Chapter 3

Spectrum

3-1 Problem Solutions

P-3.1 **DSP First2e**

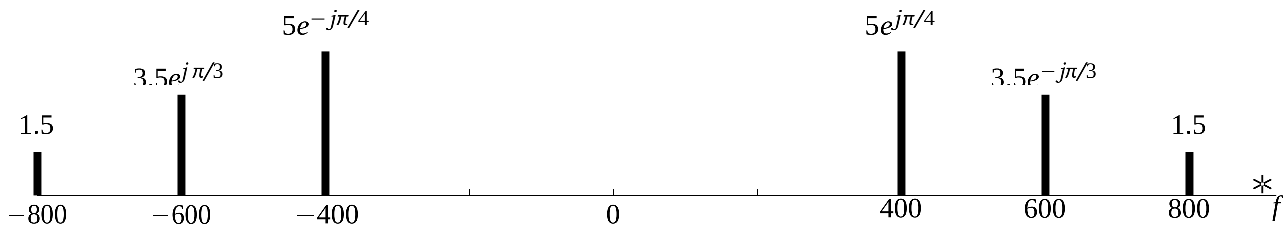
(a) $x(t) = 11 + 14\cos(100\pi t - \pi/3) + 8\cos(350\pi t - \pi/2)$

(b) Since the gcd of 50 and 175 is 25, $x(t)$ is periodic with period $T_0 = 1/25 = 0.04$ s.

(c) Negative frequencies are implicit in the cosine terms. They are needed to give a real signal when combined with their corresponding positive-frequency terms.

P-3.2 **DSP First2e**

(a) A single plot labeled with complex amplitudes is sufficient. The spectrum consists of the lines $\{(400, 5e^{j\pi/4}), (-400, 5e^{-j\pi/4}), (600, 3.5e^{-j\pi/3}), (-600, 3.5e^{j\pi/3}), (800, 1.5), (-800, 1.5)\}$ where the frequencies are in Hz.



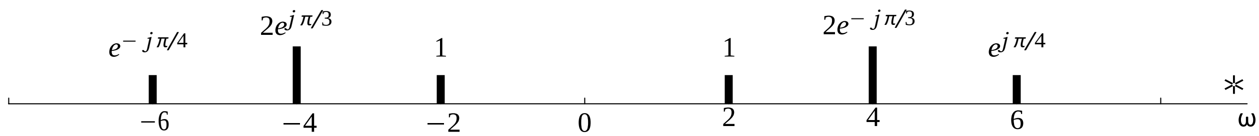
(b) The signal $x(t)$ is periodic with fundamental frequency 200 Hz or period $1/200 = 0.005$ s since the gcd of $\{400, 600, 800\}$ is 200.

(c) The spectrum has the added components $\{(500, 2.5e^{j\pi/2}), (500, 2.5e^{-j\pi/2})\}$. Now we seek the gcd of $\{400, 500, 600, 800\}$ so the fundamental frequency changes to 100 Hz and the period is 0.01 s.

P-3.3 **DSP First2e**

(a) $x(t) = 2\cos(2t) + 4\cos(4t - \pi/3) + 2\cos(6t + \pi/4)$

(b) The spectrum is $\{(2, 1), (-2, 1), (4, 2e^{-j\pi/3}), (-4, 2e^{j\pi/3}), (6, e^{j\pi/4}), (-6, e^{-j\pi/4})\}$
The frequencies are all in rad/s.



P-3.4 **DSP First2e**

(a) Determine a formula for $x(t)$ as the real part of a sum of complex exponentials.

Use Euler's formula for the sine function obtaining

$$\sin^3(27\pi t) = \left(\frac{e^{j27\pi t} - e^{-j27\pi t}}{2j}\right)^3$$

$$= \frac{1}{-8j}\left(e^{j27\pi t3} - 3e^{j27\pi t2}e^{-j27\pi t} + 3e^{j27\pi t}e^{-j27\pi t2} - e^{-j27\pi t3}\right)$$

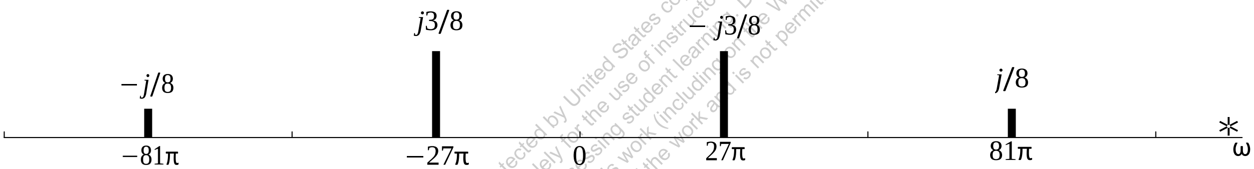
$$= \frac{3}{4}\sin(27\pi t) - \frac{1}{4}\sin(81\pi t)$$

(b) What is the fundamental period for $x(t)$?

The fundamental frequency is 27/2 so the fundamental period is 2/27.

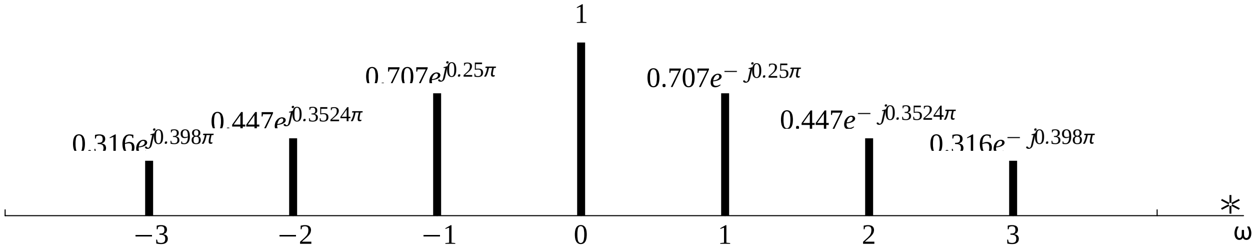
(c) Plot the *spectrum* for $x(t)$.

