INSTRUCTOR'S SOLUTIONS MANUAL

INTRODUCTION TO CRYPTOGRAPHY WITH CODING THEORY

THIRD EDITION

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Pearson

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ISBN-13: 978-0-13-487691-7
ISBN-10: 0-13-487691-1

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1. Among the shifts of *EVIRE*, there are two words: *arena* and *river*. Therefore, Anthony cannot determine where to meet Caesar.

3. The inverse of 9 mod 26 is 3. Therefore, the decryption function is $x = 3(y-2) = 3y - 6 \pmod{26}$. Now simply decrypt letter by letter as follows. $U = 20$ so decrypt U by calculating $3 * 20 - 6 \pmod{26} = 2$, and so on. The decrypted message is 'cat'.

5. Changing the plaintext to numbers yields 7, 14, 22, 0, 17, 4, 24, 14, 20. Applying $5x+7$ to each yields $5 \cdot 7 + 7 = 42 \equiv 16 \pmod{26}$, $5 \cdot 14 + 7 = 77 \equiv 25$, etc. Changing back to letters yields *QZNHOBXZD* as the ciphertext. The decryption function is $21x + 9$.

7. Let $mx + n$ be the encryption function. Since $h = 7$ and $N = 13$, we have $m \cdot 7 + n \equiv 13 \pmod{26}$. Using the second letters yields $m \cdot 0 + n \equiv 14$. Therefore $n = 14$. The first congruence now yields $7m \equiv -1 \pmod{26}$. This yields $m = 11$. The encryption function is therefore $11x + 14$.

9. Let the decryption function be $x = ay + b$. The first letters tell us that $7 \equiv a \cdot 2 + b \pmod{26}$. The second letters tell us that $0 \equiv a \cdot 17 + b$.Subtracting yields $7 \equiv a \cdot (-15) \equiv 11a$. Since $11^{-1} \equiv 19 \pmod{26}$, we have $a \equiv 19 \cdot 7 \equiv 3 \pmod{26}$. The first congruence now tells us that $7 \equiv 3 \cdot 2 + b$, so $b = 1$. The decryption function is therefore $x \equiv 3y + 1$. Applying this to *CRWWZ* yields *happy* for the plaintext.

11. Let $mx + n$ be one affine function and $ax + b$ be another. Applying the first then the second yields the function $a(mx+n)+b = (am)x+(an+b)$, which is an affine function. Therefore, successively encrypting with two affine functions is the same as encrypting with a single affine function. There is therefore no advantage of doing double encryption in this case. (Technical point: Since $\gcd(a, 26) = 1$ and $\gcd(m, 26) = 1$, it follows that $\gcd(am, 26) = 1$, so the affine function we obtained is still of the required form.)

13. For an affine cipher $mx + n \pmod{27}$, we must have $\gcd(27, m) = 1$, and we can always take $1 \le m \le 27$. So we must exclude all multiples of 3, which leaves 18 possibilities for m. All 27 values of n are possible, so we have $18 \cdot 27 = 486$ keys. When we work mod 29, all values $1 \le m \le 28$ are allowed, so we have $28 \cdot 29 = 812$ keys.

15. (a) In order for α to be valid and lead to a decryption algorithm, we need $\gcd(\alpha, 30) = 1$. The possible values for α are $1, 7, 11, 13, 17, 19, 23, 29$.

(b) We need to find two x such that $10x \pmod{30}$ gives the same value.

There are many such possible answers, for example $x = 1$ and $x = 4$ will work. This corresponds to the letters 'b' and 'e'.

17. If $x_1 = x_2 + (26/d)$, then $\alpha x_1 + \beta = \alpha x_2 + \beta + (\alpha/d)26$. Since $d = \gcd(\alpha, 26)$ divides α, the number $\alpha/26$ is an integer. Therefore $(\alpha/d)26$ is a multiple of 26, which means that $\alpha x_1 + \beta \equiv \alpha x_2 + \beta \pmod{26}$. Therefore x_1 and x_2 encrypt to the same ciphertext, so unique decryption is impossible.

19. (a) In order to find the most probable key length, we write the ciphertext down on two strips and shift the second strip by varying amounts. The shift with the most matches is the most likely key length. As an example, look at the shift by 1:

B	A	B	A	B	A	A	A	B	A	
	B	A	B	A	B	A	A	A	B	A
						*	*			

This has a total of 2 matches. A shift by 2 has 6 matches, while a shift by 3 has 2 matches. Thus, the most likely key length is 2.

(b) To decrypt, we use the fact that the key length is 2 and extract off every odd letter to get BBBAB, and then every even letter to get AAAAA. Using a frequency count on each of these yields that a shift of 0 is the most likely scenario for the first character of the Vigenere key, while a shift of 1 is the most likely case for the second character of the key. Thus, the key is AB. Decrypting each subsequence yields BBBAB and BBBBB. Combining them gives the original plaintext BBBBBBABBB.

21. If we look at shifts of 1, 2, and 3 we have 2, 3, and 1 matches. This certainly rules out 3 as the key length, but the key length may be 1 or 2.

In the ciphertext, there are 3 A's, 5 B's, and 2 C's. If the key length were 1, this should approximate the frequencies .7, .2, .1 of the plaintext in some order, which is not the case. So we rule out 1 as the key length.

Let's consider a key length of 2. The first, third, fifth, ... letters are $ACABA$. There are 3 A's, 1 B, and 1C. These frequencies of .6, .2, .2 is a close match to .7, .2, .1 shifted by 0 positions. The first element of the key is probably A. The second, fourth, ... letters of the ciphertext are $BBBBC$. There are 0 A's, 4 B's, and 1 C. These frequencies .0, .8, .2 and match .7, .2, .1 with a shift by 1. Therefore the second key element is probably B.

Since the results for length 2 match the frequencies most closely, we conclude that the key is probably AB.

23. Since the entries of $\mathbb{A}_i$ are the same as those in $\mathbb{A}_0$ (shifted a few places) the two vectors have the same length. Therefore

$$\mathbb{A}_0 \cdot \mathbb{A}_i = |\mathbb{A}_0||\mathbb{A}_i| \cos\theta = |\mathbb{A}_0|^2 \cos\theta.$$

Note that $\cos\theta \leq 1$, and equals 1 exactly when $\theta = 0$. But $\theta = 0$ exactly when the two vectors are equal. So we see that the largest value of the cosine is when $\mathbb{A}_0 = \mathbb{A}_i$. Therefore the largest value of the dot product is when $i = 0$.

25. (a) The ciphertext will be one letter repeated a few hundred times, so the plaintext must also be a repeated letter. But the plaintext could be the shift of any letter, so the key and the plaintext cannot be deduced.

(b) The ciphertext will be one letter repeated a few hundred times, so the plaintext must also be a repeated letter. But the plaintext could be the encryption of any letter, so the key and the plaintext cannot be deduced.
(c) The ciphertext will be a sequence of letters repeated many times. This means that the plaintext consists of a pattern of letters that is repeated (for example, if the key length is 6, then three letters repeated many times would cause the ciphertext to repeat every 6 letters). If the key is a word, then the ciphertext is this word repeated several times. But Eve cannot be sure of this, so the key and the plaintext cannot be deduced with certainty.

27. EK IO IR NO AN HF YG BZ YB LF GM ZN AG ND OD VC MK

29. AAXFFGDGAFAX

31. (a) Since $2^{128} = 2^{64} \cdot 2^{64}$, it will take 2^{64} days, which is approximately 5×10^{16} years.
(b) It will take around 5×10^{14} years (the offset by 10 years is insignificant), which is much less that the time in part (a).