^{21} times larger than the number of electrons in the universe.

56.

$\text{earth's diameter} = \dfrac{\text{sun's diameter}}{110} = \dfrac{1.4\times10^{9}\text{ m}}{110} = 1.27272\times10^{7}\text{ m}$ which is rounded to 1.3×10^{7} m.

57.

$2^{30} = 1\ 073\ 741\ 824 = 1.073\ 741\ 824\times10^{9} \approx 1\times10^{9}$

58.

$\dfrac{7.5\times10^{-15}\text{s}}{\text{addition}}\times5.6\times10^{6}\text{additions} = 4.2\times10^{-8}\text{s}$

59.

We have a ratio of $\dfrac{+0.6\ °\text{C}}{50\text{ years}} = \dfrac{+0.6\ °\text{C}}{50\ \cancel{\text{years}}}\left(\dfrac{1\ \cancel{\text{y}}}{365\text{ days}}\right) = 0.000\ 0329\ °\text{C/day}.$

In scientific notation, we would express the final result as: $0.000\ 0329\ °\text{C/day} = 3.29\times10^{-5}\ °\text{C/day}.$

60.

$d = 2.998\times10^{8}\dfrac{\text{m}}{\text{s}}\times0.078\text{ s} = 2.3\times10^{7}$

61.

(a)

$1\text{ day}\times\dfrac{24\text{ h}}{\text{day}}\times\dfrac{60\text{ min}}{\text{h}}\times\dfrac{60\text{ s}}{\text{min}} = 86400\text{ s} = 8.64\times10^{4}\text{ s}$

(b)

$100\text{ year}\times\dfrac{365.25\text{ day}}{\text{year}}\times\dfrac{24\text{ h}}{\text{day}}\times\dfrac{60\text{ min}}{\text{h}}\times\dfrac{60\text{ s}}{\text{min}} = 3\ 155\ 760\ 000\text{ s} = 3.155\ 760\ 0\times10^{9}\text{ s}$

62.

$\dfrac{1.66\times10^{-27}\text{ kg}}{\text{amu}}\times\dfrac{1.6\times10^{1}\text{ amu}}{\text{oxygen atoms}}\times1.25\times10^{8}\text{ oxygen atoms} = 3.32\times10^{-18}\text{ kg}$

63.

$W = kT^{4}$

$W = 5.7\times10^{-8}\text{ W/K}^{4}\times(3.03\times10^{2}\text{ K})^{4}$

$W = 5.7\times10^{-8}\text{ W/K}^{4}\times8.428\ 892\ 481\times10^{9}\text{ K}^{4}$

$W = 4.804\ 468\ 714\ 17\times10^{2}\text{ W}$

$W = 4.8\times10^{2}\text{ W}$

64.

$$R=\frac{k}{d^2}=\frac{2.196\times10^{-8}\ \Omega\cdot\text{m}^2}{\left(7.998\times10^{-5}\text{m}\right)^2}=\frac{2.196\times10^{-8}\ \Omega\cdot\text{m}^2}{6.396\ 800\ 4\times10^{-9}\text{m}^2}=3.432\ 966\ 268\ 57\ \Omega=3.433\ \Omega$$

65.

$$\frac{1.496\times10^{8}\ \text{km}}{\text{AU}}\times\frac{\text{AU}}{4.99\times10^{2}\text{s}}=2.997\ 995\ 991\ 98\times10^{5}\ \text{km/s}=2.998\times10^{5}\ \text{km/s}$$

This is the same speed mentioned in Question 60 $\left(2.998\times10^{5}\ \text{km/s}=2.998\times10^{8}\ \text{m/s}\right)$.

1.6 Roots and Radicals

1.
$-\sqrt[3]{64} = -\sqrt[3]{(4)^3} = -4$

2.
$\sqrt{(15)(5)}$
Neither 15 nor 5 is a perfect square, so this expression is not as useful. However, if we further factor the 15 to $\sqrt{(3)(5)(5)} = \sqrt{3(5)^2} = 5\sqrt{3}$, the result can still be obtained.

3.
$\sqrt{16\times 9} = \sqrt{144} = \sqrt{12^2} = 12$

4.
$-\sqrt{-64}$ is still imaginary because an even root (in this case $n = 2$) of a negative number is imaginary, regardless of the numerical factor placed in front of the root.

5.
$\sqrt{49} = \sqrt{7\cdot 7} = 7$

6.
$\sqrt{343} = \sqrt{(7\cdot 7)\cdot 7} = 7\sqrt{7}$

7.
$-\sqrt{45} = -\sqrt{(3\cdot 3)\cdot 5} = -3\sqrt{5}$

8.
$-\sqrt{72} = -\sqrt{2\cdot 6\cdot 6} = -\left(2\cdot 6^2\right)^{1/2} = -2^{1/2}6^1 = -6\sqrt{2}$

9.
$-\sqrt{64} = -\sqrt{(8)(8)} = -8$

10.
$\sqrt{0.25} = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2} = 0.5$

11.
$\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{\sqrt{9}}{\sqrt{100}} = \frac{3}{10} = 0.3$

12.
$-\sqrt{900} = -\sqrt{(9)(100)} = -\sqrt{9}\times\sqrt{100} = -3\times 10 = -30$

13.
$\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

14.
$\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

15.
$\sqrt[3]{-216} = \sqrt{(-6)^3} = -6$

16.
$\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$

17.
$\left(\sqrt{5}\right)^2 = \sqrt{5} \times \sqrt{5} = 5$

18.
$\left(\sqrt[3]{31}\right)^3 = \sqrt[3]{31} \times \sqrt[3]{31} \times \sqrt[3]{31} = 31$

19.
$\left(-\sqrt[3]{-47}\right)^3 = \left(-1\right)^3 \left(\sqrt[3]{-47}\right)^3 = (-1)(-47) = 47$

20.
$\left(\sqrt[5]{-23}\right)^5 = -23$

21.
$\left(-\sqrt[4]{53}\right)^4 = \left(-1\right)^4 \left(\sqrt[4]{53}\right)^4 = (1)(53) = 53$

22.
$-\sqrt{32} = -\sqrt{(16)(2)} = -\sqrt{16} \times \sqrt{2} = -4\sqrt{2}$

23.
$\sqrt{18} = \sqrt{(9)(2)} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

24.
$\sqrt{1200} = \sqrt{(100)(4)(3)} = \sqrt{100} \times \sqrt{4} \times \sqrt{3} = 10 \times 2 \times \sqrt{3} = 20\sqrt{3}$

25.
$2\sqrt{84} = 2\sqrt{(4)(21)} = 2 \times \sqrt{4} \times \sqrt{21} = 2 \times 2 \times \sqrt{21} = 4\sqrt{21}$

26.
$$\frac{\sqrt{108}}{2} = \frac{\sqrt{(36)(3)}}{2} = \frac{\sqrt{36} \times \sqrt{3}}{2} = \frac{6 \times \sqrt{3}}{2} = 3\sqrt{3}$$

27.
$$\sqrt{\frac{80}{7-3}} = \sqrt{\frac{80}{4}} = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2 \times \sqrt{5} = 2\sqrt{5}$$

28.

$\sqrt{81\times10^2} = \sqrt{81}\times\sqrt{10^2} = 9\times10 = 90$

29.

$\sqrt[3]{8^2} = \sqrt[3]{64} = \sqrt[3]{4^3} = 4$

30.

$\sqrt[4]{9^2} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$

31.

$$\frac{5^3\sqrt{8}}{3^4\sqrt[3]{125}} = \frac{5^3\left(2^3\right)^{1/2}}{3^4\left(5^3\right)^{1/3}} = \frac{5^3 2^{3/2}}{3^4 5^1} = \frac{5^2 2^{3/2}}{3^4} = \frac{5^2 2^1 2^{1/2}}{3^4} = \frac{50\sqrt{2}}{81}$$

32.

$$\frac{2^4\sqrt[4]{512}}{7\sqrt{121}} = \frac{2^4(2^9)^{1/4}}{7\cdot\left(11^2\right)^{1/2}} = \frac{2^4 2^{9/4}}{7\cdot11} = \frac{2^4 2^2 2^{1/4}}{77} = \frac{2^6\sqrt[4]{2}}{77} = \frac{64\sqrt[4]{2}}{77}$$

33.

$\sqrt{36+64} = \sqrt{100} = \sqrt{10^2} = 10$

34.

$\sqrt{25+144} = \sqrt{169} = \sqrt{13^2} = 13$

35.

$\sqrt{3^2+9^2} = \sqrt{9+81} = \sqrt{90} = \sqrt{(9)(10)} = \sqrt{9}\times\sqrt{10} = 3\sqrt{10}$

36.

$\sqrt{8^2-4^2} = \sqrt{64-16} = \sqrt{48} = \sqrt{(16)(3)} = \sqrt{16}\times\sqrt{3} = 4\sqrt{3}$

37.

$\sqrt{85.4} = 9.24121204171$, which is rounded to 9.24

38.

$\sqrt{3762} = 61.3351449007$, which is rounded to 61.34

39.

$\sqrt{0.8152} = 0.9028842672$, which is rounded to 0.9029

40.

$\sqrt{0.0627} = 0.25039968051$, which is rounded to 0.250

41.
(a)
$\sqrt{1296+2304}=\sqrt{3600}=60$, which is expressed as 60.00
(b)
$\sqrt{1296}+\sqrt{2304}=36+48=84$, which is expressed as 84.00

42.
(a)
$\sqrt{10.6276+2.1609}=\sqrt{12.7885}=3.57610122899$,which is rounded to 3.57610
(b)
$\sqrt{10.6276}+\sqrt{2.1609}=3.26+1.47=4.73$,which is expressed as 4.7300

43.
(a)
$$\begin{aligned}\sqrt{0.0429^2-0.0183^2}&=\sqrt{0.00184041-0.00033489}\\&=\sqrt{0.00150552}\\&=0.03880103091\\&=0.0388\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{0.0429^2}-\sqrt{0.0183^2}&=0.0429-0.0183\\&=0.0246\end{aligned}$$

44.
(a)
$$\begin{aligned}\sqrt{3.625^2+0.614^2}&=\sqrt{13.140625+0.376996}\\&=\sqrt{13.517621}\\&=3.67663174658\\&=3.677\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{3.625^2}+\sqrt{0.614^2}&=3.625+0.614\\&=4.239\end{aligned}$$

45.
$\sqrt{207s}=\sqrt{(207)(46)}=\sqrt{9522}=97.5807358037=98\text{ km/h}$

46.
$$\begin{aligned}\sqrt{Z^2-X^2}&=\sqrt{(5.362\ \Omega)^2-(2.875\ \Omega)^2}\\&=\sqrt{28.751044\ \Omega^2-8.265625\ \Omega^2}\\&=\sqrt{20.485419\ \Omega^2}\\&=4.52608208056\ \Omega\\&=4.526\ \Omega\end{aligned}$$

47.

$$\sqrt{\frac{B}{d}} = \sqrt{\frac{2.18\times10^9\ \text{Pa}}{1.03\times10^3\ \text{kg/m}^3}}$$
$$= \sqrt{2116504.85436\ \frac{\text{N/m}^2}{\text{kg/m}^3}}$$
$$= \sqrt{2116504.85436\ \frac{\left(\text{kg}\cdot\text{m/s}^2\right)/\text{m}^2}{\text{kg/m}^3}}$$
$$= \sqrt{2116504.85436\ \text{m}^2/\text{s}^2}$$
$$= 1454.82124481\ \text{m/s}$$
$$= 1450\ \text{m/s}$$

48.

$$\sqrt{40m} = \sqrt{(40)(75)}$$
$$= \sqrt{3000}$$
$$= 54.7722557505$$
$$= 55\ \text{m/s}$$

49.

$$\sqrt{w^2+h^2} = \sqrt{(93.0\ \text{cm})^2+(52.1\ \text{cm})^2}$$
$$= \sqrt{8649\ \text{cm}^2+2714.41\ \text{cm}^2}$$
$$= \sqrt{11363.41\ \text{cm}^2}$$
$$= 106.599296432\ \text{cm}$$
$$= 107\ \text{cm}$$

50.

$$100\left(1-\sqrt{\frac{V}{C}}\right) = 100\left(1-\sqrt{\frac{15000}{22000}}\right)$$
$$= 100\left(1-\sqrt{0.68181818181}\right)$$
$$= 100\left(1-0.82572282684\right)$$
$$= 100(0.17427717615)$$
$$= 17.427717615\ \%$$
$$= 17\ \%$$

51.

$$\sqrt{gd} = \sqrt{(9.8)(3500)}$$
$$= \sqrt{34300}$$
$$= 185.20259$$
$$= 190\ \text{m/s}$$

52.

$\sqrt{1.27\times10^4 h+h^2} = \sqrt{1.27\times10^4(9500)+(9500)^2}$

$=\sqrt{1.2065\times10^8+9.025\times10^7}$

$=\sqrt{2.109\times10^8}$

$=14522.3965$

which is rounded to 15000 km

53.

$\sqrt{a^2}=a$ is not necessarily true for negative values of a because a^2 will be a positive number, regardless whether a is negative or positive. The principal root calculated is assumed to be positive, but there are always two solutions to a square root, $\sqrt{a^2}=\pm a$ since $(+a)^2=a^2$ and $(-a)^2=a^2$ (see the introduction to this chapter section), so it is sometimes true and sometimes false for negative values of a, depending on which root solution is desired. If *only principal roots* are considered, then it will *not* be true for negative values of a. For example, $\sqrt{(-4)^2}=\sqrt{16}=4\neq-4$.

54.

(a) $x>\sqrt{x}$ when $x>1$. Any number greater than 1 will have a square root that is smaller than itself. For example, $2>\sqrt{2}=1.41$

(b) $x=\sqrt{x}$ when $x=1$ or $x=0$ because the only numbers that are their own squares are 0 and 1 (i.e., $0^2=0$ and $1^2=1$).

(c) $x<\sqrt{x}$ when $0<x<1$. Any number between 0 and 1 will have a square root larger than itself. For example, $0.25<\sqrt{0.25}=0.5$

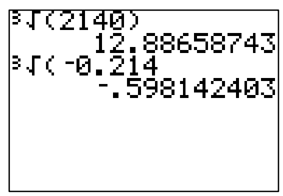
55.

(a)

$\sqrt[3]{2140}=12.8865874254$, which is rounded to 12.9

(b)

$\sqrt[3]{-0.214}=-0.59814240297$, which is rounded to –0.598



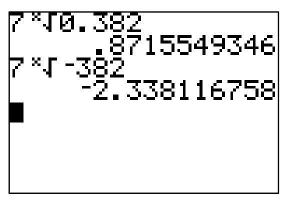
56.

(a)

$\sqrt[7]{0.382}=0.87155493458$, which is rounded to 0.872

(b)

$\sqrt[7]{-382}=-2.33811675837$, which is rounded to –2.34



57.

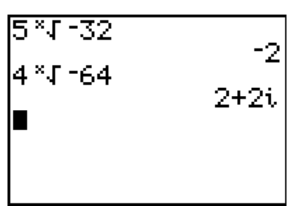
(a) $\sqrt[4]{-81}$ is imaginary since it is an even root of a negative number.

(b) $\sqrt[7]{-128}$ is real since it is an odd root of a negative number.

58.

(a) $\sqrt[5]{-32}=-2$ is real since it is an odd root of a negative number.

(b) $\sqrt[4]{-64}$ is imaginary since it is an even root of a negative number.



59.

$$
\begin{aligned}
f &= \frac{1}{2\pi\sqrt{LC}} \\
&= \frac{1}{2(3.1416)\sqrt{0.250(40.52\times10^{-6})}} \\
&= \frac{1}{6.2832\sqrt{10.0625\times10^{-6}}} \\
&= \frac{1}{6.2832(0.003172144385)} \\
&= \frac{1}{0.0199312175998} \\
&= 50.172549
\end{aligned}
$$

which is rounded to 50.2 Hz

60.

$$
\begin{aligned}
\text{standard deviation} &= \sqrt{\text{variance}} \\
&= \sqrt{80.5\ \text{kg}^2} \\
&= 8.972179222\ \text{kg}
\end{aligned}
$$

which is rounded to 8.97 kg

1.7 Addition and Subtraction of Algebraic Expressions

1.
$3x+2y-5y=3x-3y$

2.
$3c-(2b-c)=3c-2b+c=-2b+4c$

3.
$$\begin{aligned}3ax-[(ax-5s)-2ax]&=3ax-[ax-5s-2ax]\\&=3ax-[-ax-5s]\\&=3ax+ax+5s\\&=4ax+5s\end{aligned}$$

4.
$$\begin{aligned}3a^2b-\{a-[2a^2b-(a+2b)]\}&=3a^2b-\{a-[2a^2b-a-2b]\}\\&=3a^2b-\{a-2a^2b+a+2b\}\\&=3a^2b-\{2a-2a^2b+2b\}\\&=3a^2b-2a+2a^2b-2b\\&=5a^2b-2a-2b\end{aligned}$$

5.
$5x+7x-4x=8x$

6.
$6t-3t-4t=-t$

7.
$2y-y+4x=y+4x$

8.
$4C+L-6C=-2C+L$

9.
$2F-2T-2+3F-T=5F-3T-2$

10.
$x-2y+3x-y+z=4x-3y+z$

11.
$a^2b-a^2b^2-2a^2b=-a^2b-a^2b^2$

12.
$xy^2-3x^2y^2+2xy^2=3xy^2-3x^2y^2$

13.
$s+(3s-4-s)=s+(2s-4)=s+2s-4=3s-4$

14.
$5+(3-4n+p)=5+3-4n+p=-4n+p+8$

15.
$v-(4-5x+2v)=v-4+5x-2v=-v+5x-4$

16.
$2a-(b-a)=2a-b+a=3a-b$

17.
$2-3-(4-5a)=-1-4+5a=5a-5$

18.
$\sqrt{A}+(h-2\sqrt{A})-3\sqrt{A}=\sqrt{A}+h-2\sqrt{A}-3\sqrt{A}=-4\sqrt{A}+h$

19.
$(a-3)+(5-6a)=a-3+5-6a=-5a+2$

20.
$(4x-y)-(-2x-4y)=4x-y+2x+4y=6x+3y$

21.
$-(t-2u)+(3u-t)=-t+2u+3u-t=-2t+5u$

22.
$2(x-2y)+(5x-y)=2x-4y+5x-y=7x-5y$

23.
$3(2r+s)-(-5s-r)=6r+3s+5s+r=7r+8s$

24.
$3(a-b)-2(a-2b)=3a-3b-2a+4b=a+b$

25.
$-7(6-3j)-2(j+4)=-42+21j-2j-8=19j-50$

26.
$-(5t+a^2)-2(3a^2-2st)=-5t-a^2-6a^2+4st=-7a^2+4st-5t$

27.
$$\begin{aligned}-[(4-6n)-(n-3)]&=-[4-6n-n+3]\\&=-[-7n+7]\\&=7n-7\end{aligned}$$

28.
$$\begin{aligned}-[(h-k)-(k-h)]&=-[h-k-k+h]\\&=-[2h-2k]\\&=-2h+2k\end{aligned}$$

29.

$$\begin{aligned}2[4-(t^2-5)] &= 2[4-t^2+5] \\ &= 2[-t^2+9] \\ &= -2t^2+18\end{aligned}$$

30.

$$\begin{aligned}3[-3-(a-4)] &= 3[-3-a+4] \\ &= 3[-a+1] \\ &= -3a+3\end{aligned}$$

31.

$$\begin{aligned}-2[-x-2a-(a-x)] &= -2[-x-2a-a+x] \\ &= -2[-3a] \\ &= 6a\end{aligned}$$

32.

$$\begin{aligned}-2[-3(x-2y)+4y] &= -2[-3x+6y+4y] \\ &= -2[-3x+10y] \\ &= 6x-20y\end{aligned}$$

33.

$$\begin{aligned}aZ-[3-(aZ+4)] &= aZ-[3-aZ-4] \\ &= aZ-[-aZ-1] \\ &= aZ+aZ+1 \\ &= 2aZ+1\end{aligned}$$

34.

$$\begin{aligned}9v-[6-(v-4)+4v] &= 9v-[6-v+4+4v] \\ &= 9v-[3v+10] \\ &= 9v-3v-10 \\ &= 6v-10\end{aligned}$$

35.

$$\begin{aligned}8c-\{5-[2-(3+4c)]\} &= 8c-\{5-[2-3-4c]\} \\ &= 8c-\{5-2+3+4c\} \\ &= 8c-\{6+4c\} \\ &= 8c-6-4c \\ &= 4c-6\end{aligned}$$

36.

$$\begin{aligned}7y-\{y-[2y-(x-y)]\} &= 7y-\{y-[2y-x+y]\} \\ &= 7y-\{y-[3y-x]\} \\ &= 7y-\{y-3y+x\} \\ &= 7y-\{-2y+x\} \\ &= 7y+2y-x \\ &= -x+9y\end{aligned}$$

37.