The spectrum is $\{(27\pi, -j3/8), (-27\pi, j3/8), (81\pi, j/8), (-81\pi, -j/8)\}$, where the frequencies are in rad/s.



P-3.5 **DSP First2e**

There are seven spectral components:
$\{(-3, 1/(1-j3)), (-2, 1/(1-j2)), (-1, 1/(1-j)), (0, 1), (1, 1/(1+j)), (2, 1/(1+j2)), (3, 1/(1+j3))\}$,
where the frequencies are all in rad/s.
Putting all the complex numbers in polar form gives the following plot:



P-3.6 **DSP First2e**

(a) In this case we need to find the gcd of 36 and 84, which is 12. Thus, the fundamental frequency is $\omega_0 = 1.2\pi$ rad/s.

(b) The fundamental period is $T_0 = 2\pi/\omega_0 = 1/0.6 = 5/3\,$s.

(c) The DC value is -7.

(d) The a_k coefficients are nonzero for $k = 0, \pm3, \pm7$.
Here is the list of the nonzero Fourier series coefficients in a table.

k	-7	-3	0	3	7
a_k	$3e^{-j\pi/4}$	$4e^{j\pi/3}$	$7e^{j\pi}$	$4e^{-j\pi/3}$	$3e^{j\pi/4}$

P-3.7 **DSP First2e**

(a) The phasor representation is $z(t) = Ae^{j2\pi(f_c - f_\Delta)t} + Be^{j2\pi(f_c + f_\Delta)t}$

(b)

$$\begin{aligned} z(t) &= e^{j2\pi f_c t}\left(Ae^{-j2\pi f_\Delta t} + Be^{j2\pi f_\Delta t}\right) \\ &= e^{j2\pi f_c t}\left(A\cos(2\pi f_\Delta t) - jA\sin(2\pi f_\Delta t) + B\cos(2\pi f_\Delta t) - jB\sin(2\pi f_\Delta t)\right) \\ &= e^{j2\pi f_c t}\left[(A+B)\cos(2\pi f_\Delta t) - j(A-B)\sin(2\pi f_\Delta t)\right] \end{aligned}$$

Therefore, the real part is

$$x(t) = \Re e\{z(t)\} = (A+B)\cos(2\pi f_\Delta t)\cos(2\pi f_c t) + (A-B)\sin(2\pi f_\Delta t)\sin(2\pi f_c t)$$

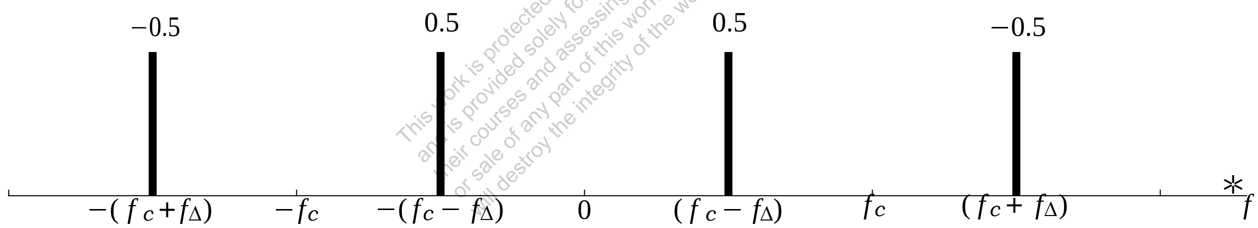
so $C = A + B$ and $D = A - B$. If $A = B = 1$, $C = 2$ and $D = 0$, so using the trigonometric identity $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$, it follows that

$$x(t) = 2\cos(2\pi f_\Delta t)\cos(2\pi f_c t) = 2\left[\tfrac{1}{2}\cos(2\pi(f_c - f_\Delta)t) + \tfrac{1}{2}\cos(2\pi(f_c + f_\Delta)t)\right]$$

(c) The values are $A = 1$ and $B = -1$. In this case,

$$\begin{aligned} x(t) &= 2\left(\frac{e^{j2\pi f_\Delta t} - e^{-j2\pi f_\Delta t}}{2j}\right)\left(\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}\right) \\ &= -\tfrac{1}{2}\left(e^{j2\pi(f_c+f_\Delta)t} - e^{j2\pi(f_c-f_\Delta)t} - e^{-j2\pi(f_c-f_\Delta)t} + e^{-j2\pi(f_c+f_\Delta)t}\right) \end{aligned}$$

The spectrum is $\{(-f_c - f_\Delta, -0.5), (-f_c + f_\Delta, 0.5), (f_c - f_\Delta, 0.5), (f_c + f_\Delta, -0.5)\}$, and the plot is



P-3.8 **DSP First2e**

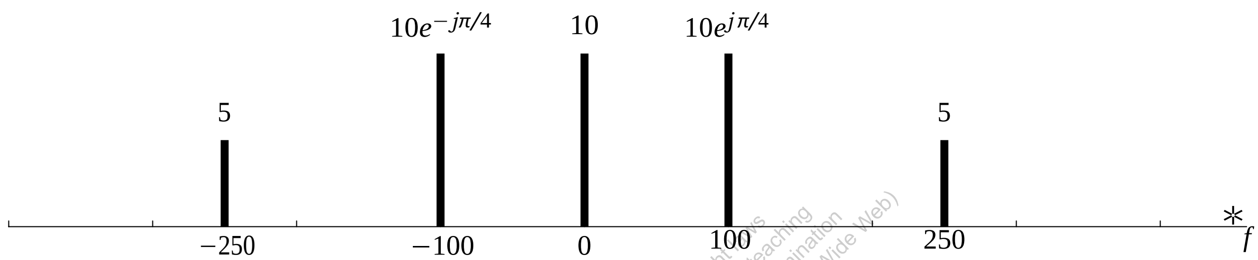
(a) Using Euler's relation we get

$$x(t) = 10 + 10e^{j\pi/4}e^{j2\pi(100)t} + 10e^{-j\pi/4}e^{-j2\pi(100)t} + 5e^{j2\pi(250)t} + 5e^{-j2\pi(250)t}$$

The gcd of 100 and 250 is 50 so $f_0 = 50$ and therefore $N = 5$. The nonzero Fourier coefficients are, therefore, $a_{-5} = 5$, $a_{-2} = 10e^{-j\pi/4}$, $a_0 = 10$, $a_2 = 10e^{j\pi/4}$, and $a_5 = 5$.