$$\begin{aligned}5p-(q-2p)-[3q-(p-q)] &= 5p-q+2p-[3q-p+q]\\ &= 5p-q+2p-[4q-p]\\ &= 7p-q-4q+p\\ &= 8p-5q\end{aligned}$$

38.

$$\begin{aligned}-(4-\sqrt{LC})-[(5\sqrt{LC}-7)-(6\sqrt{LC}+2)] &= -4+\sqrt{LC}-[5\sqrt{LC}-7-6\sqrt{LC}-2]\\ &= -4+\sqrt{LC}-[-\sqrt{LC}-9]\\ &= -4+\sqrt{LC}+\sqrt{LC}+9\\ &= 2\sqrt{LC}+5\end{aligned}$$

39.

$$\begin{aligned}-2\{-(4-x^2)-[3+(4-x^2)]\} &= -2\{-4+x^2-[3+4-x^2]\}\\ &= -2\{-4+x^2-3-4+x^2\}\\ &= -2\{2x^2-11\}\\ &= -4x^2+22\end{aligned}$$

40.

$$\begin{aligned}-\{-[-(x-2a)-b]-(a-x)\} &= -\{-[-x+2a-b]-a+x\}\\ &= -\{x-2a+b-a+x\}\\ &= -\{-3a+b+2x\}\\ &= 3a-b-2x\end{aligned}$$

41.

$$\begin{aligned}5V^2-(6-(2V^2+3)) &= 5V^2-(6-2V^2-3)\\ &= 5V^2-(-2V^2+3)\\ &= 5V^2+2V^2-3\\ &= 7V^2-3\end{aligned}$$

42.

$$\begin{aligned}-2F+2((2F-1)-5) &= -2F+2(2F-1-5)\\ &= -2F+2(2F-6)\\ &= -2F+4F-12\\ &= 2F-12\end{aligned}$$

43.

$$\begin{aligned}-(3t-(7+2t-(5t-6))) &= -(3t-(7+2t-5t+6))\\ &= -(3t-(-3t+13))\\ &= -(3t+3t-13)\\ &= -(6t-13)\\ &= -6t+13\end{aligned}$$

44.

$$\begin{aligned} a^2-2(x-5-(7-2(a^2-2x)-3x)) &= a^2-2(x-5-(7-2a^2+4x-3x)) \\ &= a^2-2(x-5-(7-2a^2+x)) \\ &= a^2-2(x-5-7+2a^2-x) \\ &= a^2-2(2a^2-12) \\ &= a^2-4a^2+24 \\ &= -3a^2+24 \end{aligned}$$

45.

$$\begin{aligned} -4[4R-2.5(Z-2R)-1.5(2R-Z)] &= -4[4R-2.5Z+5R-3R+1.5Z] \\ &= -4[6R-Z] \\ &= -24R+4Z \end{aligned}$$

46.

$$\begin{aligned} 3\{2.1e-1.3[f-2(e-5f)]\} &= 3\{2.1e-1.3[f-2e+10f]\} \\ &= 3\{2.1e-1.3[-2e+11f]\} \\ &= 3\{2.1e+2.6e-14.3f\} \\ &= 3\{4.7e-14.3f\} \\ &= 14.1e-42.9f \end{aligned}$$

47.

$$3D-(D-d)=3D-D+d=2D+d$$

48.

$$i_1-(2-3i_2)+i_2=i_1-2+3i_2+i_2=i_1+4i_2-2$$

49.

$$\begin{aligned} \left[\left(B+\frac{4}{3}\alpha\right)+2\left(B-\frac{2}{3}\alpha\right)\right]-\left[\left(B+\frac{4}{3}\alpha\right)-\left(B-\frac{2}{3}\alpha\right)\right] &= \left[B+\frac{4}{3}\alpha+2B-\frac{4}{3}\alpha\right]-\left[B+\frac{4}{3}\alpha-B+\frac{2}{3}\alpha\right] \\ &= [3B]-\left[\frac{6}{3}\alpha\right] \\ &= 3B-2\alpha \end{aligned}$$

50.

$$\begin{aligned} \text{Distance} &= 30\text{ km/h}\times(t-1)\text{h}+40\text{ km/h}\times(t+2)\text{ h} \\ &= 30(t-1)\text{ km}+40(t+2)\text{ km} \\ &= (30t-30+40t+80)\text{ km} \\ &= (70t+50)\text{ km} \end{aligned}$$

51.

$$\begin{aligned} \text{Memory} &= x(4\text{ terabytes}) + (x+25)(8\text{ terabytes}) \\ &= (4x+8x+200)\text{ terabytes} \\ &= (12x+200)\text{ terabytes} \end{aligned}$$

52.
We find the value of all shingles at $30 for one supplier, and subtract from it the value of all shingles at $20 for one supplier. Since we need to include the difference in value for both suppliers, we multiply this difference by 2.

$$\begin{aligned}\text{Difference} &= 2[(2n+1)(\$30)-(n-2)(\$20)]\\ &= \$\ 2[60n+30-20n+40]\\ &= \$\ 2[40n+70]\\ &= \$\ (80n+140)\end{aligned}$$

53.
(a)

$$\begin{aligned}(2x^2-y+2a)+(3y-x^2-b) &= 2x^2-y+2a+3y-x^2-b\\ &= x^2+2y+2a-b\end{aligned}$$

(b)

$$\begin{aligned}(2x^2-y+2a)-(3y-x^2-b) &= 2x^2-y+2a-3y+x^2+b\\ &= 3x^2-4y+2a+b\end{aligned}$$

54.

$$\begin{aligned}(3a^2+b-c^3)+(2c^3-2b-a^2)-(4c^3-4b+3) &= 3a^2+b-c^3+2c^3-2b-a^2-4c^3+4b-3\\ &= 2a^2+3b-3c^3-3\end{aligned}$$

55.

$$\begin{aligned}|a-b| &= |-(-a+b)|\\ &= |-(b-a)|\\ &= |-1\times(b-a)|\\ &= |-1|\times|(b-a)|\\ &= 1\times|(b-a)|\\ &= |(b-a)|\end{aligned}$$

56.
$(a-b)-c=a-b-c$
However,
$a-(b-c)=a-b+c$
Since they are not equivalent, subtraction is not associative.
For example, $(10-5)-2=5-2=3$ is not the same as $10-(5-2)=10-3=7$.

1.8 Multiplication of Algebraic Expressions

1.

$$\begin{aligned} 2s^3(-st^4)^3(4s^2t) &= 2s^3(-1)^3s^3t^{12}(4s^2t) \\ &= -2s^6t^{12}(4s^2t) \\ &= -8s^8t^{13} \end{aligned}$$

2.

$$\begin{aligned} -2ax(3ax^2-4yz) &= (-2ax)(3ax^2)-(-2ax)(4yz) \\ &= (-6a^2x^3)-(-8axyz) \\ &= -6a^2x^3+8axyz \end{aligned}$$

3.

$$\begin{aligned} (x-2)(x-3) &= x(x)+x(-3)+(-2)(x)+(-2)(-3) \\ &= x^2-3x-2x+6 \\ &= x^2-5x+6 \end{aligned}$$

4.

$$\begin{aligned} (2a-b)^2 &= (2a-b)(2a-b) \\ &= (2a)(2a)+(2a)(-b)+(2a)(-b)+(-b)(-b) \\ &= 4a^2-2ab-2ab+b^2 \\ &= 4a^2-4ab+b^2 \end{aligned}$$

5.

$$(a^2)(ax) = a^3x$$

6.

$$(2xy)(x^2y^3) = 2x^3y^4$$

7.

$$-a^2c^2(a^2cx^3) = -a^4c^3x^3$$

8.

$$\begin{aligned} (-2cs^2)(-4cs)^2 &= (-2cs^2)(-4cs)(-4cs) \\ &= (-2cs^2)(16c^2s^2) \\ &= -32c^3s^4 \end{aligned}$$

9.

$$\begin{aligned} (2ax^2)^2(-2ax) &= (2ax^2)(2ax^2)(-2ax) \\ &= (4a^2x^4)(-2ax) \\ &= -8a^3x^5 \end{aligned}$$

10.

$$\begin{aligned}(6pq^3)(3pq^2)^2 &= (6pq^3)(3pq^2)(3pq^2)\\ &= (6pq^3)(9p^2q^4)\\ &= 54p^3q^7\end{aligned}$$

11.

$$\begin{aligned}i^2(R+2r) &= (i^2)(R)+(i^2)(2r)\\ &= i^2R+2i^2r\end{aligned}$$

12.

$$\begin{aligned}2x(-p-q) &= (2x)(-p)-(2x)(q)\\ &= -2px-2qx\end{aligned}$$

13.

$$-5s(3s^3-2t) = (-5s)(3s^3)+(-5s)(-2t) = -15s^4+10st$$

14.

$$\begin{aligned}-3b(2b^2-b) &= (-3b)(2b^2)+(-3b)(-b)\\ &= -6b^3+3b^2\end{aligned}$$

15.

$$\begin{aligned}5m(m^2n+3mn) &= (5m)(m^2n)+(5m)(3mn)\\ &= 5m^3n+15m^2n\end{aligned}$$

16.

$$\begin{aligned}a^2bc(2ac-3b^2c) &= (a^2bc)(2ac)+(a^2bc)(-3b^2c)\\ &= 2a^3bc^2-3a^2b^3c^2\end{aligned}$$

17.

$$\begin{aligned}3M(-M-N+2) &= (3M)(-M)+(3M)(-N)+(3M)(2)\\ &= -3M^2-3MN+6M\end{aligned}$$

18.

$$\begin{aligned}-4c^2(-9gc-2c+g^2) &= (-4c^2)(-9cg)+(-4c^2)(-2c)+(-4c^2)(g^2)\\ &= 36c^3g+8c^3-4c^2g^2\end{aligned}$$

19.

$$\begin{aligned}ax(cx^2)(x+y^3) &= acx^3(x+y^3)\\ &= (acx^3)(x)+(acx^3)(y^3)\\ &= acx^4+acx^3y^3\end{aligned}$$

20.

$$\begin{aligned}-2(-3st^3)(3s-4t) &= 6st^3(3s-4t)\\ &= (6st^3)(3s)+(6st^3)(-4t)\\ &= 18s^2t^3-24st^4\end{aligned}$$

21.

$$\begin{aligned}(x-3)(x+5) &= (x)(x)+(x)(5)+(-3)(x)+(-3)(5)\\ &= x^2+5x-3x-15\\ &= x^2+2x-15\end{aligned}$$

22.

$$\begin{aligned}(a+7)(a+1) &= (a)(a)+(a)(1)+(7)(a)+(7)(1)\\ &= a^2+a+7a+7\\ &= a^2+8a+7\end{aligned}$$

23.

$$\begin{aligned}(x+5)(2x-1) &= (x)(2x)+(x)(-1)+(5)(2x)+(5)(-1)\\ &= 2x^2-x+10x-5\\ &= 2x^2+9x-5\end{aligned}$$

24.

$$\begin{aligned}(4t_1+t_2)(2t_1-3t_2) &= (4t_1)(2t_1)+(4t_1)(-3t_2)+(t_2)(2t_1)+(t_2)(-3t_2)\\ &= 8t_1^{\ 2}-12t_1t_2+2t_1t_2-3t_2^{\ 2}\\ &= 8t_1^{\ 2}-10t_1t_2-3t_2^{\ 2}\end{aligned}$$

25.

$$\begin{aligned}(y+8)(y-8) &= (y)(y)+(y)(-8)+(8)(y)+(8)(-8)\\ &= y^2-8y+8y-64\\ &= y^2-64\end{aligned}$$

26.

$$\begin{aligned}(z-4)(z+4) &= (z)(z)+(z)(4)+(-4)(z)+(-4)(4)\\ &= z^2+4z-4z-16\\ &= z^2-16\end{aligned}$$

27.

$$\begin{aligned}(2a-b)(3a-2b) &= (2a)(3a)+(2a)(-2b)+(-b)(3a)+(-b)(-2b)\\ &= 6a^2-4ab-3ab+2b^2\\ &= 6a^2-7ab+2b^2\end{aligned}$$

28.

$$\begin{aligned}(-3+4w^2)(3w^2-1) &= (-3)(3w^2)+(-3)(-1)+(4w^2)(3w^2)+(4w)(-1)\\ &= -9w^2+3+12w^4-4w^2\\ &= 12w^4-13w^2+3\end{aligned}$$

29.

$$\begin{aligned}(2s+7t)(3s-5t) &= (2s)(3s)+(2s)(-5t)+(7t)(3s)+(7t)(-5t)\\ &= 6s^2-10st+21st-35t^2\\ &= 6s^2+11st-35t^2\end{aligned}$$

30.

$$\begin{aligned}(5p-2q)(p+8q) &= (5p)(p)+(5p)(8q)+(-2q)(p)+(-2q)(8q)\\ &= 5p^2+40pq-2pq-16q^2\\ &= 5p^2+38pq-16q^2\end{aligned}$$

31.

$$\begin{aligned}(x^2-1)(2x+5) &= (x^2)(2x)+(x^2)(5)+(-1)(2x)+(-1)(5)\\ &= 2x^3+5x^2-2x-5\end{aligned}$$

32.

$$\begin{aligned}(3y^2+2)(2y-9) &= (3y^2)(2y)+(3y^2)(-9)+(2)(2y)+(-9)(2)\\ &= 6y^3-27y^2+4y-18\end{aligned}$$

33.

$$\begin{aligned}&(x-2y-4)(x-2y+4)\\ &= (x)(x)+(x)(-2y)+(x)(4)+(-2y)(x)+(-2y)(-2y)+(-2y)(4)+(-4)(x)+(-4)(-2y)+(-4)(4)\\ &= x^2-2xy+4x-2xy+4y^2-8y-4x+8y-16\\ &= x^2+4y^2-4xy-16\end{aligned}$$

34.

$$\begin{aligned}&(2a+3b+1)(2a+3b-1)\\ &= (2a)(2a)+(2a)(3b)+(2a)(-1)+(3b)(2a)+(3b)(3b)+(3b)(-1)+(1)(2a)+(1)(3b)+(1)(-1)\\ &= 4a^2+6ab-2a+6ab+9b^2-3b+2a+3b-1\\ &= 4a^2+9b^2+12ab-1\end{aligned}$$

35.

$$\begin{aligned}2(a+1)(a-9) &= 2[(a)(a)+(a)(-9)+(1)(a)+(-9)(1)]\\ &= 2[a^2-9a+a-9]\\ &= 2[a^2-8a-9]\\ &= 2a^2-16a-18\end{aligned}$$

36.

$$\begin{aligned}-5(y-3)(y+6) &= -5[(y)(y)+(y)(6)+(-3)(y)+(-3)(6)]\\ &= -5[y^2+6y-3y-18]\\ &= -5[y^2+3y-18]\\ &= -5y^2-15y+90\end{aligned}$$

37.

$$\begin{aligned}-3(3-2T)(3T+2) &= -3[(3)(3T)+(3)(2)+(-2T)(3T)+(-2T)(2)]\\ &= -3[-6T^2+9T-4T+6]\\ &= -3[-6T^2+5T+6]\\ &= 18T^2-15T-18\end{aligned}$$

38.

$$\begin{aligned}2n(5-n)(6n+5) &= 2n[(5)(6n)+(5)(5)+(-n)(6n)+(-n)(5)]\\ &= 2n[-6n^2+30n-5n+25]\\ &= 2n[-6n^2+25n+25]\\ &= -12n^3+50n^2+50n\end{aligned}$$

39.

$$\begin{aligned}2L(L+1)(4-L) &= 2L[(L)(4)+(L)(-L)+(1)(4)+(1)(-L)]\\ &= 2L[-L^2+4L-L+4]\\ &= 2L[-L^2+3L+4]\\ &= -2L^3+6L^2+8L\end{aligned}$$

40.

$$\begin{aligned}ax(x+4)(7-x^2) &= ax[(x)(7)+(x)(-x^2)+(4)(7)+(4)(-x^2)]\\ &= ax[-x^3-4x^2+7x+28]\\ &= -ax^4-4ax^3+7ax^2+28ax\end{aligned}$$

41.

$$\begin{aligned}(2x-5)^2 &= (2x-5)(2x-5)\\ &= (2x)(2x)+(2x)(-5)+(-5)(2x)+(-5)(-5)\\ &= 4x^2-10x-10x+25\\ &= 4x^2-20x+25\end{aligned}$$

42.

$$\begin{aligned}(x-3y)^2 &= (x-3y)(x-3y)\\ &= (x)(x)+(x)(-3y)+(-3y)(x)+(-3y)(-3y)\\ &= x^2-3xy-3xy+9y^2\\ &= x^2-6xy+9y^2\end{aligned}$$

43.

$$\begin{aligned}(x_1+3x_2)^2 &= (x_1+3x_2)(x_1+3x_2)\\ &= (x_1)(x_1)+(x_1)(3x_2)+(3x_2)(x_1)+(3x_2)(3x_2)\\ &= x_1^{\,2}+3x_1x_2+3x_1x_2+9x_2^{\,2}\\ &= x_1^{\,2}+6x_1x_2+9x_2^{\,2}\end{aligned}$$

44.

$$\begin{aligned}(7m+1)^2 &= (7m+1)(7m+1)\\ &= (7m)(7m)+(7m)(1)+(1)(7m)+(1)(1)\\ &= 49m^2+7m+7m+1\\ &= 49m^2+14m+1\end{aligned}$$

45.

$$\begin{aligned}(xyz-2)^2 &= (xyz-2)(xyz-2)\\ &= (xyz)(xyz)+(xyz)(-2)+(-2)(xyz)+(-2)(-2)\\ &= x^2y^2z^2-2xyz-2xyz+4\\ &= x^2y^2z^2-4xyz+4\end{aligned}$$

46.

$$\begin{aligned}(b-6x^2)^2 &= (b-6x^2)(b-6x^2)\\ &= (b)(b)+(b)(-6x^2)+(-6x^2)(b)+(-6x^2)(-6x^2)\\ &= b^2-6bx^2-6bx^2+36x^4\\ &= b^2-12bx^2+36x^4\end{aligned}$$

47.

$$\begin{aligned}2(x+8)^2 &= 2[(x+8)(x+8)]\\ &= 2[(x)(x)+(x)(8)+(8)(x)+(8)(8)]\\ &= 2[x^2+8x+8x+64]\\ &= 2[x^2+16x+64]\\ &= 2x^2+32x+128\end{aligned}$$

48.

$$\begin{aligned}3(3R+4)^2 &= 3[(3R+4)(3R+4)]\\ &= 3[(3R)(3R)+(3R)(4)+(4)(3R)+(4)(4)]\\ &= 3[9R^2+12R+12R+16]\\ &= 3[9R^2+24R+16]\\ &= 27R^2+72R+48\end{aligned}$$

49.

$$\begin{aligned}(2+x)(3-x)(x-1) &= [(6-2x+3x-x^2)](x-1)\\ &= (x-1)[-x^2+x+6]\\ &= (x)(-x^2)+(x)(x)+(6)(x)+(-1)(-x^2)+(-1)(x)+(-1)(6)\\ &= -x^3+x^2+6x+x^2-x-6\\ &= -x^3+2x^2+5x-6\end{aligned}$$

50.

$$\begin{aligned}(3x-c^2)^3 &= (3x-c^2)(3x-c^2)(3x-c^2)\\ &= [(3x)(3x)-3c^2x-3c^2x+c^4)](3x-c^2)\\ &= (3x-c^2)[9x^2-6c^2x+c^4]\\ &= (3x)(9x^2)+(3x)(-6c^2x)+(3x)(+c^4)+(-c^2)(9x^2)+(-c^2)(-6c^2x)+(-c^2)(+c^4)\\ &= 27x^3-18c^2x^2+3c^4x-9c^2x^2+6c^4x-c^6\\ &= -c^6+9c^4x-27c^2x^2+27x^3\end{aligned}$$

51.

$$\begin{aligned}3T(T+2)(2T-1) &= 3T[(T)(2T)+(T)(-1)+(2)(2T)+(2)(-1)]\\ &= 3T[2T^2-T+4T-2]\\ &= 3T[2T^2-T+4T-2]\\ &= 3T[2T^2+3T-2]\\ &= 6T^3+9T^2-6T\end{aligned}$$

52.

$$\begin{aligned}&[(x-2)^2(x+2)]^2\\ &=[(x-2)(x-2)(x+2)][(x-2)(x-2)(x+2)]\\ &=[(x-2)[(x)(x)+(-2)(x)+(2)(x)+(-2)(2)]][(x-2)[(x)(x)+(-2)(x)+(2)(x)+(-2)(2)]]\\ &=[(x-2)[x^2-2x+2x-4]][(x-2)[x^2-2x+2x-4]]\\ &=[(x-2)[x^2-4]][(x-2)[x^2-4]]\\ &=[(x)(x^2)+(-4)(x)+(-2)(x^2)+(-2)(-4)][(x)(x^2)+(-4)(x)+(-2)(x^2)+(-2)(-4)]\\ &=[x^3-2x^2-4x+8][x^3-2x^2-4x+8]\\ &=(x^3)(x^3)+(x^3)(-2x^2)+(x^3)(-4x)+(x^3)(8)+(-2x^2)(x^3)+(-2x^2)(-2x^2)+(-2x^2)(-4x)+(-2x^2)(8)\\ &\quad+(-4x)(x^3)+(-4x)(-2x^2)+(-4x)(-4x)+(-4x)(8)+(8)(x^3)+(8)(-2x^2)+(8)(-4x)+(8)(8)\\ &=x^6-2x^5-4x^4+8x^3-2x^5+4x^4+8x^3-16x^2-4x^4+8x^3+16x^2-32x+8x^3-16x^2-32x+64\\ &=x^6-4x^5-4x^4+32x^3-16x^2-64x+64\end{aligned}$$

53.