(b) The signal is periodic because all the frequencies are multiples of 50 Hz. Therefore, the fundamental period is $T_0 = 1/50 = 0.02$s.

(c) Here is the spectrum plot of this signal versus f in Hz.



P-3.9 **DSP First2e**

(a) Use phasors to show that $x(t)$ can be expressed in the form

$$x(t) = A_1 \cos(\omega_1 t + \varphi_1) + A_2 \cos(\omega_2 t + \varphi_2) + A_3 \cos(\omega_3 t + \varphi_3)$$

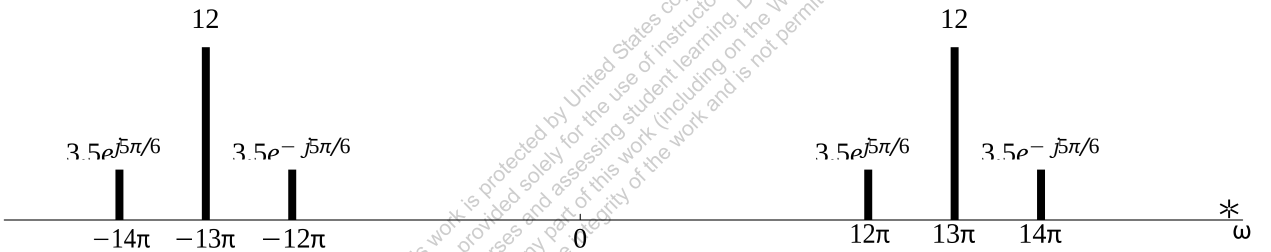
where $\omega_1 < \omega_2 < \omega_3$; i.e., find values of the parameters $A_1, A_2, A_3, \varphi_1, \varphi_2, \varphi_3, \omega_1, \omega_2, \omega_3$.

Using Euler's relation we get

$$x(t) = \left(12 - 3.5 je^{-j\pi/3}e^{j\pi t} + 3.5 je^{j\pi/3}e^{-j\pi t}\right)\left(0.5e^{j13\pi t} + .5e^{-j13\pi t}\right)$$
$$= 12\cos(13\pi t) - 1.75 je^{-j\pi/3}(e^{j14\pi t} + e^{-j12\pi t}) + 1.75 je^{j\pi/3}(e^{j12\pi t} + e^{-j14\pi t})$$
$$= 12\cos(13\pi t) + 1.75e^{-j\pi/2}e^{-j\pi/3}(e^{j14\pi t} + e^{-j12\pi t}) + 1.75e^{j\pi/2}e^{j\pi/3}(e^{j12\pi t} + e^{-j14\pi t})$$
$$= 12\cos(13\pi t) + 1.75e^{-j5\pi/6}e^{j14\pi t} + 1.75e^{-j5\pi/6}e^{-j12\pi t} + 1.75e^{j5\pi/6}e^{j12\pi t} + 1.75e^{j5\pi/6}e^{-j14\pi t}$$
$$= 12\cos(13\pi t) + 3.5\cos(14\pi t - 5\pi/6) + 3.5\cos(12\pi t + 5\pi/6)$$

The requested parameters are easily picked off from this equation.

(b) Sketch the two-sided spectrum of this signal on a frequency axis. Be sure to label important features of the plot. Label your plot in terms of the numerical values of A_i, φ_i, and ω_i.



P-3.10 **DSP First2e**

(a) Assume without limitation that $\omega_2 - \omega_1 > 0$. For periodicity with period T_0 we require that $\omega_0 = 2\pi/T_0$. This means that $k_1\omega_0 = \omega_2 - \omega_1$ and $k_2\omega_0 = \omega_2 + \omega_1$, where k_1 and k_2 are integers and $k_2 > k_1$.

(b) Part (a) gives two equations for ω_1 and ω_2. If we solve them in terms of ω_0 we get $\omega_1 = (k_2 - k_1)\omega_0/2$ and $\omega_2 = (k_2 + k_1)\omega_0/2$, so the main condition is that both ω_1 and ω_2 are integer multiples of $\omega_0/2$. This is the most general condition.

Therefore, the relationship between ω_2 and ω_1 is

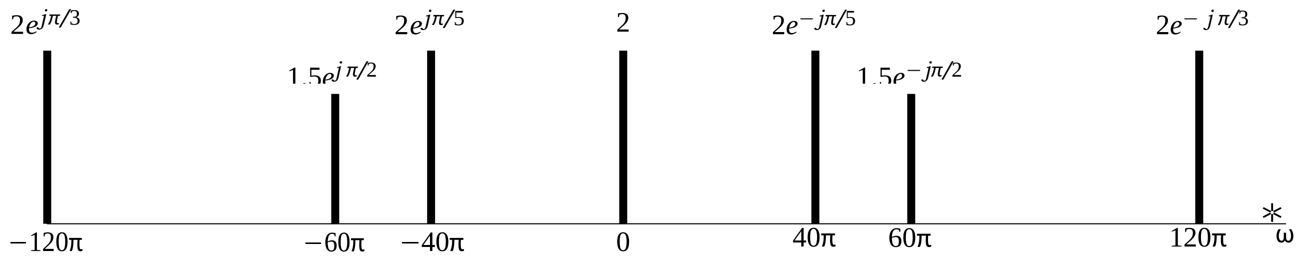
$$\omega_2 = \frac{k_2 + k_1}{k_2 - k_1}\omega_1$$

if $x(t + T_0) = x(t)$. Thus, ω_2 could be an integer multiple of ω_1 if $k_2 - k_1$ divides into $k_2 + k_1$ with no remainder, but that is not necessary for periodicity of $x(t)$.