(a)

$(x+y)^2=(3+4)^2=7^2=49$

$x^2+y^2=3^2+4^2=9+16=25$

$(x+y)^2\neq x^2+y^2$

$49\neq 25$

(b)

$(x-y)^2=(3-4)^2=(-1)^2=1$

$x^2-y^2=3^2-4^2=9-16=-7$

$(x-y)^2\neq x^2-y^2$

$1\neq -7$

54.

$$\begin{aligned}(98)(102) &= (100-2)(100+2)\\ &= (100)(100)+(100)(2)+(-2)(100)+(2)(-2)\\ &= 10000+200-200-4\\ &= 9996\end{aligned}$$

55.

$(4)^2-1=16-1=15=(3)(5)$

If we let x equal an integer between 1 and 9, $1<x<9$, then x^2-1 can be factored to $(x-1)(x+1)$:

$$\begin{aligned}(x-1)(x+1) &= (x)(x)+(1)(x)+(-1)(x)+(1)(-1)\\ &= x^2-x+x-1\\ &= x^2-1\end{aligned}$$

$(x-1)$ is the number before x, and $(x+1)$ is the number after x.

56.

$$\begin{aligned}(x-2)(x+2)(x+3)(x-3) &= [x^2+2x-2x-4][x^2+3x-3x-9]\\ &= (x^2-4)(x^2-9)\\ &= (x^2)(x^2)+(x^2)(-9)+(x^2)(-4)+(-4)(-9)\\ &= x^4-9x^2-4x^2+36\\ &= x^4-13x^2+36\end{aligned}$$

If we group the $(x + 2)$ with the $(x-2)$, then the $2x$ and $-2x$ will cancel each other out when we multiply the terms out. Likewise with the $(x + 3)$ and the $(x - 3)$.

57.

$$\begin{aligned}x^3+y^3 &\neq (x+y)^3\\ &\neq (x+y)(x+y)(x+y)\\ &\neq (x+y)[(x)(x)+(x)(y)+(y)(x)+(y)(y)]\\ &\neq (x+y)[x^2+xy+xy+y^2]\\ &\neq (x+y)[x^2+2xy+y^2]\\ &\neq (x)(x^2)+(x)(2xy)+(x)(y^2)+(y)(x^2)+(y)(2xy)+(y)(y^2)\\ &\neq x^3+2x^2y+y^2x+x^2y+2y^2x+y^3\\ &\neq x^3+3x^2y+3y^2x+y^3\end{aligned}$$

58.

$$\begin{aligned}&(x+y)(x^2-xy+y^2)\\ &=(x)(x^2)+(x)(-xy)+(x)(y^2)+(y)(x^2)+(y)(-xy)+(y)(y^2)\\ &=x^3-x^2y+y^2x+x^2y-y^2x+y^3\\ &=x^3+y^3\end{aligned}$$

59.

$$\begin{aligned}P(1+0.01r)^2 &= P(1+0.01r)(1+0.01r)\\ &= P[(1)(1)+(1)(0.01r)+(0.01r)(1)+(0.01r)(0.01r)]\\ &= P[1+0.01r+0.01r+0.0001r^2]\\ &= 0.0001r^2P+0.02rP+P\end{aligned}$$

60.

$$\begin{aligned}w(1-x)(4-x^2) &= w[(1)(4)+(1)(-x^2)+(-x)(4)+(-x)(-x^2)]\\ &= w[4-x^2-4x+x^3]\\ &= x^3w-x^2w-4wx+4w\end{aligned}$$

61.

$$\begin{aligned}(2R-X)^2-(R^2+X^2) &= (2R-X)(2R-X)-(R^2+X^2)\\ &= [(2R)(2R)+(2R)(-X)+(2R)(-X)+(-X)(-X)]-(R^2+X^2)\\ &= 4R^2-2RX-2RX+X^2-R^2-X^2\\ &= 3R^2-4RX\end{aligned}$$

62.

$$\begin{aligned}(2T^3+3)(T^2-T-3) &= (2T^3)(T^2)+(2T^3)(-T)+(2T^3)(-3)+(3)(T^2)+(3)(-T)+(3)(-3)\\ &= 2T^5-2T^4-6T^3+3T^2-3T-9\end{aligned}$$

63.

$$\begin{aligned}\text{Number of switches} &= n^2 \\ &= (n+100)^2 \\ &= (n+100)(n+100) \\ &= (n)(n)+(n)(100)+(100)(n)+(100)(100) \\ &= n^2+100n+100n+10\ 000 \\ &= n^2+200n+10\ 000\end{aligned}$$

64.

$$\begin{aligned}(T^2-100)(T-10)(T+10) &= (T^2-100)\left[T^2+10T-10T-100\right] \\ &= (T^2-100)\left[T^2-100\right] \\ &= T^4-100T^2-100T^2+10\ 000 \\ &= T^4-200T^2+10\ 000\end{aligned}$$

65.

$$\begin{aligned}(R_1+R_2)^2-2R_2(R_1+R_2) &= \left[(R_1+R_2)(R_1+R_2)\right]-2R_2(R_1+R_2) \\ &= \left[(R_1)(R_1)+(R_1)(R_2)+(R_2)(R_1)+(R_2)(R_2)\right]-2R_1R_2-2R_2^{\,2} \\ &= \left[R_1^{\,2}+R_1R_2+R_1R_2+R_2^{\,2}\right]-2R_1R_2-2R_2^{\,2} \\ &= R_1^{\,2}-R_2^{\,2}\end{aligned}$$

66.

$$\begin{aligned}&27x^2-24(x-6)^2-(x-12)^3 \\ &= 27x^2-24(x-6)(x-6)-(x-12)(x-12)(x-12) \\ &= 27x^2-24\left[x^2-6x-6x+36\right]-(x-12)\left[x^2-12x-12x+144\right] \\ &= 27x^2-24\left[x^2-12x+36\right]-(x-12)\left[x^2-24x+144\right] \\ &= 27x^2-24x^2+288x-864-\left[(x)(x^2)+(x)(-24x)+(x)(144)+(-12)(x^2)+(-12)(-24x)+(-12)(144)\right] \\ &= 3x^2+288x-864-x^3+24x^2-144x+12x^2-288x+1728 \\ &= -x^3+39x^2-144x+864\end{aligned}$$

1.9 Division of Algebraic Expressions

1.

$$\frac{-6a^2xy^2}{-2a^2xy^5}=\left(\frac{-6}{-2}\right)\frac{a^{2-2}x^{1-1}}{y^{5-2}}=\frac{3}{y^3}$$

2.

$$\begin{aligned}\frac{4x^3y-8x^3y^2+2x^2y}{2xy^2}&=\frac{4x^3y}{2xy^2}-\frac{8x^3y^2}{2xy^2}+\frac{2x^2y}{2xy^2}\\&=\frac{2x^{3-1}}{y^{2-1}}-4x^{3-1}y^{2-2}+\frac{x^{2-1}}{y^{2-1}}\\&=\frac{2x^2}{y}-4x^2+\frac{x}{y}\end{aligned}$$

3.

$$\begin{array}{r|l} & 3x-2 \\ 2x-1 & 6x^2-7x+2 \\ & \underline{6x^2-3x} \\ & \quad -4x+2 \\ & \quad \underline{-4x+2} \\ & \qquad 0 \end{array}$$

4.

$$\begin{array}{r|l} & 2x-1 \\ 4x^2-1 & 8x^3-4x^2+0x+3 \\ & \underline{8x^3 \qquad\quad -2x} \\ & \quad -4x^2+2x+3 \\ & \quad -\underline{4x^2 \qquad\quad 1} \\ & \qquad\quad 2x+2 \end{array}$$

$$\frac{8x^3-4x^2+3}{4x^2-1}=2x-1+\frac{2x+2}{4x^2-1}$$

5.

$$\frac{8x^3y^2}{-2xy}=-4x^{3-1}y^{2-1}=-4x^2y$$

6.

$$\frac{-18b^7c^3}{bc^2}=-18b^{7-1}c^{3-2}=-18b^6c$$

7.

$$\frac{-16r^3t^5}{-4r^5t}=\frac{4t^{5-1}}{r^{5-3}}=\frac{4t^4}{r^2}$$

8.

$$\frac{51mn^5}{17m^2n^2}=\frac{3n^{5-2}}{m^{2-1}}=\frac{3n^3}{m}$$

9.

$$\frac{-5x^{-2}y^6}{10x^3y^2}=\left(\frac{-5}{10}\right)\cdot x^{-2-3}y^{6-2}=\frac{-1}{2}\cdot x^{-5}\cdot y^4=-\frac{y^4}{2x^5}$$

10.

$$\frac{-20b^5c^{-5}}{4b^{-3}c^{-2}}=\left(\frac{-20}{4}\right)b^{5-(-3)}c^{-5-(-2)}=-5b^8c^{-3}=\frac{-5b^8}{c^3}$$

11.

$$\frac{-200m^3n^5}{-10m^{10}n^{-10}}=\left(\frac{-200}{-10}\right)m^{3-10}n^{5-(-10)}=20m^{-7}n^{15}=\frac{20n^{15}}{m^7}$$

12.

$$\frac{-81r^7k^{-3}p^{10}}{135r^{-2}k^{-3}p^{-5}}=\left(\frac{-81}{135}\right)r^{7-(-2)}k^{-3-(-3)}p^{10-(-5)}=\frac{-3}{5}r^9k^0p^{15}=\frac{-3r^9p^{15}}{5}$$

13.

$$\frac{(15x^2)(4bx)(2y)}{30bxy}=\frac{120x^3by}{30bxy}=4x^{3-1}b^{1-1}y^{1-1}=4x^2$$

14.

$$\frac{(5sT)(8s^2T^3)}{10s^3T^2}=\frac{40s^3T^4}{10s^3T^2}=4s^{3-3}T^{4-2}=4T^2$$

15.

$$\frac{6(ax)^2}{-ax^2}=\frac{6a^2x^2}{-ax^2}=-6a^{2-1}x^{2-2}=-6a$$

16.

$$\frac{12a^2b}{(3ab^2)^2}=\frac{12a^2b}{9a^2b^4}=\frac{4a^{2-2}}{3b^{4-1}}=\frac{4}{3b^3}$$

17.

$$\frac{3a^2x+6xy}{3x}=\frac{3a^2x}{3x}+\frac{6xy}{3x}=\frac{3a^2x^{1-1}}{3}+\frac{6x^{1-1}y}{3}=a^2+2y$$

18.

$$\frac{2m^2n-6mn}{2m}=\frac{2m^2n}{2m}-\frac{6mn}{2m}=m^{2-1}n-3m^{1-1}n=mn-3n$$

19.

$$\frac{3rst-6r^2st^2}{3rs}=\frac{3rst}{3rs}-\frac{6r^2st^2}{3rs}=r^{1-1}s^{1-1}t-2r^{2-1}s^{1-1}t^2=-2rt^2+t$$

20.

$$\frac{-5a^2n-10an^2}{5an}=\frac{-5a^2n}{5an}-\frac{10an^2}{5an}=-a^{2-1}n^{1-1}-2a^{1-1}n^{2-1}=-a-2n$$

21.

$$\begin{aligned}\frac{4pq^3+8p^2q^2-16pq^5}{4pq^2}&=\frac{4pq^3}{4pq^2}+\frac{8p^2q^2}{4pq^2}-\frac{16pq^5}{4pq^2}\\&=p^{1-1}q^{3-2}+2p^{2-1}q^{2-2}-4p^{1-1}q^{5-2}\\&=-4q^3+2p+q\end{aligned}$$

22.

$$\begin{aligned}\frac{a^2x_1x_2^2+ax_1^3-ax_1}{ax_1}&=\frac{a^2x_1x_2^2}{ax_1}+\frac{ax_1^3}{ax_1}-\frac{ax_1}{ax_1}\\&=a^{2-1}x_1^{1-1}x_2^2+a^{1-1}x_1^{3-1}-a^{1-1}x_1^{1-1}\\&=ax_2^2+x_1^2-1\end{aligned}$$

23.

$$\begin{aligned}\frac{2\pi fL-\pi fR^2}{\pi fR}&=\frac{2\pi fL}{\pi fR}-\frac{\pi fR^2}{\pi fR}\\&=\frac{2f^{1-1}L}{R}-f^{1-1}R^{2-1}\\&=\frac{2L}{R}-R\end{aligned}$$

24.

$$\begin{aligned}\frac{9(aB)^4-6aB^4}{3aB^3}&=\frac{9(aB)^4}{3aB^3}-\frac{6aB^4}{3aB^3}\\&=\frac{9a^4B^4}{3aB^3}-\frac{6aB^4}{3aB^3}\\&=3a^{4-1}B^{4-3}-2a^{1-1}B^{4-3}\\&=3a^3B-2B\end{aligned}$$

25.

$$\begin{aligned}\frac{-7a^2b+14ab^2-21a^3}{14a^2b^2}&=-\frac{7a^2b}{14a^2b^2}+\frac{14ab^2}{14a^2b^2}-\frac{21a^3}{14a^2b^2}\\&=-\frac{a^{2-2}}{2b^{2-1}}+\frac{b^{2-2}}{a^{2-1}}-\frac{3}{2}a^{3-2}b^{-2}\\&=-\frac{1}{2b}+\frac{1}{a}-\frac{3a}{2b^2}\end{aligned}$$

26.

$$\begin{aligned}\frac{2x^{n+2}+4ax^n}{2x^n} &= \frac{2x^{n+2}}{2x^n}+\frac{4ax^n}{2x^n}\\ &= x^{n-n+2}+2ax^{n-n}\\ &= x^2+2a\end{aligned}$$

27.

$$\begin{aligned}\frac{6y^{2n}-4ay^{n+1}}{2y^n} &= \frac{6y^{2n}}{2y^n}-\frac{4ay^{n+1}}{2y^n}\\ &= 3y^{2n-n}-2ay^{n-n+1}\\ &= 3y^n-2ay\end{aligned}$$

28.

$$\begin{aligned}\frac{3a(F+T)b^2-(F+T)}{a(F+T)} &= \frac{3a(F+T)b^2}{a(F+T)}-\frac{(F+T)}{a(F+T)}\\ &= \frac{3a^{1-1}\,\cancel{(F+T)}b^2}{\cancel{(F+T)}}-\frac{\cancel{(F+T)}}{a\,\cancel{(F+T)}}\\ &= 3b^2-\frac{1}{a}\end{aligned}$$

29.

$$\begin{array}{r} x+5 \\ x+4\overline{)\,x^2+9x+20} \\ \underline{x^2+4x} \\ 5x+20 \\ \underline{5x+20} \\ 0 \end{array}$$

$$\frac{x^2+9x+20}{x+4}=x+5$$

30.

$$\begin{array}{r} x+9 \\ x-2\overline{)\,x^2+7x-18} \\ \underline{x^2-2x} \\ 9x-18 \\ \underline{9x-18} \\ 0 \end{array}$$

$$\frac{x^2+7x-18}{x-2}=x+9$$

31.

$$\begin{array}{r} 2x+1 \\ x+3\overline{)2x^2+7x+3} \\ \underline{2x^2+6x} \\ x+3 \\ \underline{x+3} \\ 0 \end{array}$$

$$\frac{2x^2+7x+3}{x+3}=2x+1$$

32.

$$\begin{array}{r} 3t-4 \\ t-1\overline{)3t^2-7t+4} \\ \underline{3t^2-3t} \\ -4t+4 \\ \underline{-4t+4} \\ 0 \end{array}$$

$$\frac{3t^2-7t+4}{t-1}=3t-4$$

33.

$$\begin{array}{r} x-1 \\ x-2\overline{)x^2-3x+2} \\ \underline{x^2-2x} \\ -x+2 \\ \underline{-x+2} \\ 0 \end{array}$$

$$\frac{x^2-3x+2}{x-2} = x-1$$

34.

$$\begin{array}{r} 2x-7 \\ x+1\overline{)2x^2-5x-7} \\ \underline{2x^2+2x} \\ -7x-7 \\ \underline{-7x-7} \\ 0 \end{array}$$

$$\frac{2x^2-5x-7}{x+1}=2x-7$$

35.

$$\begin{array}{r} 4x^2 - x - 1 \\ 2x-3\overline{)8x^3 - 14x^2 + x + 0} \\ \underline{8x^3 - 12x^2} \\ -2x^2 + x \\ \underline{-2x^2 + 3x} \\ -2x + 0 \\ \underline{-2x + 3} \\ -3 \end{array}$$

$$\frac{8x^3 - 14x^2 + x}{2x-3} = 4x^2 - x - 1 - \frac{3}{2x-3}$$

36.

$$\begin{array}{r} 3y + 2 \\ 2y+1\overline{)6y^2 + 7y + 6} \\ \underline{6y^2 + 3y} \\ 4y + 6 \\ \underline{4y + 2} \\ 4 \end{array}$$

$$\frac{6y^2 + 7y + 6}{2y+1} = 3y + 2 + \frac{4}{2y+1}$$

37.

$$\begin{array}{r} Z - 2 \\ 4Z+3\overline{)4Z^2 - 5Z - 7} \\ \underline{4Z^2 + 3Z} \\ -8Z - 7 \\ \underline{-8Z - 6} \\ -1 \end{array}$$

$$\frac{4Z^2 - 5Z - 7}{4Z+3} = Z - 2 - \frac{1}{4Z+3}$$

38.

$$\begin{array}{r} 2x + 1 \\ 3x-4\overline{)6x^2 - 5x - 9} \\ \underline{6x^2 - 8x} \\ 3x - 9 \\ \underline{3x - 4} \\ -5 \end{array}$$

$$\frac{6x^2 - 5x - 9}{3x-4} = 2x + 1 - \frac{5}{3x-4}$$

39.

$$
\begin{array}{r}
x^2+x-6 \\
x+2\overline{)\,x^3+3x^2-4x-12} \\
\underline{x^3+2x^2} \\
x^2-4x \\
\underline{x^2+2x} \\
-6x-12 \\
\underline{-6x-12} \\
0
\end{array}
$$

$$\frac{x^3+3x^2-4x-12}{x+2}=x^2+x-6$$

40.

$$
\begin{array}{r}
x^2+7x+9 \\
3x-2\overline{)\,3x^3+19x^2+13x-20} \\
\underline{3x^3-2x^2} \\
21x^2+13x \\
\underline{21x^2-14x} \\
27x-20 \\
\underline{27x-18} \\
-2
\end{array}
$$

$$\frac{3x^3+19x^2+13x-20}{3x-2}=x^2+7x+9-\frac{2}{3x-2}$$

41.

$$
\begin{array}{r}
2a^2+8 \\
a^2-2\overline{)\,2a^4+0a^3+4a^2+0a-16} \\
\underline{2a^4\qquad-4a^2} \\
8a^2\qquad-16 \\
\underline{8a^2\qquad-16} \\
0
\end{array}
$$

$$\frac{2a^4+4a^2-16}{a^2-2}=2a^2+8$$

42.

$$\begin{array}{r} 2T+1 \\ 3T^2-T+2\overline{)6T^3+T^2+0T+2} \\ \underline{6T^3-2T^2+4T} \\ 3T^2-4T+2 \\ \underline{3T^2-T+2} \\ -3T \end{array}$$

$$\frac{6T^3+T^2+2}{3T^2-T+2}=2T+1-\frac{3T}{3T^2-T+2}$$

43.

$$\begin{array}{r} x^2-2x+4 \\ x+2\overline{)x^3+0x^2+0x+8} \\ \underline{x^3+2x^2} \\ -2x^2+0x \\ \underline{-2x^2-4x} \\ 4x+8 \\ \underline{4x+8} \\ 0 \end{array}$$

$$\frac{x^3+8}{x+2}=x^2-2x+4$$

44.

$$\begin{array}{r} D^2+D+1 \\ D-1\overline{)D^3+0D^2+0D-1} \\ \underline{D^3-1D^2} \\ D^2+0D \\ \underline{D^2-1D} \\ D-1 \\ \underline{D-1} \\ 0 \end{array}$$

$$\frac{D^3-1}{D-1}=D^2+D+1$$

45.

$$\begin{array}{r} x-y \\ x-y\overline{)x^2-2xy+y^2} \\ \underline{x^2-xy} \\ -xy+y^2 \\ \underline{-xy+y^2} \\ 0 \end{array}$$

$$\frac{x^2-2xy+y^2}{x-y}=x-y$$

46.

$$\begin{array}{r} 3r+4R \\ r-3R\overline{\smash{\big)}3r^2-5rR+2R^2} \\ \underline{3r^2-9rR} \\ 4rR+2R^2 \\ \underline{4rR-12R^2} \\ 14R^2 \end{array}$$

$$\frac{3r^2-5rR+2R^2}{r-3R}=3r+4R+\frac{14R^2}{r-3R}$$

47.

$$\begin{array}{r} t-2 \\ t^2+2t+4\overline{\smash{\big)}t^3+0t^2+0t-8} \\ \underline{t^3+2t^2+4t} \\ -2t^2-4t-8 \\ \underline{-2t^2-4t-8} \\ 0 \end{array}$$

$$\frac{t^3-8}{t^2+2t+4}=t-2$$

48.

$$\begin{array}{r} a^2+2ab+2b^2 \\ a^2-2ab+2b^2\overline{\smash{\big)}a^4+0a^3b+0a^2b^2+0ab^3+b^4} \\ \underline{a^4-2a^3b+2a^2b^2} \\ 2a^3b-2a^2b^2+0ab^3 \\ \underline{2a^3b-4a^2b^2+4ab^3} \\ 2a^2b^2-4ab^3+b^4 \\ \underline{2a^2b^2-4ab^3+4b^4} \\ -3b^4 \end{array}$$

$$\frac{a^4+b^4}{a^2-2ab+2b^2}=a^2+2ab+2b^2-\frac{3b^4}{a^2-2ab+2b^2}$$

49.