P-3.11 **DSP First2e**

(a) The gcd of 40, 60, 120 is 20 so $\omega_0 = 20\pi$ and the fundamental period is $T_0 = 2\pi/\omega_0 = 0.1$s. The finite Fourier series has components indexed by 0, ±2, ±3, ±6 so $N = 6$. The coefficients are $a_0 = 2$, $a_{\pm2} = 2e^{\mp j\pi/5}$, $a_{\pm3} = 1.5e^{\mp j\pi/2}$, $a_{\pm6} = 2e^{\mp j\pi/3}$

(b) The spectrum is
$\{(-120\pi, 2e^{j\pi/3}), (-60\pi, 1.5e^{j\pi/2}), (-40\pi, 2e^{j\pi/5}), \ldots$
$(0, 2), (40\pi, 2e^{-j\pi/5}), (60\pi, 1.5e^{-j\pi/2}), (120\pi, 2e^{-j\pi/3})\}$



(c) Now the fundamental frequency is 10π rad/s because the gcd of 20, 40, 50, and 120 is 10. Therefore, the period is $T_0 = 2\pi/10\pi = 1/5 = 0.2$s. The spectrum is the same as in part (b) except there are two additional components at $\pm50\pi$ rad/s: $(-50\pi, 5e^{j\pi/6})$ and $(50\pi, 5e^{-j\pi/6})$.

P-3.12 DSP First2e

(a) Make a table of the frequencies of the tones of the octave beginning with middle C, assuming that the A above middle C is tuned to 440 Hz.

Note name	*C*	$C^{\#}$	*D*	E^{b}	*E*	*F*	$F^{\#}$
Note number	40	41	42	43	44	45	46
Frequency	262	277	294	311	330	349	370
Note name	$F^{\#}$	*G*	$G^{\#}$	*A*	B^{b}	*B*	*C*
Note number	46	47	48	49	50	51	52
Frequency	370	392	415	440	466	494	523

(b) The formula for the frequency f as a function of note number n is

$$f = 440 \cdot 2^{(n-49)/12}$$

(c) The spectrum would have the form:

$$\{(-440, a_3^*), (-370, a_2^*), (-294, a_1^*), (294, a_1), (370, a_2), (440, a_3)\}$$

To sound like a musical chord, the coefficients should have similar magnitudes, but the phases could be arbitrarily chosen. A chord from a real instrument would have overtones (higher harmonics) of each individual note.

P-3.13 **DSP First2e**

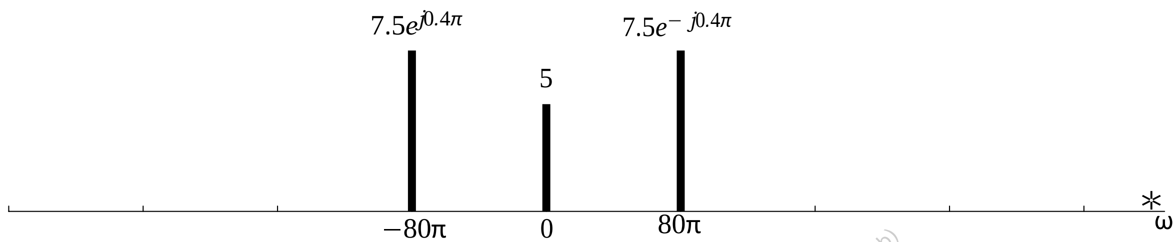
(a) The frequency of the DC component is by definition 0. The waveform is periodic with period 25 ms so the frequency is $1/0.025 = 40$ Hz.

(b) The DC level is $(20 - 10)/2 = 5$, the amplitude of the cosine is $(20 + 10)/2 = 15$, and the cosine is delayed by 0.005 s, so

$$x(t) = 5 + 15\cos(2\pi(40)(t - .005)) = 5 + 15\cos(80\pi t - 0.4\pi)$$

(c) $x(t) = 5 + 7.5e^{j(80\pi t - 0.4\pi)} + 7.5e^{-j(80\pi t - 0.4\pi)} = 7.5e^{j0.4\pi}e^{-j80\pi t} + 5 + 7.5e^{-j0.4\pi}e^{j80\pi t}$

(d) Plot of the two-sided spectrum of the signal $x(t)$.



P-3.14 **DSP First2e**

(a) Using symmetry we obtain

$$X_{-1} = \sqrt{2} - j\sqrt{2} = 2e^{-j\pi/4} \qquad X_2 = 8e^{j\pi/3} \qquad \omega_1 = 70\pi \qquad \omega_2 = -100\pi$$

(b) $x(t) = 20 + 4\cos(70\pi t + \pi/4) + 16\cos(100\pi t + \pi/3)$

(c) The gcd of 70 and 100 is 10, so the fundamental frequency of the signal is $f_0 = 5$ Hz and the fundamental period is $T_0 = 1/5 = 0.2$ s.

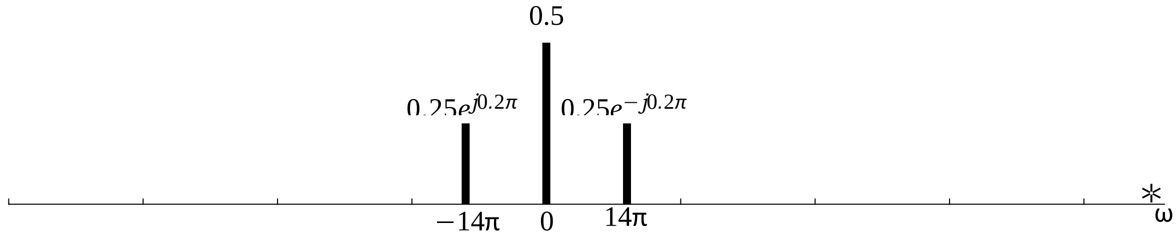
(d) Note that the $-20 \le 4\cos(70\pi t + \pi/4) + 16\cos(100\pi t + \pi/3) \le 20$ since the individual terms satisfy $-4 \le 4\cos(70\pi t + \pi/4) \le 4$ and $-16 \le 16\cos(100\pi t + \pi/3) \le 16$. The value ± 20 would be attained only if the phases of the two cosines are such that $4\cos(70\pi(t - t_0)) + 16\cos(100\pi(t - t_0))$.

P-3.15 **DSP First2e**

(a) $x(t) = \cos^2(7\pi t - 0.1\pi)$

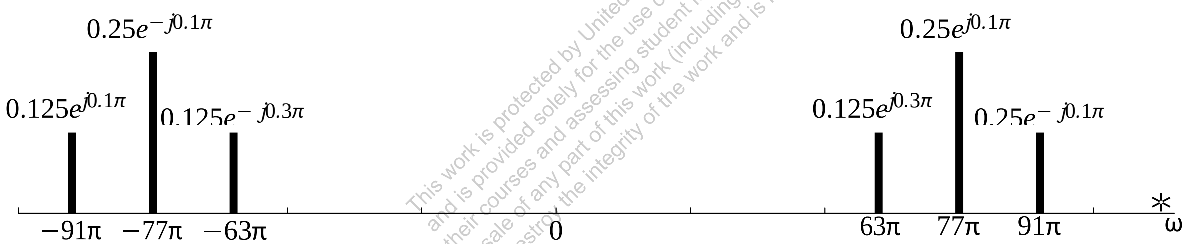
We need to express $x(t)$ in terms of complex exponentials

$$x(t) = \left(0.5e^{j(7\pi t - 0.1\pi)} + 0.5e^{-j(7\pi t - 0.1\pi)}\right)^2 = 0.25e^{j0.2\pi}e^{-j14\pi t} + 0.5 + 0.25e^{-j0.2\pi}e^{j14\pi t}$$



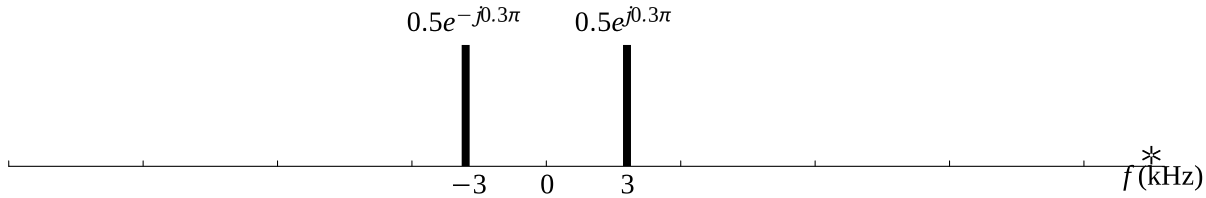
(b) $y(t) = \cos^2(7\pi t - 0.1\pi)\cos(77\pi t + 0.1\pi)$

$$x(t) = \left(0.25e^{j0.2\pi}e^{-j14\pi t} + 0.5 + 0.25e^{-j0.2\pi}e^{j14\pi t}\right)\left(0.5e^{j0.1\pi}e^{j77\pi t} + 0.5e^{-j0.1\pi}e^{-j77\pi t}\right)$$
$$= 0.125e^{j0.1\pi}e^{-j91\pi t} + 0.25e^{-j0.1\pi}e^{-j77\pi t} + 0.125e^{-j0.3\pi}e^{-j63\pi t}$$
$$+ 0.125e^{j0.3\pi}e^{j63\pi t} + 0.25e^{j0.1\pi}e^{j77\pi t} + 0.125e^{-j0.1\pi}e^{j91\pi t}$$

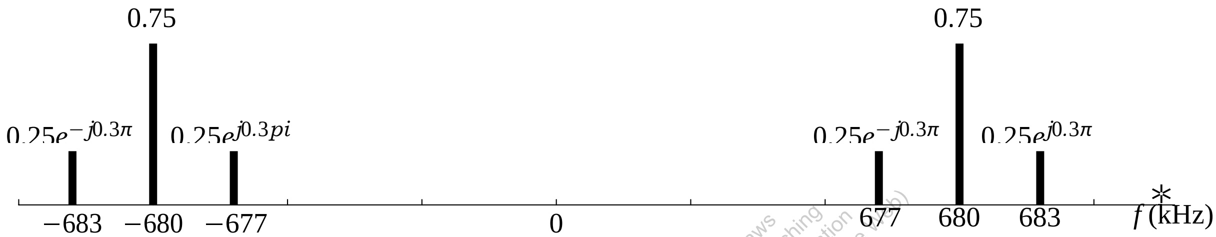


Note: ω axis is not to scale.

(a) Plotting spectrum of $v(t)$ versus f in kHz,



(b) The spectrum for $x(t)$ versus f in kHz.



Note: f axis is not to scale.

P-3.17 **DSP First2e**

(a) The gcd of 105 and 180 is 15, so the given frequencies are the 7th and 12th harmonics of $f_0 = 15$ Hz.

(b) $x(t) = 22\cos(2\pi(105)t - 0.4\pi) + 14\cos(2\pi(180)t - 0.6\pi)$

(c) Simplify the numerical values for the complex amplitudes, i.e., phases should be in $[-\pi, \pi]$.

$$\begin{aligned}
x_2(t) &= 22\cos(2\pi(105)(t-0.05) - 0.4\pi) + 14\cos(2\pi(180)(t-0.05) - 0.6\pi) \\
&= 22\cos(2\pi(105)t - 10.5\pi - 0.4\pi) + 14\cos(2\pi(180)t - 18\pi - 0.6\pi) \\
&= 22\cos(2\pi(105)t - 10\pi - 0.9\pi) + 14\cos(2\pi(180)t - 18\pi - 0.6\pi) \\
&= 22\cos(2\pi(105)t - 0.9\pi) + 14\cos(2\pi(180)t - 0.6\pi)
\end{aligned}$$

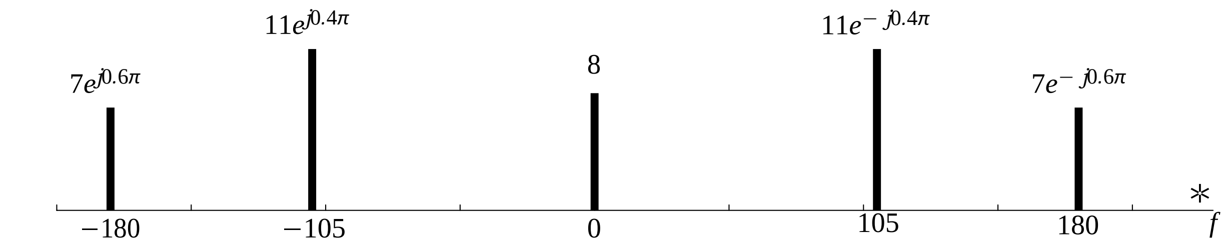
Note that even multiples of 2π rad can be dropped from the equation. Thus, the spectrum is: $\{(-180, 7e^{j0.6\pi}), (-105, 11e^{j0.9\pi}), (105, 11e^{-j0.9\pi}), (180, 7e^{-j0.6\pi})\}$ where the frequencies are in hertz. Therefore, the plot of the spectrum looks just like Fig. P-3.17 except the phase is different at frequencies ± 105 Hz.