We know that $2x+1$ multiplied by $x+c$ will give us $2x^2-9x-5$, so $2x^2-9x-5$ divided by $2x+1$ will give us $x+c$:

$$\begin{array}{r} x-5 \\ 2x+1\overline{)2x^2-9x-5} \\ \underline{2x^2+x} \\ -10x-5 \\ \underline{-10x-5} \\ 0 \end{array}$$

$$\begin{aligned} x+c &= x-5 \\ c &= -5 \end{aligned}$$

50.

$$\begin{array}{r} 2x-3 \\ 3x+4\overline{)6x^2-x+k} \\ \underline{6x^2+8x} \\ -9x+k \\ \underline{-9x-12} \\ 0 \end{array}$$

$$\begin{aligned} k-(-12) &= 0 \\ k+12 &= 0 \\ k &= -12 \end{aligned}$$

51.

$$\begin{array}{r} x^3-x^2+x-1 \\ x+1\overline{)x^4+0x^3+0x^2+0x+1} \\ \underline{x^4+x^3} \\ -x^3+0x^2 \\ \underline{-x^3-x^2} \\ x^2+0x \\ \underline{x^2+x} \\ -x+1 \\ \underline{-x-1} \\ 2 \end{array}$$

$$\frac{x^4+1}{x+1}=x^3-x^2+x-1+\frac{2}{x+1}\neq x^3$$

52.

$$\begin{array}{r} x^2 - xy + y^2 \\ x + y\overline{)x^3 + 0x^2y + 0y^2x + 0x + y^3} \\ \underline{x^3 + x^2y} \\ -x^2y + 0y^2x \\ \underline{-x^2y - y^2x} \\ y^2x + y^3 \\ \underline{y^2x + y^3} \\ 0 \end{array}$$

$$\frac{x^3 + y^3}{x + y} = x^2 - xy + y \neq x^2 + y^2$$

53.

$$\begin{aligned} \frac{8A^5 + 4A^3\mu^2E^2 - A\mu^4E^4}{8A^4} &= \frac{8A^5}{8A^4} + \frac{4A^3\mu^2E^2}{8A^4} - \frac{A\mu^4E^4}{8A^4} \\ &= A^{5-4} + \frac{\mu^2E^2}{2A^{4-3}} - \frac{\mu^4E^4}{8A^{4-1}} \\ &= A + \frac{\mu^2E^2}{2A} - \frac{\mu^4E^4}{8A^3} \end{aligned}$$

54.

$$\begin{aligned} \frac{6R_1 + 6R_2 + R_1R_2}{6R_1R_2} &= \frac{6R_1}{6R_1R_2} + \frac{6R_2}{6R_1R_2} + \frac{R_1R_2}{6R_1R_2} \\ &= \frac{\cancel{R_1}}{\cancel{R_1}R_2} + \frac{\cancel{R_2}}{R_1\cancel{R_2}} + \frac{\cancel{R_1R_2}}{6\cancel{R_1R_2}} \\ &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{6} \end{aligned}$$

55.

$$\begin{aligned} \frac{GMm[(R + r) - (R - r)]}{2rR} &= \frac{GMm[R + r - R + r]}{2rR} \\ &= \frac{GMm[2r]}{2rR} \\ &= \frac{GMm\,[\cancel{2r}]}{\cancel{2r}R} \\ &= \frac{GMm}{R} \end{aligned}$$

56.

$$\begin{array}{r}
3T^2-2T-4 \\
T-2\overline{)3T^3-8T^2+0T+8} \\
\underline{3T^3-6T^2} \qquad\qquad \\
-2T^2+0T \qquad \\
\underline{-2T^2+4T} \qquad \\
-4T+8 \\
\underline{-4T+8} \\
0
\end{array}$$

57.

$$\left(\frac{s^2-2s-2}{s^4+4}\right)^{-1}=\frac{s^4+4}{s^2-2s-2}$$

$$\begin{array}{r}
s^2+2s+6 \\
s^2-2s-2\overline{)s^4+0s^3+0s^2+0s+4} \\
\underline{s^4-2s^3-2s^2} \qquad\qquad \\
2s^3+2s^2+0s \qquad \\
\underline{2s^3-4s^2-4s} \qquad \\
6s^2+4s+4 \\
\underline{6s^2-12s-12} \\
16s+16
\end{array}$$

$$\frac{s^4+4}{s^2-2s-2}=s^2+2s+6+\frac{16s+16}{s^2-2s-2}$$

58.

$$\begin{array}{r}
t^2-3t+5 \\
2t+100\overline{)2t^3+94t^2-290t+500} \\
\underline{2t^3+100t^2} \qquad\qquad\qquad \\
-6t^2-290t \qquad \\
\underline{-6t^2-300t} \qquad \\
10t+500 \\
\underline{10t+500} \\
0
\end{array}$$

1.10 Solving Equations

1.

(a)

$$\begin{aligned} x-3&=-12\\ x-3+3&=-12+3\\ x&=-9 \end{aligned}$$

(b)

$$\begin{aligned} x+3&=-12\\ x+3-3&=-12-3\\ x&=-15 \end{aligned}$$

(c)

$$\begin{aligned} \frac{x}{3}&=-12\\ 3\left(\frac{x}{3}\right)&=3(-12)\\ x&=-36 \end{aligned}$$

(d)

$$\begin{aligned} 3x&=-12\\ \frac{3x}{3}&=\frac{-12}{3}\\ x&=-4 \end{aligned}$$

2.

$$\begin{aligned} 7-2t&=9\\ 7-7-2t&=9-7\\ -2t&=2\\ \frac{-2t}{-2}&=\frac{2}{-2}\\ t&=-1 \end{aligned}$$

Check:

$$\begin{aligned} 7-2t&=9\\ 7-2(-1)&=9\\ 7-(-2)&=9\\ 9&=9 \end{aligned}$$

3.

$$\begin{aligned} x-7&=3x-(8-6x)\\ x-7&=3x-8+6x\\ x-7&=9x-8\\ -8x&=-1\\ x&=\frac{1}{8} \end{aligned}$$

4.

$$\frac{L}{3.80}=\frac{7}{4}$$

$$3.80\left(\frac{L}{3.80}\right)=3.80\left(\frac{7}{4}\right)$$

$$L=6.65\text{ m}$$

5.

$$x-2=7$$
$$x=7+2$$
$$x=9$$

6.

$$x-4=-1$$
$$x=-1+4$$
$$x=3$$

7.

$$x+5=4$$
$$x=4-5$$
$$x=-1$$

8.

$$-s-6=14$$
$$-s=14+6$$
$$s=-20$$

9.

$$-\frac{t}{2}=5$$
$$t=(-2)(5)$$
$$t=-10$$

10.

$$\frac{x}{-4}=2$$
$$x=-4(2)$$
$$x=-8$$

11.

$$\frac{y-8}{3}=4$$
$$y-8=4(3)$$
$$y=12+8$$
$$y=20$$

12.

$$\frac{7-r}{6}=3$$
$$7-r=6(3)$$
$$-r=18-7$$
$$-r=11$$
$$r=-11$$

13.

$$4E=-20$$
$$E=\frac{-20}{4}$$
$$E=-5$$

14.

$$2x=12$$
$$x=\frac{12}{2}$$
$$x=6$$

15.

$$3t+5=-4$$
$$3t=-4-5$$
$$t=\frac{-9}{3}$$
$$t=-3$$

16.

$$5D-2=13$$
$$5D=13+2$$
$$D=\frac{15}{5}$$
$$D=3$$

17.

$$5-2y=-3$$
$$-2y=-3-5$$
$$y=\frac{-8}{-2}$$
$$y=4$$

18.

$$\begin{aligned} 8-5t &= 18 \\ -5t &= 18-8 \\ t &= \frac{10}{-5} \\ t &= -2 \end{aligned}$$

19.

$$\begin{aligned} 3x+7 &= x \\ x-3x &= 7 \\ -2x &= 7 \\ x &= -\frac{7}{2} \end{aligned}$$

20.

$$\begin{aligned} 6+4L &= 5-3L \\ 4L+3L &= 5-6 \\ 7L &= -1 \\ L &= -\frac{1}{7} \end{aligned}$$

21.

$$\begin{aligned} 2(3q+4) &= 5q \\ 6q+8 &= 5q \\ 6q-5q &= -8 \\ q &= -8 \end{aligned}$$

22.

$$\begin{aligned} 3(4-n) &= -n \\ -n &= 12-3n \\ -n+3n &= 12 \\ 2n &= 12 \\ n &= \frac{12}{2} \\ n &= 6 \end{aligned}$$

23.

$$\begin{aligned} -(r-4) &= 6+2r \\ -r+4 &= 6+2r \\ -r-2r &= 2 \\ -3r &= 2 \\ r &= -\frac{2}{3} \end{aligned}$$

24.

$$\begin{aligned}5-(x+2)&=5x\\5x&=5-x-2\\5x+x&=3\\6x&=3\\x&=\frac{3}{6}=\frac{1}{2}\end{aligned}$$

25.

$$\begin{aligned}8(y-5)&=-2y\\8y-40&=-2y\\8y+2y&=40\\10y&=40\\y&=\frac{40}{10}\\y&=4\end{aligned}$$

26.

$$\begin{aligned}6(8-3F)&=-10\\48-18F&=-10\\-18F&=-10-48\\F&=\frac{-58}{-18}=\frac{29}{9}\end{aligned}$$

27.

$$\begin{aligned}0.1x-0.5(x-2)&=2\\x-5(x-2)&=2(10)\\x-5x+10&=20\\-4x&=20-10\\x&=\frac{10}{-4}=-\frac{5}{2}\end{aligned}$$

28.

$$\begin{aligned}1.5x-0.3(x-4)&=6\\15x-3(x-4)&=6(10)\\15x-3x+12&=60\\12x&=60-12\\x&=\frac{48}{12}\\x&=4\end{aligned}$$

29.

$$7-3(1-2p)=4+2p$$
$$7-3+6p=4+2p$$
$$4+6p-2p=4$$
$$4p=4-4$$
$$p=\frac{0}{4}$$
$$p=0$$

30.

$$3-6(2-3t)=t-5$$
$$3-12+18t=t-5$$
$$-9+18t-t=-5$$
$$17t=4$$
$$t=\frac{4}{17}$$

31.

$$\frac{4x-2(x-4)}{3}=8$$
$$4x-2x+8=3(8)$$
$$2x=24-8$$
$$x=\frac{16}{2}$$
$$x=8$$

32.

$$2x=\frac{-5(7-3x)+2}{4}$$
$$4(2x)=-35+15x+2$$
$$8x-15x=-33$$
$$-7x=-33$$
$$x=\frac{-33}{-7}=\frac{33}{7}$$

33.

$$|x|-9=2$$
$$|x|=2+9=11$$
$$x=11 \text{ or } x=-11$$

34.

$$\begin{aligned}2-|x| &= 4\\ -|x| &= 4-2\\ |x| &= \frac{2}{-1}\\ |x| &= -2\end{aligned}$$

There is no real solution for x

35.

$$\begin{aligned}5.8-0.3(x-6.0) &= 0.5x\\ 0.5x &= 5.8-0.3x+1.8\\ 0.5x+0.3x &= 7.6\\ 0.8x &= 7.6\\ x &= \frac{7.6}{0.8}\\ x &= 9.5\end{aligned}$$

36.

$$\begin{aligned}1.9t &= 0.5(4.0-t)-0.8\\ 1.9t &= 2.0-0.5t-0.8\\ 1.9t+0.5t &= 1.2\\ 2.4t &= 1.2\\ t &= \frac{1.2}{2.4}\\ t &= 0.50\end{aligned}$$

37.

$$\begin{aligned}-0.24(C-0.50) &= 0.63\\ -0.24C+0.12 &= 0.63\\ -0.24C &= 0.63-0.12\\ -0.24C &= 0.51\\ C &= \frac{0.51}{-0.24}\\ C &= -2.125\\ C &= -2.1\end{aligned}$$

38.

$$27.5(5.17-1.44x)=73.4$$
$$142.175-39.6x=73.4$$
$$-39.6x=73.4-142.175$$
$$-39.6x=-68.775$$
$$x=\frac{-68.775}{-39.6}$$
$$x=1.736742424$$
$$x=1.74$$

39.

$$\frac{x}{2.0}=\frac{17}{6.0}$$
$$x=2.0\left(\frac{17}{6.0}\right)$$
$$x=5.6666666...$$
$$x=5.7$$

40.

$$\frac{3.0}{7.0}=\frac{R}{42}$$
$$R=42\left(\frac{3.0}{7.0}\right)$$
$$R=18$$

41.

$$\frac{165}{223}=\frac{13V}{15}$$
$$\frac{15}{13}\left(\frac{165}{223}\right)=\frac{15}{13}\left(\frac{13V}{15}\right)$$
$$V=\frac{2475}{2899}$$
$$V=0.85374267$$
$$V=0.85$$

42.

$$\frac{276x}{17.0}=\frac{1360}{46.4}$$
$$276x=17\left(\frac{1360}{46.4}\right)$$
$$x=\frac{498.2758621}{276}$$
$$x=1.805347326$$
$$x=1.81$$

43.

$$\frac{7.4}{x} = \frac{19.1}{42.7}$$

$$7.4 = x\left[\frac{19.1}{42.7}\right]$$

$$\frac{42.7}{19.1}[7.4] = x$$

$$x = 16.5$$

44.

$$\frac{10}{12.5} = \frac{-2.5}{R}$$

$$R\left[\frac{10}{12.5}\right] = -2.5$$

$$R = \frac{12.5}{10}[-2.5]$$

$$R = -3.125 \sim -3.1$$

45.

(a)

$$2x+3 = 3+2x$$

$$2x+3 = 2x+3$$

It is an identity, since it is true for all values of x.

(b)

$$2x-3 = 3-2x$$

$$4x = 6$$

$$x = \frac{6}{4} = \frac{3}{2}$$

It is conditional as x has one answer only.

46.

$$2x+a = 2x$$

$$2x-2x+a = 2x-2x$$

$$a = 0$$

Since a only has one value, this equation is conditional on a being 0.

Check:

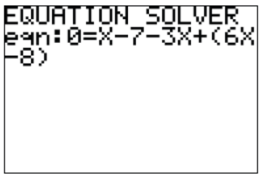
$$2x+a = 2x$$

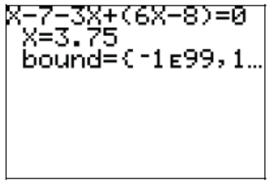
$$2x+0 = 2x$$

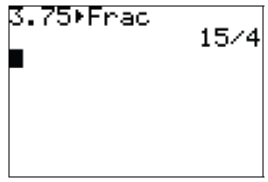
$$2x = 2x$$

47.

$$x-7=3x-(6x-8)$$
$$0=3x-6x+8-x+7$$
$$0=-4x+15$$
$$x=3.75$$

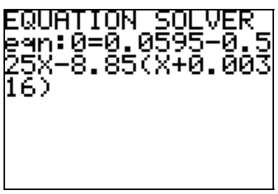


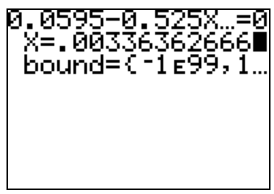




48.

$$0.0595-0.525i-8.85(i+0.0316)=0$$
$$0.595-0.525i-8.85i-0.27966=0$$
$$-9.375i+0.31534=0$$
$$i=0.033636266$$
$$i=0.0336$$





49.

$$2.0v+40=2.5(v+5.0)$$
$$2.0v+40=2.5v+12.5$$
$$40-12.5=2.5v-2.0v$$
$$27.5=0.5v$$
$$v=\frac{27.5}{0.5}$$
$$v=55 \text{ km/h}$$

50.

$$15(5.5+v)=24(5.5-v)$$
$$82.5+15v=132-24v$$
$$15v+24v=132-82.5$$
$$39v=49.5$$
$$v=\frac{49.5}{39}$$
$$v=1.269230769 \text{ km/h}$$
$$v=1.3 \text{ km/h}$$

51.

$$\begin{aligned}1.1 &= \frac{(T-76)}{40}\\ 40(1.1) &= T-76\\ 44 &= T-76\\ T &= 44+76\\ T &= 120\ °\text{C}\end{aligned}$$

52.

$$\begin{aligned}1.12V-0.67(10.5-V) &= 0\\ 1.12V-7.035+0.67V &= 0\\ 1.79V-7.035 &= 0\\ 1.79V &= 7.035\\ V &= \frac{7.035}{1.79}\\ V &= 3.930167598\ \text{V}\\ V &= 3.9\ \text{V}\end{aligned}$$

53.

$$\begin{aligned}0.14n+0.06(2000-n) &= 0.09(2000)\\ 0.14n+120-0.06n &= 180\\ 0.14n-0.06n &= 180-120\\ 0.08n &= 60\\ n &= \frac{60}{0.08}\\ n &= 750\ \text{L}\end{aligned}$$

54.

$$\begin{aligned}215(3x) &= 55.3x+38.5(8.25-3x)\\ 645x &= 55.3x+317.625-115.5x\\ 645x-55.3x+115.5x &= 317.625\\ 705.2x &= 317.625\\ x &= \frac{317.625}{705.2}\\ x &= 0.45040414\ \text{m}\\ x &= 0.450\ \text{m}\end{aligned}$$

55.

$$\begin{aligned}\frac{2050\ \text{km}}{55\ \text{L}} &= \frac{x}{22\ \text{L}}\\ x &= 22\ \text{L}\left(\frac{2050\ \text{km}}{55\ \text{L}}\right)\\ x &= 820\ \text{km}\end{aligned}$$

56.

$$\frac{1.8\text{ m}}{20\text{ mm}}=\frac{x}{16\text{ mm}}$$

$$x=16\text{ mm}\left(\frac{1.8\text{ m}}{20\text{ mm}}\right)$$

$$x=1.44\text{ mm}$$

$$x=1.4\text{ mm}$$

57.

$$\frac{20\text{ min}}{250\text{ cal}}=\frac{x}{400\text{ cal}}$$

$$x=400\text{ cal}\left(\frac{20\text{ min}}{250\text{ cal}}\right)$$

$$x=32\text{ min}$$

1.11 Formulas and Literal Equations

1.

$$v = v_0 + at$$
$$v - v_0 = at$$
$$a = \frac{v - v_0}{t}$$

2.

$$W = \frac{L(wL + 2P)}{8}$$
$$8W = L(wL + 2P)$$
$$8W = wL^2 + 2LP$$
$$wL^2 = 8W - 2LP$$
$$w = \frac{8W - 2LP}{L^2}$$

3.

$$V = V_0[1 + b(T - T_0)]$$
$$V = V_0[1 + bT - bT_0]$$
$$V = V_0 + bTV_0 - bT_0V_0$$
$$bT_0V_0 = V_0 + bTV_0 - V$$
$$T_0 = \frac{V_0 + bTV_0 - V}{bV_0}$$

4.

$$R = R_0 + R_0\alpha T$$
$$R - R_0 = R_0\alpha T$$
$$\alpha = \frac{R - R_0}{R_0 T}$$

5.

$$E = IR$$
$$R = \frac{E}{I}$$

6.

$$pV = nRT$$
$$T = \frac{pV}{nR}$$

7.

$$rL = g_2 - g_1$$
$$g_1 + rL = g_2$$
$$g_1 = g_2 - rL$$

8.

$$W = S_d T - Q$$
$$Q + W = S_d T$$
$$Q = S_d T - W$$

9.

$$B = \frac{nTWL}{12}$$
$$12B = nTWL$$
$$n = \frac{12B}{TWL}$$

10.

$$P = 2\pi Tf$$
$$T = \frac{P}{2\pi f}$$

11.

$$p = p_a + dgh$$
$$p - p_a = dgh$$
$$h = \frac{p - p_a}{dg}$$

12.

$$2Q = 2I + A + S$$
$$2I = 2Q - A - S$$
$$I = \frac{2Q - A - S}{2}$$

13.

$$F_c = \frac{mv^2}{r}$$
$$rF_c = mv^2$$
$$r = \frac{mv^2}{F_c}$$

14.

$$P = \frac{4F}{\pi D^2}$$
$$P\pi D^2 = 4F$$
$$F = \frac{P\pi D^2}{4}$$

15.

$$S_T = \frac{A}{5T} + 0.05d$$

$$S_T - 0.05d = \frac{A}{5T}$$

$$A = 5T\left(S_T - 0.05d\right)$$

16.

$$A = LH + \frac{1}{2}BH + \frac{\pi L^2}{8}$$

$$A - \frac{\pi L^2}{8} = LH + \frac{1}{2}BH$$

$$A - \frac{\pi L^2}{8} = H(L + 0.5B)$$

$$H = \frac{\frac{8A - \pi L^2}{8}}{L + 0.5B}$$

$$H = \frac{8A - \pi L^2}{8(L + 0.5B)}$$

$$H = \frac{8A - \pi L^2}{8L + 4B}$$

17.

$$ct^2 = 0.3t - ac$$

$$ac + ct^2 = 0.3t$$

$$ac = 0.3t - ct^2$$

$$a = \frac{-ct^2 + 0.3t}{c}$$

18.

$$2p + dv^2 = 2d(C - W)$$

$$2p + dv^2 = 2Cd - 2dW$$

$$2Cd = dv^2 + 2p + 2dW$$

$$C = \frac{dv^2 + 2dW + 2p}{2d}$$

19.

$$T = \frac{c + d}{v}$$

$$c + d = Tv$$

$$d = Tv - c$$

20.

$$B=\frac{\mu_0 I}{2\pi R}$$

$$BR=\frac{\mu_0 I}{2\pi}$$

$$R=\frac{\mu_0 I}{2\pi B}$$

21.

$$\frac{K_1}{K_2}=\frac{m_1+m_2}{m_1}$$

$$K_2(m_1+m_2)=K_1m_1$$

$$K_2m_1+K_2m_2=K_1m_1$$

$$K_2m_2=K_1m_1-K_2m_1$$

$$m_2=\frac{K_1m_1-K_2m_1}{K_2}$$

22.

$$f=\frac{F}{d-F}$$

$$f(d-F)=F$$

$$fd-fF=F$$

$$fd=F+fF$$

$$d=\frac{F+fF}{f}$$

23.

$$a=\frac{2mg}{M+2m}$$

$$a(M+2m)=2gm$$

$$aM+2am=2gm$$

$$aM=2gm-2am$$

$$M=\frac{2gm-2am}{a}$$

24.

$$v=\frac{V(m+M)}{m}$$

$$mv=mV+MV$$

$$MV=mv-mV$$

$$M=\frac{mv-mV}{V}$$

25.

$$C_0^2 = C_1^2(1+2V)$$
$$C_0^2 = C_1^2 + 2C_1^2V$$
$$2C_1^2V = C_0^2 - C_1^2$$
$$V = \frac{C_0^2 - C_1^2}{2C_1^2}$$

26.

$$A_1 = A(M+1)$$
$$A_1 = AM + A$$
$$AM = A_1 - A$$
$$M = \frac{A_1 - A}{A}$$

27.

$$N = r(A-s)$$
$$N = Ar - rs$$
$$rs + N = Ar$$
$$rs = Ar - N$$
$$s = \frac{Ar - N}{r}$$

28.

$$T = 3(T_2 - T_1)$$
$$T = 3T_2 - 3T_1$$
$$3T_1 + T = 3T_2$$
$$3T_1 = 3T_2 - T$$
$$T_1 = \frac{3T_2 - T}{3}$$

29.

$$T_2 = T_1 - \frac{h}{100}$$
$$100T_2 = 100T_1 - h$$
$$h + 100T_2 = 100T_1$$
$$h = 100T_1 - 100T_2$$

30.

$$p_2 = p_1 + rp_1(1-p_1)$$
$$p_2 - p_1 = rp_1(1-p_1)$$
$$r = \frac{p_2 - p_1}{p_1(1-p_1)}$$

31.

$$Q_1 = P(Q_2 - Q_1)$$
$$Q_1 = PQ_2 - PQ_1$$
$$PQ_2 = Q_1 + PQ_1$$
$$Q_2 = \frac{Q_1 + PQ_1}{P}$$

32.

$$p - p_a = dg\left(y_2 - y_1\right)$$
$$y_2 - y_1 = \frac{p - p_a}{dg}$$
$$y_2 = \frac{p - p_a}{dg} + y_1$$

33.

$$N = N_1T - N_2(1 - T)$$
$$N_1T = N + N_2(1 - T)$$
$$N_1 = \frac{N + N_2 - N_2T}{T}$$

34.

$$t_a = t_c + (1 - h)t_m$$
$$t_a = t_c + t_m - ht_m$$
$$t_a + ht_m = t_c + t_m$$
$$ht_m = t_c + t_m - t_a$$
$$h = \frac{t_c + t_m - t_a}{t_m}$$

35.

$$L = \pi(r_1 + r_2) + 2x_1 + 2x_2$$
$$L = \pi r_1 + \pi r_2 + 2x_1 + 2x_2$$
$$\pi r_1 = L - \pi r_2 - 2x_1 - 2x_2$$
$$r_1 = \frac{L - \pi r_2 - 2x_1 - 2x_2}{\pi}$$

36.

$$I = \frac{VR_2 + VR_1\left(1 + \mu\right)}{R_1R_2}$$
$$IR_1R_2 = VR_2 + VR_1 + VR_1\mu$$
$$VR_1\mu = IR_1R_2 - VR_2 + VR_1$$
$$\mu = \frac{IR_1R_2 - VR_2 + VR_1}{VR_1}$$

37.

$$P = \frac{V_1(V_2 - V_1)}{gJ}$$
$$gJP = V_1V_2 - V_1^2$$
$$V_1V_2 = V_1^2 + gJP$$
$$V_2 = \frac{V_1^2 + gJP}{V_1}$$

38.

$$W = T\left(S_1 - S_2\right) - Q$$
$$W + Q = TS_1 - TS_2$$
$$TS_2 = TS_1 - W - Q$$
$$S_2 = \frac{TS_1 - W - Q}{T}$$

39.

$$C = \frac{2eAk_1k_2}{d(k_1 + k_2)}$$
$$Cd(k_1 + k_2) = 2eAk_1k_2$$
$$e = \frac{Cd(k_1 + k_2)}{2Ak_1k_2}$$

40.

$$d = \frac{3LPx^2 - Px^3}{6EI}$$
$$6dEI = 3LPx^2 - Px^3$$
$$3LPx^2 = 6dEI + Px^3$$
$$L = \frac{6dEI + Px^3}{3Px^2}$$

41.

$$V = C\left(1 - \frac{n}{N}\right)$$
$$V = C - \frac{Cn}{N}$$
$$V + \frac{Cn}{N} = C$$
$$\frac{Cn}{N} = C - V$$
$$Cn = CN - NV$$
$$n = \frac{CN - NV}{C}$$

42.

$$\frac{p}{P}=\frac{AI}{B+AI}$$
$$p(B+AI)=AIP$$
$$pB+AIp=AIP$$
$$pB=AIP-AIp$$
$$B=\frac{AIP-AIp}{p}$$

43.

$$\eta=\frac{T_2}{T_1+T_2}$$
$$\eta(T_1+T_2)=T_2$$
$$\eta T_1+\eta T_2=T_2$$
$$\eta T_1=T_2-\eta T_2$$
$$T_1=\frac{T_2-\eta T_2}{\eta}$$
$$T_1=\frac{875\text{ K}-0.450(875\text{ K})}{0.450}$$
$$T_1=\frac{875\text{ K}-393.75\text{ K}}{0.450}$$
$$T_1=\frac{481.25\text{ K}}{0.450}$$
$$T_1=1069.444444\text{ K}$$
$$T_1=1070\text{ K}$$

44.

$$P_t=P_c(1+0.500m^2)$$
$$P_c=\frac{P_t}{1+0.500m^2}$$
$$P_c=\frac{685\text{ W}}{1+0.500(0.925)^2}$$
$$P_c=\frac{685\text{ W}}{1+0.500(0.855625)}$$
$$P_c=\frac{685\text{ W}}{1+0.4278125}$$
$$P_c=\frac{685\text{ W}}{1.4278125}$$
$$P_c=479.7548698\text{ W}$$
$$P_c=4.80\times10^2\text{ W}$$

45.

$$F = \frac{9}{5}C + 32$$
$$90.2 = \frac{9}{5}C + 32$$
$$\frac{5}{9}(90.2 - 32) = C$$
$$C = \frac{5}{9} \times 58.2$$
$$C = 32.3^\circ\text{C}$$

46.

$$V = \frac{1}{2}L(B + b)$$
$$2V = BL + bL$$
$$bL = 2V - BL$$
$$b = \frac{2V - BL}{L}$$
$$b = \frac{2(1.09 \text{ m}^3) - (0.244 \text{ m}^2)(4.91 \text{ m})}{(4.91 \text{ m})}$$
$$b = \frac{2.18 \text{ m}^3 - 1.19804 \text{ m}^3}{4.91 \text{ m}}$$
$$b = \frac{0.98186 \text{ m}^3}{4.91 \text{ m}}$$
$$b = 0.199991853 \text{ m}^2$$
$$b = 0.200 \text{ m}^2$$

47.

$$V_1 = \frac{VR_1}{R_1 + R_2}$$
$$V_1(R_1 + R_2) = VR_1$$
$$R_1 + R_2 = \frac{VR_1}{V_1}$$
$$R_2 = \frac{VR_1}{V_1} - R_1$$
$$R_2 = \frac{(12.0 \text{ V})(3.56\ \Omega)}{6.30 \text{ V}} - (3.56\ \Omega)$$
$$R_2 = 6.780952381\ \Omega - 3.56\ \Omega$$
$$R_2 = 3.220952381\ \Omega$$
$$R_2 = 3.22\ \Omega$$

48.

$$\eta = \frac{1}{q + p(1-q)}$$
$$\eta\left[q + p(1-q)\right] = 1$$
$$\eta q + \eta p(1-q) = 1$$
$$\eta p(1-q) = 1 - \eta q$$
$$p = \frac{1-\eta q}{\eta(1-q)}$$
$$p = \frac{1-(0.66)(0.83)}{0.66(1-0.83)}$$
$$p = \frac{1-0.5478}{0.66(0.17)}$$
$$p = \frac{0.4522}{0.1122}$$
$$p = 4.03030303$$
$$p = 4 \text{ processors}$$

49.

$$d = v_2 t_2 + v_1 t_1$$
$$d = v_2(4 \text{ h}) + v_1(t + 2 \text{ h})$$
$$tv_1 + v_1(2 \text{ h}) = d - v_2(4 \text{ h})$$
$$tv_1 = d - v_2(4 \text{ h}) - v_1(2 \text{ h})$$
$$t = \frac{d - v_2(4 \text{ h}) - v_1(2 \text{ h})}{v_1}$$

50.

$$C = x + 15y$$
$$15y = C - x$$
$$y = \frac{C - x}{15}$$

1.12 Applied Word Problems

1.
Let x = number of 25-W lights.
Let 31 − x = number of 40-Wlights.

$$\begin{aligned} 25x+40(31-x)&=1000 \\ 25x+1240-40x&=1000 \\ -15x&=1000-1240 \\ -15x&=-240 \\ x&=16 \end{aligned}$$

There are 16 of the 25-W lights and (31 – 16) = 15 of the 40-W lights.
Check:

$$\begin{aligned} 25\cdot 16+40(31-16)&=1000 \\ 400+40(15)&=1000 \\ 400+600&=1000 \\ 1000&=1000 \end{aligned}$$

2.
Let x = the number of slides with 5 mg.
Let x−3 = the number of slides with 6 mg.

$$\begin{aligned} (5\text{ mg})x&=(6\text{ mg})(x-3) \\ (5\text{ mg})x&=(6\text{ mg})x-18\text{ mg} \\ -x&=-18 \\ x&=18\text{ slides} \end{aligned}$$

There are 18 slides with 5 mg and (18−3) =15 slides with 6 mg.
Check:

$$\begin{aligned} 5\text{ mg}(18)&=6\text{ mg}(15) \\ 90\text{ mg}&=90\text{ mg} \end{aligned}$$

3.
Let t = the time for the shuttle to reach the satellite.

$$\begin{aligned} (29\ 500\text{ km/h})t&=6000\text{ km}+(27\ 500\text{ km/h})t \\ (2000\text{ km/h})t&=6000\text{ km} \\ t&=\frac{6000\text{ km}}{2000\text{ km/h}} \\ t&=3.000\text{ h} \end{aligned}$$

It will take the shuttle 3.000 h to reach the satellite.
Check:

$$\begin{aligned} (29\ 500\text{ km/h})(3\text{ h})&=6000\text{ km}+(27\ 500\text{ km/h})(3\text{ h}) \\ 88\ 500\text{ km}&=6000\text{ km}+82\ 500\text{ km} \\ 88\ 500\text{ km}&=88\ 500\text{ km} \end{aligned}$$

4.
Let x = the number of litres of 50% methanol blend that must be added.

$$0.0600(7250\text{ L})+0.500(x)=0.100(7250\text{ L}+x)$$
$$435\text{ L}+0.500(x)=725\text{ L}+0.100x$$
$$0.400(x)=290\text{ L}$$
$$x=\frac{290\text{ L}}{0.400}$$
$$x=725\text{ L}$$

725 L of the 50% methanol blend must be added.
Check:

$$0.0600(7250\text{ L})+0.500(725\text{ L})=0.100(7250\text{ L}+725\text{ L})$$
$$435\text{ L}+362.5\text{ L}=0.1(7975\text{ L})$$
$$797.5\text{ L}=797.5\text{ L}$$

5.

	Cost
Old model	x
New model	x + 5000
Total cost of both	70 000

$$x+(x+5000)=70\ 000$$
$$2x+5000=70\ 000$$
$$2x=65\ 000$$
$$x=32\ 500$$

The old car model was \$32 500 and the new car model is \$37 500.

6.
Let x = the flow from the first stream in m^3/s.
Let x – 45 m^3/s = the flow from the second stream in m^3/s.

$$x+(x-45\text{ m}^3/\text{s})=\frac{414\ 000\text{ m}^3}{3600\text{ s}}$$
$$2x-45\text{ m}^3/\text{s}=160\text{ m}^3/\text{s}$$
$$2x=160\text{ m}^3/\text{s}$$
$$x=\frac{160\text{ m}^3/\text{s}}{2}$$
$$x=80\text{ m}^3/\text{s}$$

The first stream flows 80 m^3/s and the second stream flows (80 m^3/s – 45 m^3/s) = 35 m^3/s.
Check:

$$80\text{ m}^3/\text{s}+(80\text{ m}^3/\text{s}-45\text{ m}^3/\text{s})=\frac{414\ 000\text{ m}^3}{3600\text{ s}}$$
$$80\text{ m}^3/\text{s}+35\text{ m}^3/\text{s}=115\text{ m}^3/\text{s}$$
$$115\text{ m}^3/\text{s}=115\text{ m}^3/\text{s}$$

7.

Let x = the number of cars recycled the first year.
Let x + 700 000 = the number of cars recycled the second year.

$$x+(x+700\ 000\text{ cars})=4\ 500\ 000\text{ cars}$$
$$2x+700\ 000\text{ cars}=4\ 500\ 000\text{ cars}$$
$$2x=3\ 800\ 000\text{ cars}$$
$$x=\frac{3\ 800\ 000\text{ cars}}{2}$$
$$x=1\ 900\ 000\text{ cars}$$

The first year, 1.9×10^5 cars were recycled, and the second year (1 900 000 + 700 000) = 2.6×10^5 cars were recycled.

Check:

$$1\ 900\ 000\text{ cars}+(1\ 900\ 000\text{ cars}+700\ 000\text{ cars})=4\ 500\ 000\text{ cars}$$
$$1\ 900\ 000\text{ cars}+2\ 600\ 000\text{ cars}=4\ 500\ 000\text{ cars}$$
$$4\ 500\ 000\text{ cars}=4\ 500\ 000\text{ cars}$$

8.

Let x = number of hits to the website on the first day.
Let $1/4x$ + 4000 = number of hits on the second day.
Let $1/4x$ = number of hits on the third day.

$$1/4x+4000\text{ hits}+1/4x=x$$
$$1/2x+4000\text{ hits}=x$$
$$1/2x=4000\text{ hits}$$
$$x=\frac{4000\text{ hits}}{1/2}$$
$$x=8000\text{ hits}$$

The first day there were 8000 hits, the second day there were (1/4(8000 hits) + 4000 hits) = 6000 hits, and the third day there were (1/4(8000 hits)) = 2000 hits.

Check:

$$1/4(8000\text{ hits})+4000\text{ hits}+1/4(8000\text{ hits})=8000\text{ hits}$$
$$2000\text{ hits}+4000\text{ hits}+2000\text{ hits}=8000\text{ hits}$$
$$8000\text{ hits}=8000\text{ hits}$$

9.

Let x = the number hectares of land leased for \$200 per hectare.
Let 140 – x = the number of hectares of land leased for \$300 per hectare.

$$\$200\ /\ \text{hectare}\ x+\$300\ /\ \text{hectare}(140\text{ hectares}-x)=\$37\ 000$$
$$-\$100\ /\ \text{hectare}\ x=-\$5\ 000$$
$$x=\frac{-\$5000}{-\$100\ /\ \text{hectare}}$$
$$x=50\text{ hectares}$$

There are 50 hectares leased at \$200 per hectare and (140 hectares – 50 hectares) = 90 hectares leased for \$300 per hectare.

Check:

$$\$200\ /\ \text{hectare}\ (50\text{ hectares})+\$300\ /\ \text{hectare}(140\text{ hectares}-(50\text{ hectares}))=\$37\ 000$$
$$\$10\ 000+\$27\ 000=\$37\ 000$$
$$\$37\ 000=\$37\ 000$$

10.
Let x = the first dose in mg.
Let x + 660 mg = the second dose in mg.

$$x + x + 660 \text{ mg} = 2000 \text{ mg}$$
$$2x = 1340 \text{ mg}$$
$$x = \frac{1340 \text{ mg}}{2}$$
$$x = 670 \text{ mg}$$

The first dose is 670 mg, and the second dose is (670 mg + 660 mg) = 1130 mg.
Check:

$$670 \text{ mg} + 670 \text{ mg} + 660 \text{ mg} = 2000 \text{ mg}$$
$$670 \text{ mg} + 1330 \text{ mg} = 2000 \text{ mg}$$
$$2000 \text{ mg} = 2000 \text{ mg}$$

11.
Let x = the amount of pollutant after modification in ppm/h.

$$(5 \text{ h})x = (3 \text{ h})150 \text{ ppm/h}$$
$$x = \frac{450 \text{ ppm}}{5 \text{ h}}$$
$$x = 90 \text{ ppm/h}$$

The amount of pollutant after modification is 90 ppm/h. The device reduced emissions by (150 ppm/h – 90 ppm/h) = 60 ppm/h.
Check:

$$(5 \text{ h})90 \text{ ppm/h} = (3 \text{ h})150 \text{ ppm/h}$$
$$450 \text{ ppm} = 450 \text{ ppm}$$

12.
Let x – 13 = the number of teeth that the first meshed spur has.
Let x = the number of teeth that the second meshed spur has.
Let x + 15 = the number of teeth that the third meshed spur has.

$$x - 13 \text{ teeth} + x + x + 15 \text{ teeth} = 107 \text{ teeth}$$
$$3x + 2 = 107 \text{ teeth}$$
$$3x = 105 \text{ teeth}$$
$$x = \frac{105 \text{ teeth}}{3}$$
$$x = 35 \text{ teeth}$$

The first spur has (35 – 13)= 22 teeth, the second spur has 35 teeth, and the third spur has (35 + 15) = 50 teeth.
Check:

$$35 \text{ teeth} - 13 \text{ teeth} + 35 \text{ teeth} + 35 \text{ teeth} + 15 \text{ teeth} = 107 \text{ teeth}$$
$$107 \text{ teeth} = 107 \text{ teeth}$$

13.
Let x = the number of 18-m girders needed.
Let x + 4 = the number of 15-m girders needed.

$$(18\text{ m})x = (15\text{ m})(x+4)$$
$$(18\text{ m})x = (15\text{ m})x + 60\text{ m}$$
$$(3\text{ m})x = 60\text{ m}$$
$$x = \frac{60\text{ m}}{3\text{ m}}$$
$$x = 20\text{ girders}$$

There would be 20 18-m girders needed or (20 girders + 4 girders) = 24 15-m girders needed.
Check:

$$(18\text{ m})20 = (15\text{ m})(\ 20+4)$$
$$360\text{ m} = 360\text{ m}$$

14.
Let x = the amount of oil used in a normal 8-week period in kL.
Let x + 20 kL / week = the amount oil used in the cold 6-week period in kL.

$$(8\text{ weeks})x = (6\text{ weeks})(x + 20\text{ kL/week})$$
$$(8\text{ weeks})x = (6\text{ weeks})x + (6\text{ weeks})20\text{ kL/week}$$
$$(2\text{ weeks})x = 120\text{ kL}$$
$$x = \frac{120\text{ kL}}{2\text{ weeks}}$$
$$x = 60\text{ kL/week}$$

Normally the amount of oil used would be 60 kL/week, which means that the fuel storage depot originally had (60 kL/week × 8 weeks) = 480 kL, which is 4.80×10^5 L.