(d) The effect of this operation is simply to increase all the frequencies by 105 Hz, or, in other words, to shift the spectrum of $x(t)$ to the right by 105 Hz.

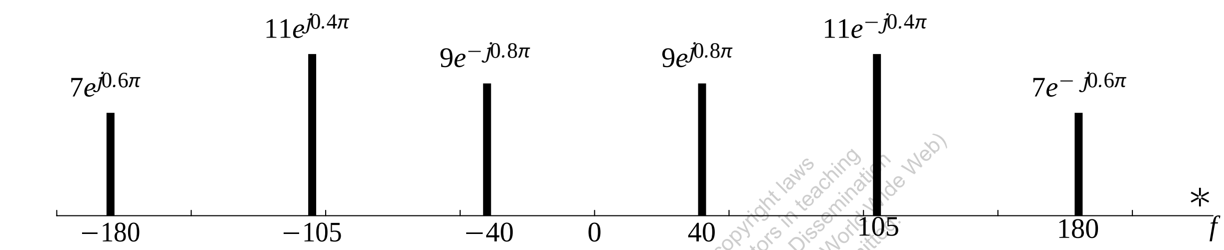
Therefore, the spectrum line at -105 Hz will move to $f = 0$, and the new DC component is equal to the value of the spectrum originally at $f = -105$ Hz, i.e., $11e^{j0.9\pi}$.

P-3.18 **DSP First2e**

(a) The spectrum of $y(t)$ is the spectrum of $x(t)$ with an added DC component of size 8.

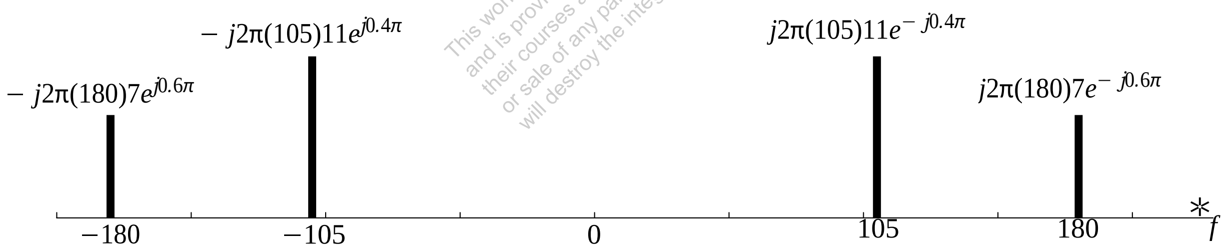


(b) The spectrum of $z(t)$ is the same as that of $x(t)$ with the addition of components of size $9e^{\pm j0.8\pi}$ at frequencies ± 40 Hz.



(c) The fundamental frequency is the gcd of 40, 105, 180, which is 5 Hz.

(d) The derivative operation multiplies each spectrum component by $j2\pi f$, where f is the frequency of the complex exponential component. So we get

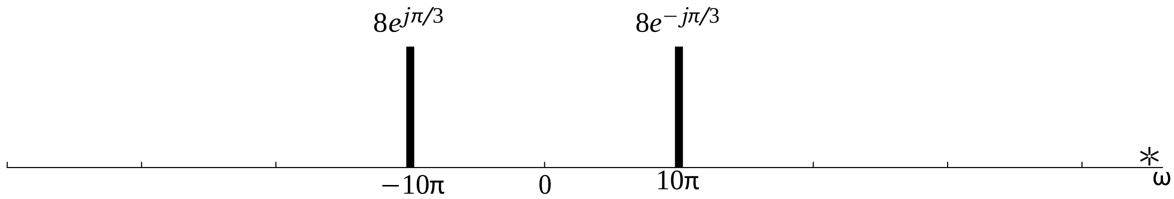


P-3.19 DSP First2e

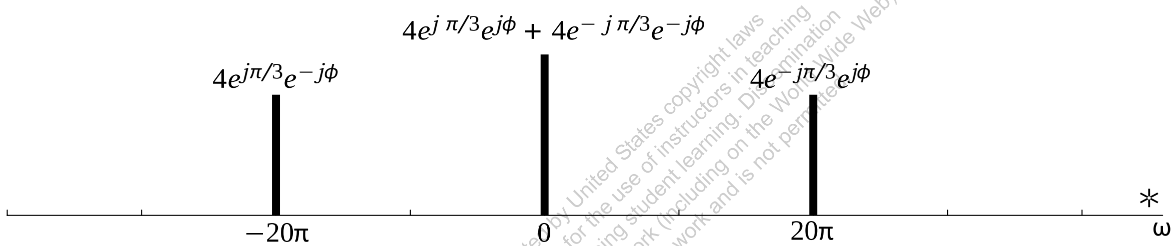
(a)

$$X_1 = 8e^{-j\pi/3} \quad \text{and} \quad \omega_1 = -10\pi$$

(b) Here is the plot of the spectrum of $x(t)$.



(c) The symmetry implies that $\omega_b = 20\pi$ and $B = +4j$. Furthermore, symmetry requires that $\omega_a = 0$. To find A, ω_c, and φ we can write $y(t)$ as $y(t) = 0.5x(t)e^{j\phi}e^{j\omega_c t} + 0.5x(t)e^{-j\phi}e^{-j\omega_c t}$, which shows that the spectrum of $y(t)$ will consist of the sum of scaled copies of the spectrum of $x(t)$ shifted right (up) by ω_c and left (down) by ω_c. In order to have only three components we must choose $\omega_c = 10\pi$ so that two of the shifted spectrum lines over lap at $\omega = 0$.