Check:

$$(8\text{ weeks})\ 60\text{ kL/week} = (6\text{ weeks})(\ 60\text{ kL/week} + 20\text{ kL/week})$$
$$480\text{ kL} = 480\text{ kL}$$

15.
Let x = the first current in μA .
Let 2x = the second current in μA .
Let x + 9.2 μA = the third current in μA

$$x + 2x + x + 9.2\ \mu\text{A} = 0\ \mu\text{A}$$
$$4x = -9.2\ \mu\text{A}$$
$$x = \frac{-9.2\ \mu\text{A}}{4}$$
$$x = -2.3\ \mu\text{A}$$

The first current is −2.3 μA ,the second current is 2(−2.3 μA) = −4.6 μA , and the third current is (−2.3 μA +9.2 μA) = 6.9 μA .
Check:

$$-2.3\ \mu\text{A} + 2(-2.3\ \mu\text{A}) + \ (\text{-}2.3)\ \mu\text{A} + 9.2\ \mu\text{A} = 0\ \mu\text{A}$$
$$-2.3\ \mu\text{A} - 4.6\ \mu\text{A} - 2.3\mu\text{A} + 9.2\ \mu\text{A} = 0\ \mu\text{A}$$
$$0\ \mu\text{A} = 0\ \mu\text{A}$$

16.
Let x = the number of trucks in the first fleet.
Let x + 5 = the number of trucks in the second fleet.

$$(8\text{ h})x+(6\text{ h})(x+5)=198\text{ h}$$
$$(8\text{ h})x+(6\text{ h})x+30\text{ h}=198\text{ h}$$
$$(14\text{ h})x=168\text{ h}$$
$$x=\frac{168\text{ h}}{14\text{ h}}$$
$$x=12\text{ trucks}$$

There are 12 trucks in the first fleet and (12 trucks + 5 trucks) = 17 trucks in the second fleet.
Check:

$$(8\text{ h})(12)+(6\text{ h})(12+5)=198\text{ h}$$
$$96\text{ h}+(6\text{ h})(17)=198\text{ h}$$
$$96\text{ h}+102\text{ h}=198\text{ h}$$
$$198\text{ h}=198\text{ h}$$

17.
Let x = the length of the first pipeline in km.
Let x + 2.6 km = the length of the 3 other pipelines.

$$x+3(x+2.6\text{ km})=35.4\text{ km}$$
$$x+3x+7.8\text{ km}=35.4\text{ km}$$
$$4x=27.6\text{ km}$$
$$x=\frac{27.6\text{ km}}{4}$$
$$x=6.9\text{ km}$$

The first pipeline is 6.9 km long, and the other three pipelines are each (6.9 km + 2.6 km) = 9.5 km long.
Check:

$$6.9\text{ km}+3(6.9\text{ km}+2.6\text{ km})=35.4\text{ km}$$
$$6.9\text{ km}+3(9.5\text{ km})=35.4\text{ km}$$
$$6.9\text{ km}+28.5\text{ km}=35.4\text{ km}$$
$$35.4\text{ km}=35.4\text{ km}$$

18.

$$1(x)+1(y)=750$$
$$0.65(x)+0.75(y)=530$$

Using the first equation: $x = 750 - y$.
Putting this in the second equation and solving:

$$0.65(750-y)+0.75y=530$$
$$487.5-0.65y+0.75y=530$$
$$0.10y=42.5$$
$$y=425$$

The second generator produces 425 MW and the first produces 750 MW-425 MW = 325 MW at 100% efficiency.

Check:

$$0.65(325\text{ MW})+0.75(425\text{ MW})=530\text{ MW}$$
$$211.25\text{ MW}+318.75\text{ MW}=530\text{ MW}$$
$$530\text{ MW}=530\text{ MW}$$

19.

$x(150)+(54-x)(180)=8760$

$150x+9720-180x=8760$

$-30x=-960$

$x=32$

They installed 32 flat panels and $54-x=54-32=22$ custom panels.

Check:

$$
\begin{aligned}
\$150(32)+\$180(54-32)&=\$8760\\
\$4800+\$180(22)&=\$8760\\
\$4800+\$3960&=\$8760\\
\$8760&=\$8760
\end{aligned}
$$

20.

The amount of lottery winnings after taxes is \$20 000 × (1 − 0.25) = \$15 000.

Let x = the amount of money invested at a 40% gain.

Let \$15 000 − x = the amount of money invested at a 10% loss.

$$
\begin{aligned}
0.40x-0.10(\$15\ 000-x)&=\$2000\\
0.40x-\$1500+0.10x&=\$2000\\
0.50x&=\$3500\\
x&=\frac{\$3500}{0.50}\\
x&=\$7000
\end{aligned}
$$

The 40% gain investment had \$7000 invested, and the 10% loss investment had (\$15 000 − \$7000) = \$8000 invested.

Check:

$$
\begin{aligned}
0.40(\$7000)-0.10(\$15\ 000-\$7000)&=\$2000\\
\$2800-\$1500+0.10(\$7000)&=\$2000\\
\$2800-\$1500+\$700&=\$2000\\
\$2000&=\$2000
\end{aligned}
$$

21.

Let x = the amount of time the skier spends on the ski lift in minutes.

Let 24 minutes −x = the amount of time the skier spends skiing down the hill in minutes.

$$
\begin{aligned}
(50\text{ m/min})x&=(150\text{ m/min})(24\text{ min}-x)\\
(50\text{ m/min})x&=3600\text{ m}-(150\text{ m/min})x\\
(200\text{ m/min})x&=3600\text{ m}\\
x&=\frac{3600\text{ m}}{200\text{ m/min}}\\
x&=18\text{ min}
\end{aligned}
$$

The length of the slope is 18 minutes × 50 m/minute = 900 m.

Check:

$$
\begin{aligned}
(50\text{ m/min})18\text{ min}&=(150\text{ m/min})(24\text{ min}-18\text{ min})\\
900\text{ m}&=3600\text{ m}-(150\text{ m/min})(18\text{ min})\\
900\text{ m}&=3600\text{ m}-2700\text{ m}\\
900\text{ m}&=900\text{ m}
\end{aligned}
$$

22.
Let x = the speed of sound.
Let x – 100 km/h = speed travelled for 1 h.
Let x + 400 km/h = the speed travelled for 3 h.

$$1\text{ h}(x-100\text{ km/h})+3\text{ h}(x+400\text{ km/h})=5740\text{ km}$$
$$(1\text{ h})x-(1\text{ h})(100\text{ km/h})+(3\text{ h})x+(3\text{ h})(400\text{ km/h})=5740\text{ km}$$
$$(1\text{ h})x-100\text{ km}+(3\text{ h})x+12000\text{ km}=5740\text{ km}$$
$$(4\text{ h})x=4640\text{ km}$$
$$x=\frac{4640\text{ km}}{4\text{ h}}$$
$$x=1160\text{ km/h}$$

The speed of sound is 1160 km/h.
Check:

$$1\text{ h}(1160\text{ km/h}-100\text{ km/h})+3\text{ h}(1160\text{ km/h}+400\text{ km/h})=5740\text{ km}$$
$$1\text{ h}(1060\text{ km/h})+3\text{ h}(1560\text{ km/h})=5740\text{ km}$$
$$1060\text{ km}+4680\text{ km}=5740\text{ km}$$
$$5740\text{ km}=5740\text{ km}$$

23.
Let x = the speed the train leaving England in km/h.
Let x + 8 km/h = speed of the train leaving France in km/h.
The distance travelled by each train is speed × time.

$$x\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)+(x+8\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)=50\text{ km}$$
$$(0.28333\text{ h})x+(x+8\text{ km/h})(0.28333\text{ h})=50\text{ km}$$
$$(0.28333\text{ h})x+(0.28333\text{ h})x+2.26667\text{ km}=50\text{ km}$$
$$(0.56666\text{ h})x=47.73333\text{ km}$$
$$x=\frac{47.73333\text{ km}}{0.56666\text{ h}}$$
$$x=84.23529421\text{ km/h}$$
$$x=84.2\text{ km/h}$$

The train leaving England was travelling at 84.2 km/h, and the train leaving France was travelling at (84.2 km/h + 8 km/h) = 92.2 km/h.
Check:

$$84.23529421\text{ km/h}\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)+(84.23529421\text{ km/h}+8\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)=50\text{ km}$$
$$23.86666\text{ km}+(92.23529421\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)=50\text{ km}$$
$$23.86666\text{ km}+26.13333\text{ km}=50\text{ km}$$
$$50\text{ km}=50\text{ km}$$

24.
Let x = time left until the appointment.
Let x – 10.0 min = time taken to get to the appointment travelling at 60.0 km/h.
Let x – 5.0 min = time taken to get to the appointment travelling at 45.0 km/h.
The distance travelled by the executive in each scenario is the same. Distance = speed × time

$$60.0\text{ km/h}\left(x-\frac{10.0\text{ min}}{60\text{ min/h}}\right)=45\text{ km/h}\left(x-\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$(60.0\text{ km/h})x-60\text{ km/h}\left(\frac{10.0\text{ min}}{60\text{ min/h}}\right)=(45\text{ km/h})x-45\text{ km/h}\left(\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$(60.0\text{ km/h})x-10\text{ km}=(45.0\text{ km/h})x-3.75\text{ km}$$
$$(15.0\text{ km/h})x=6.25\text{ km}$$
$$x=\frac{6.25\text{ km}}{15.0\text{ km/h}}$$
$$x=0.416666667\text{ h}$$
$$x=0.416666667\text{ h}\times 60\text{ min/h}$$
$$x=25\text{ min}$$

There is 25 minutes left until the executive's appointment.
Check:

$$60.0\text{ km/h}\left(0.41667\text{ h}-\frac{10.0\text{ min}}{60\text{ min/h}}\right)=45\text{ km/h}\left(0.41667\text{ h}-\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$60.0\text{ km/h}(0.25\text{ h})=45\text{ km/h}(0.33333\text{ h})$$
$$15\text{ km}=15\text{ km}$$

25.
Let x – 30.0 s = time since the first car started moving in the race in seconds.
Let x= time since the second car started the race in seconds.
The distance travelled by each car will be the same at the point where the first car overtakes the second car. Distance = speed × time.

$$79.0\text{ m/s}(x-30.0\text{ s})=73.0\text{ m/s}(x)$$
$$(79.0\text{ m/s})x-(79.0\text{ m/s})(30.0\text{ s})=(73.0\text{ m/s})$$
$$(79.0\text{ m/s})x-2370\text{ m}=(73.0\text{ m/s})x$$
$$(6.0\text{ m/s})x=2370\text{ m}$$
$$x=\frac{2370\text{ m}}{6.0\text{ m/s}}$$
$$x=395\text{ s}$$

The first car will overtake the second car after 395 s. The first car travels 79 m/s × (395 s – 30 s) = 28 835 m by this point. 8 laps around the track is 4.36 km/lap. 8 laps × 1000 m/km = 34 880 m, so the first car will already be in the lead at the end of the 8th lap.
Check:

$$79.0\text{ m/s}(395\text{ s}-30.0\text{ s})=73.0\text{ m/s}(395\text{ s})$$
$$79.0\text{ m/s}(365\text{ s})=73.0\text{ m/s}(395\text{ s})$$
$$28\,835\text{ m}=28\,835\text{ m}$$

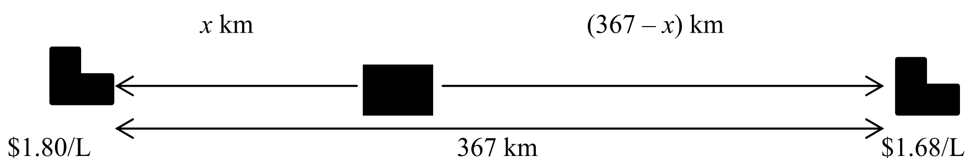
26.
Let x = the number of the first chips that is defective 0.50%.
Let $6100 - x$ = the number of the second chips that is defective 0.80%.

$$0.0050(x) + 0.0080(6100\text{ chips} - x) = 38\text{ chips}$$
$$(0.0050)x + 48.8\text{ chips} - (0.0080)x = 38\text{ chips}$$
$$-(0.0030)x = -10.8\text{ chips}$$
$$x = \frac{-10.8\text{ chips}}{-0.0030}$$
$$x = 3600\text{ chips}$$

There are 3600 chips that are 0.50% defective and (6100 chips – 3600 chips) = 2500 chips that are defective 0.80%.
Check:

$$0.0050(3600\text{ chips}) + 0.0080(6100\text{ chips} - 3600\text{ chips}) = 38\text{ chips}$$
$$18\text{ chips} + 0.0080(2500\text{ chips}) = 38\text{ chips}$$
$$18\text{ chips} + 20\text{ chips} = 38\text{ chips}$$
$$38\text{ chips} = 38\text{ chips}$$

27.



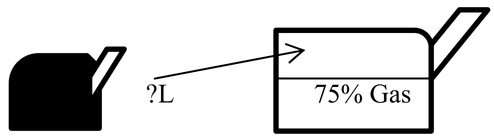
Assuming that the customer is located between the two gasoline distributors:
Let x = the distance in km to the first gasoline distributor that costs \$1.80/L.
Let 367 km $- x$ = the distance in km to the second gasoline distributor that costs \$1.68.

$$\$1.80 + \$0.0016(x) = \$1.68 + \$0.0016(367 - x)$$
$$\$1.80 + \$0.0016(x) = \$1.68 + \$0.5872 - \$0.0016(x)$$
$$\$0.0032(x) = \$0.4672$$
$$x = \frac{\$0.4672}{\$0.0032}$$
$$x = 146\text{ km}$$

The customer is 146 km away from the first gas distributor (\$1.80/L) and (367 km – 146 km) = 221 km away from the second gas distributor (\$1.68).
Check:

$$\$1.80 + \$0.0016(146\text{ km}) = \$1.68 + \$0.0016(367\text{ km} - 146\text{ km})$$
$$\$1.80 + \$0.2336 = \$1.68 + \$0.0016(221\text{ km})$$
$$\$1.80 + \$0.2336 = \$1.68 + \$0.3536$$
$$\$2.0336 = \$2.0336$$

28.



(100 % Gas) 8.0 L gas can (needs to be full of 93.75% gas/oil mixture)

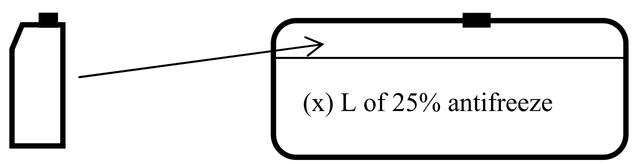
A 15:1 gas/oil mixture is 15/16 gasoline = 93.75%.
Let x = the amount of 100% gasoline added in L.
Let 8.0 L – x = the amount of 75% gasoline mixture in L.

$$1.00(x)+0.75(8.0\text{ L}-x)=0.9375(8.0\text{ L})$$
$$1.00(x)+6.0\text{ L}-0.75(x)=7.5\text{ L}$$
$$0.25(x)=1.5\text{ L}$$
$$x=\frac{1.5\text{ L}}{0.25}$$
$$x=6.0\text{ L}$$

6.0 L of 100% gasoline must be added to the 75% gas/oil mixture to make 8 L of 15:1 gasoline/ oil.
Check:

$$1.00(6.0\text{ L})+0.75(8.0\text{ L}-6.0\text{ L})=0.9375(8.0\text{ L})$$
$$6\text{ L}+0.75(2.0\text{ L})=7.5\text{ L}$$
$$6\text{ L}+1.5\text{ L}=7.5\text{ L}$$
$$7.5\text{ L}=7.5\text{ L}$$

29.



100% Antifreeze 12.0 L radiator (needs to be filled with 50% mixture)

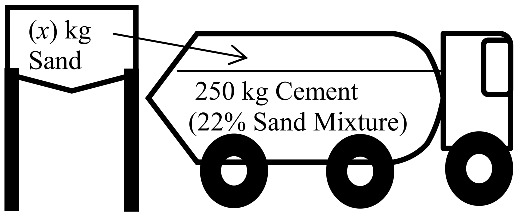
Let x = the amount in L of 25% antifreeze left in radiator
Let 12.0 L – x = the amount of 100% antifreeze added in L.

$$0.25(x)+1.00(12.0\text{ L}-x)=0.5(12.0\text{ L})$$
$$0.25(x)+12.0\text{ L}-1.00(x)=6.0\text{ L}$$
$$-0.75(x)=-6.0\text{ L}$$
$$x=\frac{-6.0\text{ L}}{-0.75}$$
$$x=8.0\text{ L}$$

There needs to be 8 L of 25% antifreeze left in radiator, so (12.0 L – 8.0 L) = 4.0 L must be drained.
Check:

$$0.25(8.0\text{ L})+1.00(12.0\text{ L}-8.0\text{ L})=0.5(12.0\text{ L})$$
$$2\ .0\text{L}+1.00(4.0\text{ L})=6.0\text{ L}$$
$$2.0\text{ L}+4.0\text{ L}=6.0\text{ L}$$
$$6.0\text{ L}=6.0\text{ L}$$

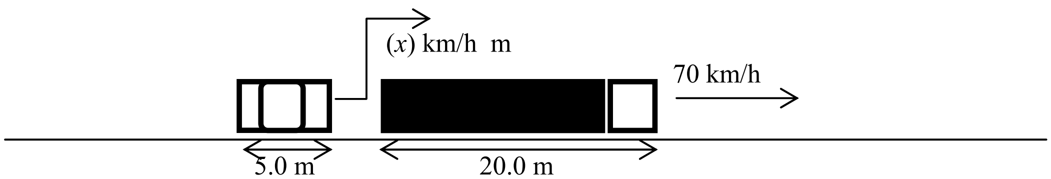
30.



Let x = the amount of sand added.
Let 250 kg + x = the amount in kg of the final 25% sand mixture.

$$1.00(x)+0.22(250\text{ kg})=0.25(250\text{ kg}+x)$$
$$1.00(x)+55\text{ kg}=62.5\text{ kg}+0.25(x)$$
$$0.75(x)=7.5\text{ kg}$$
$$x=\frac{7.5\text{ kg}}{0.75}$$
$$x=10\text{ kg}$$

31.



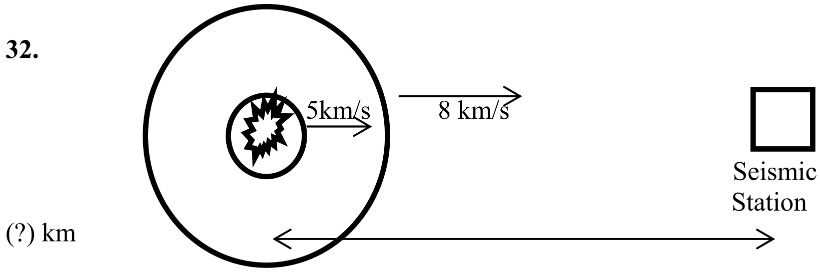
Let x = the speed the car needs to travel in km/h to pass the semi in 10 s.
Speed = distance/time. 10 s is 10s/3600 s/h = 0.002777777 h.

$$x=\frac{\text{distance needed to pass truck + distance travelled by truck in 10s}}{10s}$$
$$x=\frac{0.025\text{ km}+70\text{ km/h}\left(0.0027777\text{ h}\right)}{0.0027777\text{ h}}$$
$$x=\frac{0.025\text{ km}+0.19444\text{ km}}{0.0027777\text{ h}}$$
$$x=\frac{2.19444\text{ km}}{0.0027777\text{ h}}$$
$$x=79\text{ km/h}$$

The car needs to travel at a speed of 79 km/h to pass the semitrailer in 10 s.
Check:

$$79\text{ km/h}=\frac{0.025\text{ km}+70\text{ km/h}\left(0.0027777\text{ h}\right)}{0.0027777\text{ h}}$$
$$79\text{ km/h}=\frac{0.025\text{ km}+0.19444\text{ km}}{0.0027777\text{ h}}$$
$$79\text{ km/h}=79\text{ km/h}$$

32.



Let x = the time the first wave takes to travel to the seismic station in s.
Let x + 120 s = the time the first wave takes to travel to the seismic station in s.
Distance =speed × time. The distances travelled by both waves to the seismic station are the same. 2.0 min is (2.0 min × 60 s/min) = 120 s.

$$8.0\text{ km/s}(x) = 5.0\text{ km/s}(x + 120\text{ s})$$

$$8.0\text{ km/s}(x) = 5.0\text{ km/s}(x) + (5\text{ km/s})(120\text{ s})$$

$$3.0\text{ km/s}(x) = 600\text{ km}$$

$$x = \frac{600\text{ km}}{3.0\text{ km/s}}$$

$$x = 200\text{ s}$$

The distance to the seismic station is (200 s × 8.0 km/s) = 1600 km.
Check:

$$8.0\text{ km/s}(200\text{ s}) = 5.0\text{ km/s}(200\text{ s} + 120\text{ s})$$

$$1600\text{ km} = 5.0\text{ km/s}(320\text{ s})$$

$$1600\text{ km} = 1600\text{ km}$$

33.

$$0.1(0.5) + 4.0(0.1) = 4.1(x)$$

$$0.05 + 0.4 = 4.1x$$

$$0.45 = 4.1x$$

$$x = 0.1097 \sim 11\%$$

The resulting solution will have a concentration of 11%.

Review Exercises

1.
$(-2)+(-5)-3=-2-5-3=-10$

2.
$6-8-(-4)=6-8+4=2$

3.
$\frac{(-5)(6)(-4)}{(-2)(3)}=\frac{(20)\cancel{(6)}}{-\cancel{(6)}}=-20$

4.
$\frac{(-9)(-12)(-4)}{24}=\frac{108(-4)}{24}=\frac{-432}{24}=-18$

5.
$-5-|2(-6)|+\frac{-15}{3}=-5-|-12|+(-5)=-5-12-5=-22$

6.
$3-5|-3-2|-\frac{12}{-4}=3-5|-5|-(-3)=3-5(5)+3=6-25=-19$

7.
$\frac{18}{3-5}-(-4)^2=\frac{18}{-2}-(-4)(-4)=-9-16=-25$

8.
$-(-3)^2-\frac{-8}{(-2)-|-4|}=-(-3)(-3)-\frac{-8}{(-2)-4}=-9-\frac{-8}{-6}=-\frac{27}{3}-\frac{4}{3}=-\frac{31}{3}$

9.
$\sqrt{16}-\sqrt{64}=\sqrt{(4)(4)}-\sqrt{(8)(8)}=4-8=-4$

10.
$-\sqrt{81+144}=-\sqrt{225}=-\sqrt{(5)(5)(3)(3)}=-(3)(5)=-15$

11.
$(\sqrt{7})^2-\sqrt[3]{8}=(\sqrt{7})(\sqrt{7})-\sqrt[3]{(2)(2)(2)}=7-2=5$

12.
$-\sqrt[4]{16}+(\sqrt{6})^2=-\sqrt[4]{(2)(2)(2)(2)}+(\sqrt{6})(\sqrt{6})=-2+6=4$

13.
$(-2rt^2)^2=(-2)^2r^2t^{2\times 2}=4r^2t^4$

14.
$(3a^0b^{-2})^3=(3)^3(1)^3b^{-2\times 3}=27(1)b^{-6}=\frac{27}{b^6}$

15.

$$-3mn^{-5}t(8m^{-3}n^{4}) = -(3)(8)m^{1-3}n^{-5+4}t = -24m^{-2}n^{-1}t = -\frac{24t}{m^{2}n}$$

16.

$$\frac{15p^{4}q^{2}r}{5pq^{5}r} = \frac{3p^{4-1}\not{r}}{q^{5-2}\not{r}} = \frac{3p^{3}}{q^{3}}$$

17.

$$\frac{-16N^{-2}(NT^{2})}{-2N^{0}T^{-1}} = \frac{8N^{-2+1}T^{2+1}}{(1)} = \frac{8N^{-1}T^{3}}{(1)} = \frac{8T^{3}}{N}$$

18.

$$\frac{-35x^{-1}y(x^{2}y)}{5xy^{-1}} = \frac{-7y^{1+1+1}x^{2}}{x^{1+1}} = \frac{-7y^{3}\not{x^{2}}}{\not{x^{2}}} = -7y^{3}$$

19.

$$\sqrt{45} = \sqrt{(5)(3)(3)} = 3\sqrt{5}$$

20.

$$\sqrt{9+36} = \sqrt{45} = \sqrt{(5)(3)(3)} = 3\sqrt{5}$$

21.
(a) 8840 has three significant digits. (b) Rounded to two significant digits, it is 8800.

22.
(a) 21 450 has four significant digits. (b) Rounded to two significant digits, it is 21 000.

23.
(a) 9.040 has four significant digits. (b) Rounded to two significant digits, it is 9.0.

24.
(a) 0.700 has three significant digits. (b) Rounded to two significant digits, it is 0.70.

25.

$$\begin{aligned} 37.3-16.92(1.067)^{2} &= 37.3-16.92(1.138489) \\ &= 37.3-19.26323388 \\ &= 18.03676612 \end{aligned}$$

which rounds to 18.0.

26.

$$\begin{aligned} \frac{8.896\times10^{-12}}{3.5954+6.0449} &= \frac{8.896\times10^{-12}}{9.6403} \\ &= 9.227928591\times10^{-13} \end{aligned}$$

which rounds to 9.228×10^{-13}.

27.

$$\frac{\sqrt{0.1958+2.844}}{3.142(65)^2}=\frac{\sqrt{3.0398}}{3.142(4225)}$$
$$=\frac{1.743502223}{13274.95}$$
$$=0.000131337$$

which rounds to 1.3×10^{-4}.

28.

$$\frac{1}{0.03568}+\frac{37\ 466}{29.63^2}=28.02690583+\frac{37\ 466}{877.9369}$$
$$=28.02690583+42.67504874$$
$$=70.70195457$$

which rounds to 70.70, assuming that the 1 is exact.

29.

$$a-3ab-2a+ab=-2ab-a$$

30.

$$xy-y-5y-4xy=-3xy-6y$$

31.

$$6LC-(3-LC)=6LC-3+LC=7LC-3$$

32.

$$-(2x-b)-3(-x-5b)=-2x+b+3x+15b=16b+x$$

33.

$$(2x-1)(x+5)=(2x)(x)+(2x)(5)+(-1)(x)+(-1)(5)$$
$$=2x^2+10x-x-5$$
$$=2x^2+9x-5$$

34.

$$(C-4D)(2C-D)=(C)(2C)+(C)(-D)+(-4D)(2C)+(-4D)(-D)$$
$$=2C^2-CD-8CD+4D^2$$
$$=2C^2-9CD+4D^2$$

35.

$$(x+8)^2=(x+8)(x+8)$$
$$=(x)(x)+(x)(8)+(8)(x)+(8)(8)$$
$$=x^2+8x+8x+64$$
$$=x^2+16x+64$$

36.

$$(2r-9s)^2=(2r-9s)(2r-9s)$$
$$=(2r)(2r)+(2r)(-9s)+(-9s)(2r)+(-9s)(-9s)$$
$$=4r^2-18rs-18rs+81s^2$$
$$=4r^2-36rs+81s^2$$

37.

$$\frac{2h^3k^2-6h^4k^5}{2h^2k}=\frac{2h^3k^2}{2h^2k}-\frac{6h^4k^5}{2h^2k}$$
$$=h^{3-2}k^{2-1}-3h^{4-2}k^{5-1}$$
$$=-3h^2k^4+hk$$

38.

$$\frac{4a^2x^3-8ax^4}{-2ax^2}=\frac{4a^2x^3}{-2ax^2}-\frac{8ax^4}{-2ax^2}$$
$$=-2a^{2-1}x^{3-2}+\frac{4\cancel{a}x^{4-2}}{\cancel{a}}$$
$$=4x^2-2ax$$

39.

$$4R-\left[2r-(3R-4r)\right]=4R-\left[2r-3R+4r\right]$$
$$=4R-\left[6r-3R\right]$$
$$=4R-6r+3R$$
$$=7R-6r$$

40.

$$3b-\left[3a-(a-3b)\right]+4a=4a+3b-\left[3a-a+3b\right]$$
$$=4a+3b-\left[2a+3b\right]$$
$$=4a+3b-2a-3b$$
$$=2a$$

41.

$$2xy-\{3z-\left[5xy-(7z-6xy)\right]\}=2xy-\{3z-\left[5xy-7z+6xy\right]\}$$
$$=2xy-\{3z-\left[11xy-7z\right]\}$$
$$=2xy-\{3z-11xy+7z\}$$
$$=2xy-\{10z-11xy\}$$
$$=2xy-10z+11xy$$
$$=13xy-10z$$

42.

$$x^2+3b+\left[(b-y)-3(2b-y+z)\right]=x^2+3b+\left[b-y-6b+3y-3z\right]$$
$$=x^2+3b+\left[-5b+2y-3z\right]$$
$$=x^2+3b-5b+2y-3z$$
$$=x^2-2b+2y-3z$$

43.

$$(2x+1)(x^2-x-3)=(2x)(x^2)+(2x)(-x)+(2x)(-3)+(1)(x^2)+(1)(-x)+(1)(-3)$$
$$=2x^3-2x^2-6x+x^2-x-3$$
$$=2x^3-x^2-7x-3$$

44.

$$\begin{aligned}(x-3)(2x^2-3x+1) &= (x)(2x^2)+(x)(-3x)+(x)(1)+(-3)(2x^2)+(-3)(-3x)+(-3)(1)\\ &= 2x^3-3x^2+x-6x^2+9x-3\\ &= 2x^3-9x^2+10x-3\end{aligned}$$

45.

$$\begin{aligned}-3y(x-4y)^2 &= -3y(x-4y)(x-4y)\\ &= -3y[(x)(x)+(x)(-4y)+(-4y)(x)+(-4y)(-4y)]\\ &= -3y[x^2-4xy-4xy+16y^2]\\ &= -3y[x^2-8xy+16y^2]\\ &= -3x^2y+24xy^2-48y^3\end{aligned}$$

46.

$$\begin{aligned}-s(4s-3t)^2 &= -s(4s-3t)(4s-3t)\\ &= -s[(4s)(4s)+(4s)(-3t)+(-3t)(4s)+(-3t)(-3t)]\\ &= -s[16s^2-12st-12st+9t^2]\\ &= -s[16s^2-24st+9t^2]\\ &= -16s^3+24s^2t-9st^2\end{aligned}$$

47.

$$\begin{aligned}3p[(q-p)-2p(1-3q)] &= 3p[q-p-2p+6pq]\\ &= 3p[q-3p+6pq]\\ &= 18p^2q-9p^2+3pq\end{aligned}$$

48.

$$\begin{aligned}3x[2y-r-4(s-2r)] &= 3x[2y-r-4s+8r]\\ &= 3x[2y+7r-4s]\\ &= 21rx-12sx+6xy\end{aligned}$$

49.

$$\begin{aligned}\frac{12p^3q^2-4p^4q+6pq^5}{2p^4q} &= \frac{12p^3q^2}{2p^4q}-\frac{4p^4q}{2p^4q}+\frac{6pq^5}{2p^4q}\\ &= \frac{6q^{2-1}}{p^{4-3}}-\frac{2\cancel{p^4q}}{\cancel{p^4q}}+\frac{3q^{5-1}}{p^{4-1}}\\ &= \frac{3q^4}{p^3}+\frac{6q}{p}-2\end{aligned}$$

50.

$$\begin{aligned}\frac{27s^3t^2-18s^4t+9s^2t}{9s^2t} &= \frac{27s^3t^2}{9s^2t}-\frac{18s^4t}{9s^2t}+\frac{9s^2t}{9s^2t}\\ &= 3s^{3-2}t^{2-1}-\frac{2s^{4-2}\cancel{t}}{\cancel{t}}+\frac{\cancel{9s^2t}}{\cancel{9s^2t}}\\ &= -2s^2+3st+1\end{aligned}$$

51.

$$\begin{array}{r} 2x-5 \\ x+6\overline{)2x^2+7x-30} \\ \underline{2x^2+12x} \\ -5x-30 \\ \underline{-5x-30} \\ 0 \end{array}$$

52.

$$\begin{array}{r} 2x-7 \\ 2x+7\overline{)4x^2+0x-41} \\ \underline{4x^2+14x} \\ -14x-41 \\ \underline{-14x-49} \\ 8 \end{array}$$

$$\frac{4x^2-41}{2x+7}=2x-7+\frac{8}{2x+7}$$

53.

$$\begin{array}{r} x^2-2x+3 \\ 3x-1\overline{)3x^3-7x^2+11x-3} \\ \underline{3x^3-x^2} \\ -6x^2+11x \\ \underline{-6x^2+2x} \\ 9x-3 \\ \underline{9x-3} \\ 0 \end{array}$$

54.

$$\begin{array}{r} w^2-w+4 \\ w-3\overline{)w^3-4w^2+7w-12} \\ \underline{w^3-3w^2} \\ -w^2+7w \\ \underline{-w^2+3w} \\ 4w-12 \\ \underline{4w-12} \\ 0 \end{array}$$

55.

$$\begin{array}{r|l} & 4x^3-2x^2+6x \\ x+3 & 4x^4+10x^3+0x^2+18x-1 \\ & \underline{4x^4+12x^3} \\ & -2x^3+0x^2 \\ & \underline{-2x^3-6x^2} \\ & 6x^2+18x \\ & \underline{6x^2+18x} \\ & 0x-1 \end{array}$$

$$\frac{4x^4+10x^3+18x-1}{x+3}=4x^3-2x^2+6x-\frac{1}{x+3}$$

56.

$$\begin{array}{r|l} & 4x^2-6x+2 \\ 2x+3 & 8x^3+0x^2-14x+3 \\ & \underline{8x^3+12x^2} \\ & -12x^2-14x \\ & \underline{-12x^2-18x} \\ & 4x+3 \\ & \underline{4x+6} \\ & -3 \end{array}$$

$$\frac{8x^3-14x+3}{2x+3}=4x^2-6x+2-\frac{3}{2x+3}$$

57.

$$\begin{aligned} -3\{(r+s-t)-2[(3r-2s)-(t-2s)]\} &= -3\{r+s-t-2[3r-2s-t+2s)]\} \\ &= -3\{r+s-t-2[3r-t]\} \\ &= -3\{r+s-t-6r+2t]\} \\ &= -3\{-5r+s+t\} \\ &= 15r-3s-3t \end{aligned}$$

58.

$$\begin{aligned} &(1-2x)(x-3)-(x+4)(4-3x) \\ &=[(1)(x)+(1)(-3)+(-2x)(x)+(-2x)(-3)]-[(x)(4)+(x)(-3x)+(4)(4)+(4)(-3x) \\ &=[x-3-2x^2+6x]-[4x+-3x^2+16+-12x] \\ &=[-2x^2+7x-3]-[-3x^2-8x+16] \\ &=-2x^2+7x-3+3x^2+8x-16 \\ &=x^2+15x-19 \end{aligned}$$

59.

$$\begin{array}{r} y^2+5y-1 \\ 2y-1\overline{\big)2y^3+9y^2-7y+5} \\ \underline{2y^3-1y^2} \qquad\quad \\ 10y^2-7y \qquad \\ \underline{10y^2-5y} \qquad \\ -2y+5 \\ \underline{-2y+1} \\ 4 \end{array}$$

$$\frac{2y^3+9y^2-7y+5}{2y-1}=y^2+5y-1+\frac{4}{2y-1}$$

60.

$$\begin{array}{r} 3x+4y \\ 2x-y\overline{\big)6x^2+5xy-4y^2} \\ \underline{6x^2-3xy} \qquad \\ 8xy-4y^2 \\ \underline{8xy-4y^2} \\ 0 \end{array}$$

61.

$$\begin{aligned} 3x+1&=x-8 \\ 2x&=-9 \\ x&=-\frac{9}{2} \end{aligned}$$

62.

$$\begin{aligned} 4y-3&=5y+7 \\ -y&=10 \\ y&=-10 \end{aligned}$$

63.

$$\begin{aligned} \frac{5x}{7}&=\frac{3}{2} \\ 2(5x)&=3(7) \\ 10x&=21 \\ x&=\frac{21}{10} \end{aligned}$$

64.

$$\begin{aligned} \frac{2(N-4)}{3}&=\frac{5}{4} \\ \frac{2N-8}{3}&=\frac{5}{4} \\ 4(2N-8)&=3(5) \\ 8N-32&=15 \\ 8N&=47 \\ N&=\frac{47}{8} \end{aligned}$$

65.

$$6x-5=3(x-4)$$
$$6x-5=3x-12$$
$$3x=-7$$
$$x=-\frac{7}{3}$$

66.

$$-2(-4-y)=3y$$
$$8+2y=3y$$
$$y=8$$

67.

$$2s+4(3-s)=6$$
$$2s+12-4s=6$$
$$-2s=-6$$
$$s=\frac{-6}{-2}$$
$$s=3$$

68.

$$2|x|-1=3$$
$$2|x|=4$$
$$|x|=\frac{4}{2}$$
$$|x|=2$$
$$x=-2 \text{ and } 2$$

69.

$$3t-2(7-t)=5(2t+1)$$
$$3t-14+2t=10t+5$$
$$5t-14=10t+5$$
$$-5t=19$$
$$t=-\frac{19}{5}$$

70.

$$-(8-x)=x-2(2-x)$$
$$-8+x=x-4+2x$$
$$-8+x=3x-4$$
$$-2x=4$$
$$x=-\frac{4}{2}$$
$$x=-2$$

71.

$$\begin{aligned}2.7 + 2.0(2.1x - 3.4) &= 0.1\\ 2.7 + 4.2x - 6.8 &= 0.1\\ 4.2x - 4.1 &= 0.1\\ 4.2x &= 4.2\\ x &= \frac{4.2}{4.2}\\ x &= 1.0\end{aligned}$$

72.

$$\begin{aligned}0.250(6.721 - 2.44x) &= 2.08\\ 1.68025 - 0.610x &= 2.08\\ -0.610x &= 0.39975\\ x &= -\frac{0.39975}{0.610}\\ x &= 0.655327868\\ x &= 0.655\end{aligned}$$

73.
(a) 60 000 000 000 bytes = 6×10^{10} bytes
(b) 60 000 000 000 bytes = 60×10^{9} bytes
= 60 gigabytes

74.
(a) 40 000 km/h = 4×10^{4} km/h
(b) 40 000 km/h = 40×10^{3} km/h
= 40×10^{6} m/h
= 40 Mm/h

75.
(a) 15 400 000 000 km = 1.54×10^{10} km
(b) 15 400 000 000 km = 15.4×10^{9} km
= 15.4×10^{12} m
= 15.4 Tm

76.
(a) 1.02×10^{9} Hz = 1 020 000 000 Hz
(b) 1.02×10^{9} Hz = 1.02 GHz

77.
(a) 4.05×10^{13} km = 40 500 000 000 000 km
(b) 4.05×10^{13} km = 40.5×10^{12} km
= 40.5×10^{15} m
= 40.5 Pm
(Note that the symbol P stands for peta, which is the SI prefix associated with the multiple 10^{15}.)

78.
(a) 10^{6} m^2 = 1 000 000 m^2
(b) 10^{6} m^2 = 1 km^2

(Note that these are squared units, so 10^{6} is substituted by k.)