Now, $4e^{-j\pi/3}e^{j\phi} = 4e^{-j\pi/2}$ so $\varphi = -\pi/6$. Finally, note that the DC value can be written as $A = 8\cos(\pi/3 + \varphi) = 8\cos(\pi/6) = 8\sqrt{3}/2 = 4\sqrt{3}$.

P-3.20 **DSP First2e**

(a) The gcd of 40 and 90 is 10, so $f_0 = 10\,\text{Hz}$.

(b) The fundamental period is $T_0 = 1/f_0 = 1/10 = 0.1\,\text{s}$.

(c) From the plot, the DC value is 0.5.

(d) With $f_0 = 10$, the harmonics are $k = 0, \pm 4, \pm 9$.

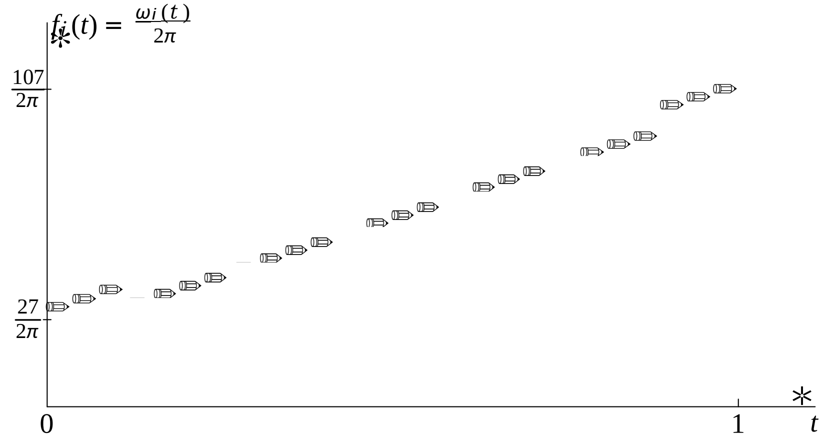
k	-9	-4	0	4	9
a_k	$0.4e^{-j2}$	$0.6e^{j1.4}$	0.5	$0.6e^{-j1.4}$	$0.4e^{j2}$

P-3.21 **DSP First2e**

(a) The instantaneous frequency is $\omega_i(t) = \frac{d\psi}{dt} = 2\alpha t + \beta$, so $\omega_1 = \omega_i(0) = \beta$ and $\omega_2 = \omega_i(T_2) = 2\alpha T_2 + \beta$.

(b) The ***instantaneous*** frequency versus time is $\omega_i(t) = 80t + 27$

(c) Here is the plot of the ***instantaneous*** frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.



P-3.22 **DSP First2e**

(a) The general form for the chirp signal is $x(t) = \cos(\alpha t^2 + \beta t + \varphi)$. The instantaneous frequency of this signal is $\omega_i(t) = 2\alpha t + \beta$. From this we observe that $\omega_1 = 2\pi f_1 = 2\pi(4800) = \omega_i(0) = \beta$. To obtain α, we note that $\omega_2 = 2\pi(800) = \omega_i(2) = 2\alpha(2) + \beta = 4\alpha + 9600\pi$ so $\alpha = -2000\pi$. Therefore, the signal is

$$x(t) = \cos(-2000\pi t^2 + 9600\pi t + \varphi)$$

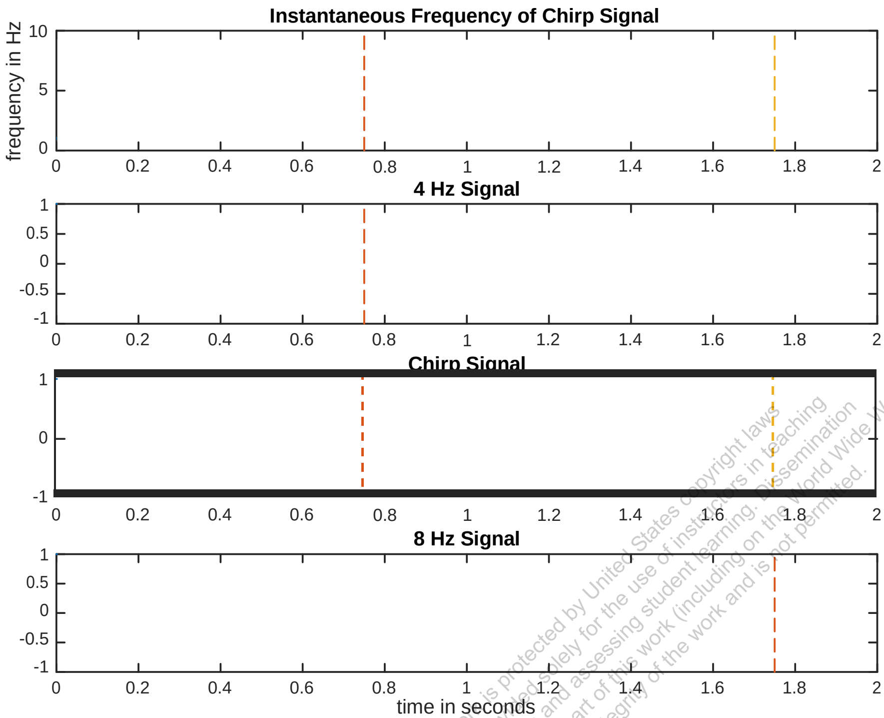
where φ is an arbitrary phase constant.

(b) The instantaneous frequency is $\omega_i = 800\pi t + 500\pi$, so $\omega_1 = \omega_i(0) = 500\pi$ and $\omega_2 = \omega_i(3) = 800\pi(3) + 500\pi = 2900\pi$ rad/s.