79.
(a) 10^{-12} W/m^2 = 0.000 000 000 001 W/m^2
(b) 10^{-12} W/m^2 = 1 pW/m^2

80.
(a) 0.000 000 15 m = 1.5×10^{-7} m
(b) 0.000 000 15 m = 150×10^{-9} m
= 150 nm

81.
(a) 1.5×10^{-1} Bq/L = 0.15 Bq/L
(b) 1.5×10^{-1} Bq/L = 150×10^{-3} mBq/L

82.
(a) 0.000 000 18 m = 1.8×10^{-7} m
(b) 0.000 000 18 m = 180×10^{-9} m
= 180 nm

83.
$$V = \pi r^2 L$$
$$L = \frac{V}{\pi r^2}$$

84.
$$R = \frac{2GM}{c^2}$$
$$c^2 R = 2GM$$
$$G = \frac{c^2 R}{2M}$$

85.
$$P = \frac{\pi^2 EI}{L^2}$$
$$L^2 P = \pi^2 EI$$
$$E = \frac{L^2 P}{\pi^2 I}$$

86.
$$f = p(c-1) - c(p-1)$$
$$f = cp - p - cp + c$$
$$f - c = -p$$
$$p = c - f$$

87.
$$Pp + Qq = Rr$$
$$Qq = Rr - Pp$$
$$q = \frac{Rr - Pp}{Q}$$

88.
$$V = IR + Ir$$
$$IR = V - Ir$$
$$R = \frac{V - Ir}{I}$$

89.

$$d = (n-1)A$$
$$d = An - A$$
$$d + A = An$$
$$n = \frac{d+A}{A}$$

90.

$$mu = (m+M)v$$
$$mu = mv + Mv$$
$$mu - mv = Mv$$
$$M = \frac{mu - mv}{v}$$

91.

$$N_1 = T(N_2 - N_3) + N_3$$
$$N_1 - N_3 = N_2T - N_3T$$
$$N_2T = N_1 - N_3 + N_3T$$
$$N_2 = \frac{N_1 - N_3 + N_3T}{T}$$

92.

$$q = \frac{KA(B-C)}{L}$$
$$Lq = ABK - ACK$$
$$ABK = Lq + ACK$$
$$B = \frac{Lq + ACK}{AK}$$

93.

$$R = \frac{A(T_2 - T_1)}{H}$$
$$HR = AT_2 - AT_1$$
$$AT_2 = HR + AT_1$$
$$T_2 = \frac{HR + AT_1}{A}$$

94.

$$Z^2\left(1 - \frac{\lambda}{2a}\right) = k$$
$$Z^2 - \frac{Z^2\lambda}{2a} = k$$
$$Z^2 - k = \frac{Z^2\lambda}{2a}$$
$$2a(Z^2 - k) = Z^2\lambda$$
$$\lambda = \frac{2aZ^2 - 2ak}{Z^2}$$

95.

$$d = kx^2[3(a+b)-x]$$
$$d = kx^2[3a+3b-x]$$
$$d = 3akx^2 + 3bkx^2 - kx^3$$
$$3akx^2 = d - 3bkx^2 + kx^3$$
$$a = \frac{d-3bkx^2+kx^3}{3kx^2}$$

96.

$$V = V_0[1+3a(T_2-T_1)]$$
$$V = V_0[1+3aT_2-3aT_1]$$
$$V = V_0 + 3aT_2V_0 - 3aT_1V_0$$
$$3aT_2V_0 = V - V_0 + 3aT_1V_0$$
$$T_2 = \frac{V-V_0+3aT_1V_0}{3aV_0}$$

97.

$$\frac{5.25\times10^{10}\text{ bytes}}{6.4\times10^4\text{ bytes}} = 82\ 0312.5$$

which rounds to 8.2×10^5. The newer computer's memory is 8.2×10^5 larger.

98.

$$t = 0.45\sqrt{22} = 2.110687092\text{ s}$$

which rounds to 2.1 s. It would take the person 2.1 s to fall 22 m.

99.

$$\frac{0.553\text{ km}}{0.442\text{ km}} = 1.251\ 131\ 222$$

which rounds to 1.25. The CN Tower is 1.25 times taller than the Willis Tower.

100.

$$t = \left(\frac{48\text{ cells}}{2650}\right)^2 = (0.018113207)^2 = 0.000328088\text{ s}$$

which rounds to 3.28×10^{-4} s. It would take the computer 3.28×10^{-4} s to check 48 memory cells.

101.

$$\frac{R_1R_2}{R_1+R_2} = \frac{(0.0275\ \Omega)(0.0590\ \Omega)}{0.0275\ \Omega + 0.0590\ \Omega}$$
$$= \frac{0.0016225\ \Omega^2}{0.0865\ \Omega}$$
$$= 0.018757225\ \Omega$$

which rounds to $0.0188\ \Omega$. The combined electric resistance is $0.0188\ \Omega$.

102.

$$1.5\times10^{11}\sqrt{\frac{m}{M}}=1.5\times10^{11}\sqrt{\frac{5.98\times10^{24}\text{ kg}}{1.99\times10^{30}\text{ kg}}}$$
$$=1.5\times10^{11}\sqrt{0.000003005}$$
$$=1.5\times10^{11}(0.0017335)$$
$$=260\ 025\ 124.4\text{ m}$$

which rounds to 2.6×10^8 m. The distance the space craft will be from the earth is 2.6×10^8 m.

103.

$$(x-2a)+100\text{ cm/m}(x+2a)=x-2a+100(x)+100(2a)$$
$$=x-2a+100x+200a$$
$$=101x+198a$$

The sum of their length is $101x + 198a$ cm.

104.

$$(Ai-R)(1+i)^2=(Ai-R)(1+i)(1+i)$$
$$=(Ai-R)[(1)(1)+(1)(i)+(i)(1)+(i)(i)]$$
$$=(Ai-R)[i^2+2i+1]$$
$$=(Ai)(i^2)+(Ai)(2i)+(Ai)(1)+(-R)(i^2)+(-R)(2i)+(-R)(1)$$
$$=Ai^3+2Ai^2+Ai-i^2R-2iR-R$$

105.

$$4(t+h)-2(t+h)^2=4t+4h-2(t+h)(t+h)$$
$$=4t+4h-2[(t)(t)+(t)(h)+(h)(t)+(h)(h)]$$
$$=4t+4h-2[t^2+2ht+h^2]$$
$$=4t+4h-2t^2-4ht-2h^2$$
$$=-2t^2-2h^2-4ht+4t+4h$$

106.

$$\frac{k^2r-2h^2k+h^2rv^2}{k^2r}=\frac{k^2r}{k^2r}-\frac{2h^2k}{k^2r}+\frac{h^2rv^2}{k^2r}$$
$$=\frac{\cancel{k^2r}}{\cancel{k^2r}}-\frac{2h^2}{k^{2-1}r}+\frac{h^2\cancel{r}v^2}{k^2\cancel{r}}$$
$$=1-\frac{2h^2}{kr}+\frac{h^2v^2}{k^2}$$

107.

$3\times18\div(9-6)=54\div(3)=18$

$3\times18\div9-6=54\div9-6=6-6=0$

Yes, the removal of the parentheses does affect the answer.

108.

$(3\times18)\div9-6=54\div9-6=6-6=0$

$3\times18\div9-6=54\div9-6=6-6=0$

No, the removal of the parentheses does not affect the answer.

109.

$$x-(3-x)=2x-3$$
$$x-3+x=2x-3$$
$$2x-3=2x-3$$

The equation is valid for all values of the unknown, so the equation is an identity.

110.

$$7-(2-x)=x+2$$
$$7-2+x=x+2$$
$$x+5=x+2$$
$$5=2$$

The equation has no values of the unknown for which it is valid, so the equation is a contradiction.

111.

$$\begin{aligned}(x-y)^3&=(x-y)(x-y)(x-y)\\&=\left(-(y-x)\right)\left(-(y-x)\right)\left(-(y-x)\right)\\&=-(y-x)(y-x)(y-x)\\&=-(y-x)^3\end{aligned}$$

112.

$$(a\div b)\div c\neq a\div(b\div c)$$
$$(8\div 4)\div 2=2\div 2=1$$
$$8\div(4\div 2)=8\div 2=4$$

Division is not associative.

113.

$$\frac{8\times10^{-3}}{2\times10^{4}}=4\times10^{-7}$$

114.

$$\frac{\sqrt{4+36}}{\sqrt{4}}=\frac{\sqrt{(2)(2)(10}}{2}=\frac{2\sqrt{10}}{2}=\sqrt{10}$$

115.

Let x = the cost of the first computer program.
Let x + \$72 = the cost of the second computer program.

$$x+(x+\$72)=\$190$$
$$2x+\$72=\$190$$
$$2x=\$118$$
$$x=\frac{\$118}{2}$$
$$x=\$59$$

The cost of the first computer program is \$59, and the other program costs (\$59 + \$72) = \$131.
Check: \$59 + \$131 = 190

116.
Let x = the cost to run the commercial on the first station.
Let x + \$1100 = the cost to run the commercial on the second station.

$$\begin{aligned} x+(x+\$1100) &= \$9500 \\ 2x+\$1100 &= \$9500 \\ 2x &= \$8400 \\ x &= \frac{\$8400}{2} \\ x &= \$4200 \end{aligned}$$

The cost of the run the commercial on the first station is \$4200, and the cost for the other station is (\$4200 + \$1100) = \$5300.
Check: \$4200 + \$5300 = \$9500

117.
Let $2x$ = the amount of oxygen produced in cm^3 by the first reaction.
Let x = the amount of oxygen produced in cm^3 by the second reaction.
Let $4x$ = the amount of oxygen produced in cm^3 by the third reaction.

$$\begin{aligned} 2x+x+4x &= 560 \text{ cm}^3 \\ 7x &= 560 \text{ cm}^3 \\ x &= \frac{560 \text{ cm}^3}{7} \\ x &= 80 \text{ cm}^3 \end{aligned}$$

The first reaction produces (2×80 cm^3) = 160 cm^3 of oxygen, the second reaction produces 80 cm^3 of oxygen, and the third reaction produces (4×80 cm^3) = 320 cm^3 of oxygen.
Check: 160 cm^3 + 80 cm^3 + 320 cm^3 = 560 cm^{3*}

118.
Let x = the speed that the river is flowing in km/h.
Let x + 5.5 km/h = the speed that the boat travels downstream.
Let $-x$ + 5.5 km/h = the speed that the boat travels upstream.
The distance that the boat travelled is the same in both experiments. Distance = speed × time.

$$\begin{aligned} (x+5.5 \text{ km/h})(5.0 \text{ h}) &= (-x+5.5 \text{ km/h})(8.0 \text{ h}) \\ (5.0 \text{ h})(x)+(5.5 \text{ km/h})(5.0 \text{ h}) &= (8.0 \text{ h})(-x)+(5.5 \text{ km/h})(8.0 \text{ h}) \\ (5.0 \text{ h})(x)+(27.5 \text{ km}) &= (-8.0 \text{ h})(x)+(44 \text{ km}) \\ (13.0 \text{ h})x &= 16.5 \text{ km} \\ x &= \frac{16.5 \text{ km}}{13 \text{ h}} \\ x &= 1.269230769 \text{ km/h} \end{aligned}$$

which rounds to 1.3 km/h. The polluted stream is flowing at 1.3 km/h.

Check:

$$\begin{aligned} (1.269230769 \text{ km/h}+5.5 \text{ km/h})(5.0 \text{ h}) &= (-1.269230769 \text{ km/h}+5.5 \text{ km/h})(8.0 \text{ h}) \\ (6.769230769 \text{ km/h})(5.0 \text{ h}) &= (4.2 \text{ km/h})(8.0 \text{ h}) \\ (33.8 \text{ km}) &= (33.8 \text{ km}) \end{aligned}$$

119.
Let x = the resistance in the first resistor in Ω.
Let $x + 1200\,\Omega$ = the resistance in the second resistor in Ω.
Voltage = current × resistance. 2.4 μA = 2.3 × 10^{-6} A. 12 mV = 0.0120 V

$$\begin{aligned}(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(x+1200\ \Omega)&=0.0120\ \text{V}\\(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(1200\ \Omega)&=0.0120\ \text{V}\\(4.8\ \times10^{-6}\text{A})(x)+(0.00288\text{V})&=0.0120\ \text{V}\\(4.0\times10^{-6}\text{A})(x)&=0.00912\ \text{V}\\x&=\frac{0.00912\ \text{V}}{4.8\times10^{-6}\text{A}}\\x&=1900\ \Omega\end{aligned}$$

The first resistor's resistance is 1900 Ω and the second resistor's is (1900 Ω + 1200 Ω) = 3100 Ω.
Check:

$$\begin{aligned}(2.4\times10^{-6}\text{A})(1900\ \Omega)+(2.4\times10^{-6}\text{A})(1900\ \Omega+1200\ \Omega)&=0.0120\ \text{V}\\0.00456\ \text{V}+0.00744\ \text{V}&=0.0120\ \text{V}\\0.0120\ \text{V}&=0.0120\ \text{V}\end{aligned}$$

120.
Let x = the concentration of the first pollutant in ppm.
Let $4x$ = the concentration of the second pollutant in ppm.

$$\begin{aligned}x+4x&=4.0\ \text{ppm}\\5x&=4.0\ \text{ppm}\\x&=\frac{4.0\ \text{ppm}}{5}\\x&=0.8\ \text{ppm}\end{aligned}$$

The concentration of the first pollutant is 0.8 ppm, and the concentration of the second is (4 × 0.8 ppm) = 3.2 ppm.
Check:

$$\begin{aligned}0.8\ \text{ppm}+4(0.8\ \text{ppm})&=4.0\ \text{ppm}\\0.8\ \text{ppm}+3.2\ \text{ppm}&=4.0\ \text{ppm}\\4.0\ \text{ppm}&=4.0\ \text{ppm}\end{aligned}$$

121.
Let x = the time taken in hours for the crew to build 250 m of road.
The crew works at a rate of 450 m/12 h, which is 37.5 m/h. Time = distance / speed.

$$\begin{aligned}x&=\frac{250\ \text{m}}{37.5\ \text{m/h}}\\x&=6.666666667\ \text{h}\end{aligned}$$

which rounds to 6.7 h.

122.
Let x = the amount of oil in L in the mixture.
Let $15x$ = the amount of gas in L in the mixture.

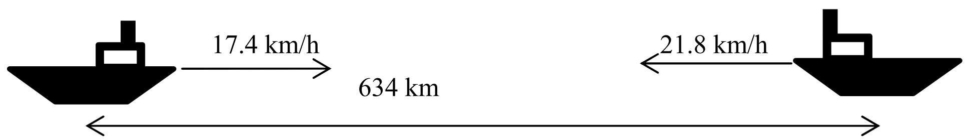
$$\begin{aligned}x+15x&=6.6\ \text{L}\\16x&=6.6\ \text{L}\\x&=\frac{6.6\ \text{L}}{16}\\x&=0.4125\ \text{L}\end{aligned}$$

which rounds to 0.41 L. There is 0.41 L of oil in the mixture and (15 × 0.41 L) = 6.2 L of gas.

Check:

$$\begin{aligned} 0.4125 \text{ L} + 15(0.4125 \text{ L}) &= 6.6 \text{ L} \\ 0.4125 \text{ L} + 6.1875 \text{ L} &= 6.6 \text{ L} \\ 6.6 \text{ L} &= 6.6 \text{ L} \end{aligned}$$

123.



Let x = the time taken by the second ship in hours.

Let x + 2 h = the amount time taken by the first ship in hours.

The distance travelled adds up to 634 km. Distance = speed × time.

$$\begin{aligned} 21.8 \text{ km/h}(x) + 17.4 \text{ km/h}(x + 2 \text{ h}) &= 634 \text{ km} \\ 21.8 \text{ km/h}(x) + 17.4 \text{ km/h}(x) + 17.4 \text{ km/h}(2 \text{ h}) &= 634 \text{ km} \\ 39.2 \text{ km/h}(x) + 34.8 \text{ km} &= 634 \text{ km} \\ 39.2 \text{ km/h}(x) &= 599.2 \text{ km} \\ x &= \frac{599.2 \text{ km}}{39.2 \text{ km/h}} \\ x &= 15.2857 \text{ h} \end{aligned}$$

which rounds to 15.3 h. The ships will pass 15.3 h after the second ship leaves Duluth.

Check:

$$\begin{aligned} 21.8 \text{ km/h}(15.2857 \text{ h}) + 17.4 \text{ km/h}(15.2857 \text{ h} + 2 \text{ h}) &= 634 \text{ km} \\ 333.23 \text{ km} + 300.77 \text{ km} &= 634 \text{ km} \\ 634 \text{ km} &= 634 \text{ km} \end{aligned}$$

124.

Let x = the time take in h for the helicopter to travel from the pond to the fire.

Let 0.5 h $-x$ = the time take in h for the helicopter to travel from the fire to the pond.
30 min / 60 min/h = 0.5 h. The distance travelled by the helicopter is the same for both trips. Distance = speed × time.

$$\begin{aligned} 175.0 \text{ km/h}(0.5 \text{ h} - x) &= 115.0 \text{ km/h}(x) \\ 87.5 \text{ km} - 175.0 \text{ km/h}(x) &= 115.0 \text{ km/h}(x) \\ 87.5 \text{ km} &= 290 \text{ km/h}(x) \\ x &= \frac{87.5 \text{ km}}{290 \text{ km/h}} \\ x &= 0.301724137 \text{ h} \end{aligned}$$

which rounds to 0.30 h. It will take the helicopter 0.30 h to fly from the pond to the fire.

Check:

$$\begin{aligned} 175.0 \text{ km/h}(0.5 \text{ h} - 0.301724137 \text{ h}) &= 115.0 \text{ km/h}(0.301724137 \text{ h}) \\ 34.69827 \text{ km} &= 34.69827 \text{ km} \end{aligned}$$

125.
Let x = the number of litres of 0.50% grade oil used.
Let 1000 L − x the number of litres of 0.75% grade oil used.

$$0.005(x)+0.0075(1000 \text{ L}-x)=0.0065(1000 \text{ L})$$
$$0.005(x)+7.5 \text{ L}-0.0075(x)=6.5 \text{ L}$$
$$-0.0025(x)=-1.0 \text{ L}$$
$$x=\frac{-1.0 \text{ L}}{-0.0025}$$
$$x=400 \text{ L}$$

It will take 400 L of the 0.50% grade oil and (1000 L – 400 L) = 600 L of the 0.75% grade oil to make 1000 L of 0.65% grade oil.
Check:

$$0.005(400 \text{ L})+0.0075(1000 \text{ L}-400 \text{ L})=0.0065(1000 \text{ L})$$
$$2 \text{ L}+4.5 \text{ L}=6.5 \text{ L}$$
$$6.5 \text{ L}=6.5 \text{ L}$$

126.
Let x = the number of mL of water added.
Let x + 20 mL = the resulting number of mL in the 45% saline solution.

$$0.60(20 \text{ mL})=0.45(x+20 \text{ mL})$$
$$12 \text{ mL}=0.45(x)+9 \text{ mL}$$
$$3 \text{ mL}=0.45(x)$$
$$x=\frac{3 \text{ mL}}{0.45}$$
$$x=6.666666667 \text{ mL}$$

which rounds to 6.67 mL. It will take 6.67 mL of water to make the 45% saline solution.
Check:

$$0.60(20 \text{ mL})=0.45(6.666666667 \text{ mL}+20 \text{ mL})$$
$$12 \text{ mL}=0.45(26.666666667 \text{ mL})$$
$$12 \text{ mL}=12 \text{ mL}$$

127.
Let x = the area of space in m^2 in the kitchen and bath.

$$\frac{\text{m}^2 \text{ of tile in the house}}{\text{m}^2 \text{ in the house}}=0.25$$
$$\frac{x+0.15(205 \text{ m}^2)}{(x+205 \text{ m}^2)}=0.25$$
$$x+30.75 \text{ m}^2=0.25(x)+(0.25)(205 \text{ m}^2)$$
$$x+30.75 \text{ m}^2=0.25(x)+51.25 \text{ m}^2$$
$$0.75x=20.5 \text{ m}^2$$
$$x=\frac{20.5 \text{ m}^2}{0.75}$$
$$x=27.33333333 \text{ m}^2$$

which rounds to 27 m^2. The kitchen and bath area is 27 m^2.
Check:

$$\frac{27.33333333 \text{ m}^2+0.15(205 \text{ m}^2)}{(27.33333333 \text{ m}^2+205 \text{ m}^2)}=0.25$$
$$\frac{58.08333333 \text{ m}^2}{232.3333333 \text{ m}^2}=0.25$$
$$0.25=0.25$$

128.
Let x = the number of grams of 9-karat gold.
Let 200 g – x = the number of grams of 18-karat gold. 9-karat gold is 9/24 gold = 0.375, 18-karat gold is 18/24 gold= 0.75, and 14-karat gold is 14/24 gold = 0.583333333.

$$\begin{aligned} 0.375(x) + 0.75(200\text{ g} - x) &= 0.583333333(200\text{ g}) \\ 0.375(x) + 150\text{ g} - 0.75(x) &= 116.6666666\text{ g} \\ -0.375(x) &= -33.3333334\text{ g} \\ x &= \frac{-33.3333334\text{ g}}{-0.375} \\ x &= 88.88888907\text{ g} \end{aligned}$$

which rounds to 89 g. There is 89 g of 9-karat gold and (200 g – 89 g) = 111 g of 18-karat gold needed to make 200 g of 14-karat gold.
Check:

$$\begin{aligned} 0.375(88.88888907\text{ g}) + 0.75(200\text{ g} - 88.88888907\text{ g}) &= 0.583333333(200\text{ g}) \\ 33.3333334\text{ g} + 83.3333332\text{ g} &= 116.6666666\text{ g} \\ 116.6666666\text{ g} &= 116.6666666\text{ g} \end{aligned}$$

129.

$$\begin{aligned} P &= P_0 + P_0 rt \\ P - P_0 &= P_0 rt \\ r &= \frac{P - P_o}{P_0 t} \\ r &= \frac{\$7625 - \$6250}{\$6250(4.000\text{ years})} \\ r &= \frac{\$1375}{25\ 000} \\ r &= 0.055 \end{aligned}$$

The rate is equal to 5.500%.

On the calculator type:
$(7625 - 6250) / (6250 \times 4.000)$