P-3.23 **DSP First2e**

(a) The instantaneous frequency is $\omega_i(t) = 2\alpha t + \beta$. Substituting the given parameters gives $\alpha = 4\pi$ and $\beta = 2\pi$, so the signal with the given parameters is $x(t) = \cos(4\pi t^2 + 2\pi t + \varphi)$.

(b–f) The solution to this problem is given in the following figure. Note that the times at which $f_i(t)$ is equal to 4 Hz and 8 Hz are indicated with dashed lines. Careful scrutiny of the plots confirms that the waveform of the chirp signal does match the waveforms of the 4 Hz and 8 Hz constant-frequency sinusoids at the corresponding two times.



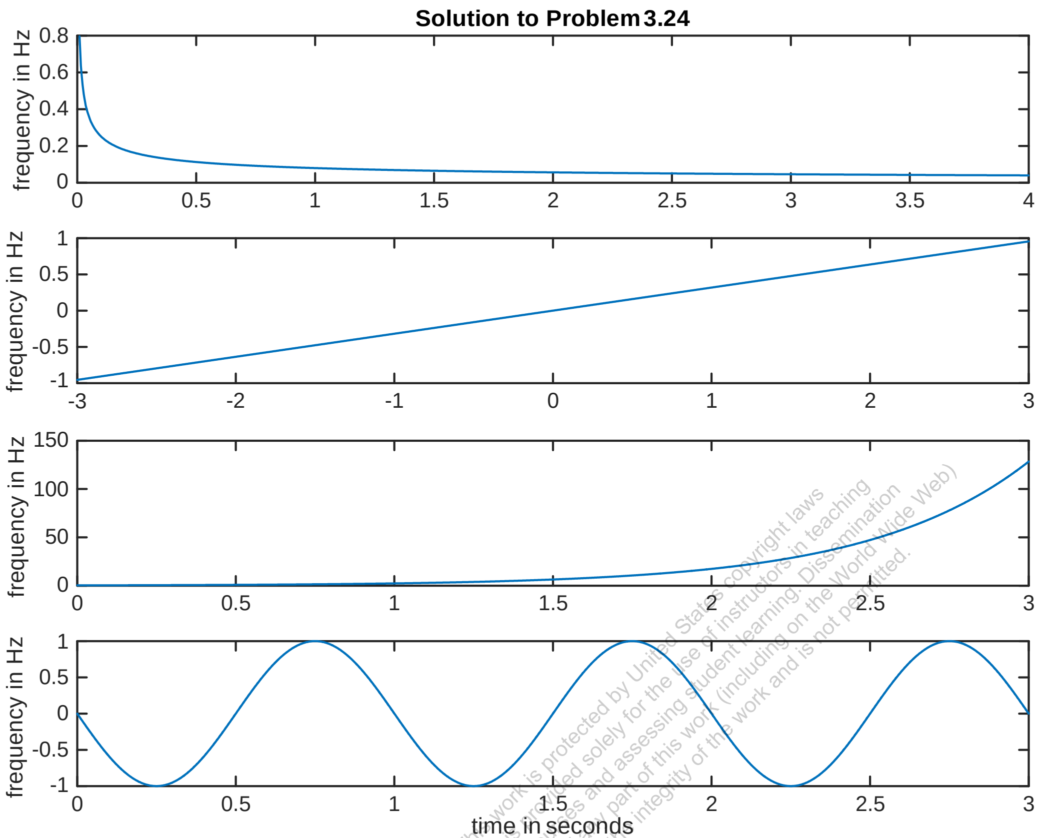
P-3.24 **DSP First2e**

(a) $f_1(t) = \dfrac{1}{2\pi\sqrt{(t)}}$

(b) $f_2(t) = t/\pi$

(c) $f_3(t) = e^{2t}/\pi$

(d) $f_4(t) = -\sin(2\pi t)$



P-3.25 **DSP First2e**

(a) Let $x(t)$ be given by the Fourier series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt}$. Then it follows that

$$x(0) = \sum_{k=-\infty}^{\infty} a_k \underbrace{e^{j(2\pi/T_0)k(0)}}_{1} = \sum_{k=-\infty}^{\infty} a_k .$$

(b) Let $f_3 = -f_2 = f_0$ and $f_4 = -f_1 = 3f_0$ so that from the spectrum we can write

$$x(t) = 12\cos(2\pi f_0 t + \pi/4) + 4\cos(6\pi f_0 t + 3\pi/4)$$

Therefore $x(0) = 12\cos(\pi/4) + 4\cos(3\pi/4) = 6\sqrt{2} - 2\sqrt{2} = 4\sqrt{2}$. Now if we add the coefficients of the Fourier series we get

$$a_1 + a_2 + a_3 + a_4 = 2e^{-j3\pi/4} + 6e^{-j\pi/4} + 6e^{j\pi/4} + 2e^{j3\pi/4} = 12\cos(\pi/4) + 4\cos(3\pi/4) = 4\sqrt{2}$$

P-3.26 **DSP First2e**

The equations corresponding to the spectra are:

The matches are

(a)	(3)
(b)	(1)
(c)	(2)
(d)	(5)
(e)	(4)

$$x_1(t) = 4\cos(4\pi t + \pi) + 4\cos(6\pi t + \pi/2)$$
$$x_2(t) = 2\cos(4\pi t + \pi/4) + 4\cos(6\pi t - 0.333\pi)$$
$$x_3(t) = -3 + 2\cos(4\pi t + \pi/4)$$
$$x_4(t) = -2 + 4\cos(4\pi t + \pi)$$
$$x_5(t) = 4\cos(2\pi t + \pi) + 4\cos(4\pi t + \pi)$$

P-3.27 **DSP First2e**

(a) $x(t) = \cos(-250\pi t^2)$
Spectrogram (2)

(b) $x(t) = \cos(100\pi t - \pi/4) + \cos(400\pi t)$
Spectrogram (5)

(c) $x(t) = \cos(1000\pi t - 250\pi t^2)$
Spectrogram (4)

(d) $x(t) = \cos(100\pi t)\cos(400\pi t)$
Spectrogram (1)

(e) $x(t) = \cos(200\pi t^2)$
Spectrogram (6)

(f) $x(t) = \cos(30e^{2t})$
Spectrogram (3)