Singiresu S. Rao, *Reliability Engineering*, Pearson, Upper Saddle River, NJ, 2015

Chapter 1

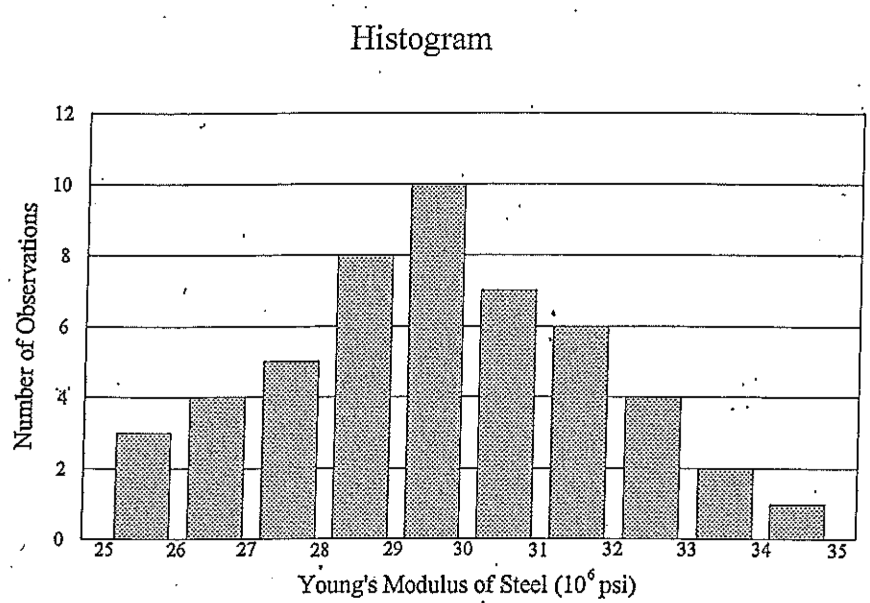
Introduction

1.1 (a)

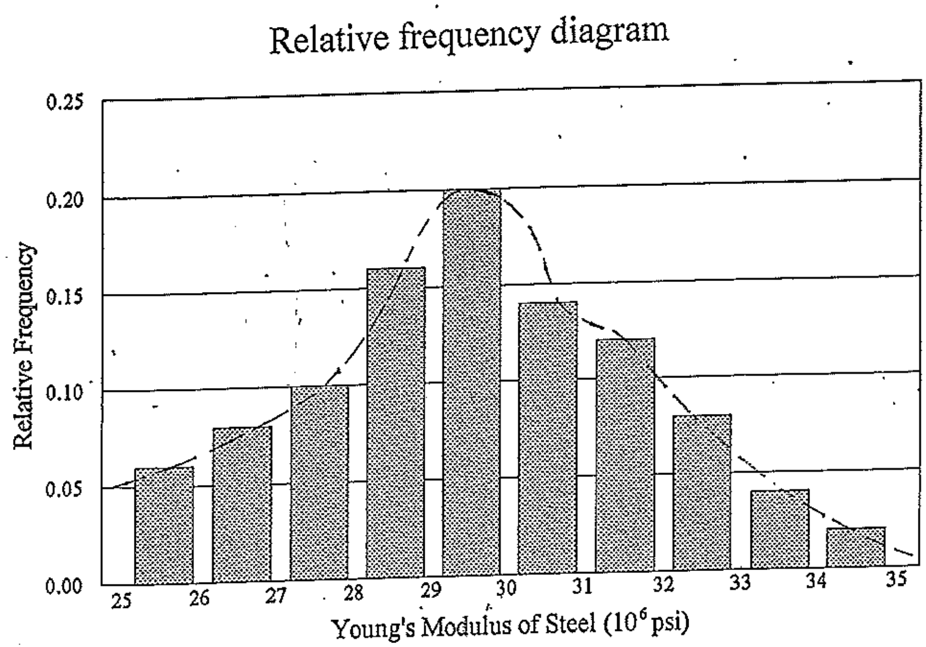
Data

25.1	27.7	29.1	30.1	31.6
25.4	27.8	29.2	30.3	31.8
25.9	28.1	29.3	30.4	31.9
26.5	28.3	29.4	30.5	32.3
26.6	28.3	29.5	30.6	32.5
26.8	28.4	29.6	30.8	32.7
26.9	28.5	29.6	30.9	32.8
27.2	28.6	29.7	31.2	33.4
27.4	28.7	29.8	31.3	33.8
27.6	28.9	29.9	31.4	34.7

Interval	Observations	Freq
25-26	3	0.060
26-27	4	0.080
27-28	5	0.100
28-29	8	0.160
29-30	10	0.200
30-31	7	0.140
31-32	6	0.120
32-33	4	0.080
33-34	2	0.040
34-35	1	0.020
	50	1.000



(b)



(c)

DATA	YOUNG'S MOD	Xi - Xave	(Xi - Xave)^2
25.1	25100000	-4476000	2.0035E+13
29.9	29900000	324000	1.0498E+11
28.1	28100000	-1476000	2.1786E+12
32.5	32500000	2924000	8.5498E+12
28.5	28500000	-1076000	1.1578E+12
29.4	29400000	-176000	3.0976E+10
25.4	25400000	-4176000	1.7439E+13
33.4	33400000	3824000	1.4623E+13
31.9	31900000	2324000	5.401E+12
26.6	26600000	-2976000	8.8566E+12
26.5	26500000	-3076000	9.4618E+12
31.2	31200000	1624000	2.6374E+12
29.2	29200000	-376000	1.4138E+11
26.9	26900000	-2676000	7.161E+12
29.3	29300000	-276000	7.6176E+10
30.5	30500000	924000	8.5378E+11
28.6	28600000	-976000	9.5258E+11
28.3	28300000	-1276000	1.6282E+12
33.8	33800000	4224000	1.7842E+13
26.8	26800000	-2776000	7.7062E+12
27.4	27400000	-2176000	4.735E+12
32.3	32300000	2724000	7.4202E+12
29.8	29800000	224000	5.0176E+10
30.3	30300000	724000	5.2418E+11
30.4	30400000	824000	6.7898E+11
31.6	31600000	2024000	4.0966E+12
29.5	29500000	-76000	5776000000
28.7	28700000	-876000	7.6738E+11
30.9	30900000	1324000	1.753E+12

27.8	27800000	-1776000	3.1542E+12
28.4	28400000	-1176000	1.383E+12
34.7	34700000	5124000	2.6255E+13
30.1	30100000	524000	2.7458E+11
25.9	25900000	-3676000	1.3513E+13
31.4	31400000	1824000	3.327E+12
32.8	32800000	3224000	1.0394E+13
30.6	30600000	1024000	1.0486E+12
29.6	29600000	24000	576000000
29.6	29600000	24000	576000000
28.9	28900000	-676000	4.5698E+11
29.1	29100000	-476000	2.2658E+11
27.2	27200000	-2376000	5.6454E+12
31.3	31300000	1724000	2.9722E+12
27.6	27600000	-1976000	3.9046E+12
32.7	32700000	3124000	9.7594E+12
28.3	28300000	-1276000	1.6282E+12
31.8	31800000	2224000	4.9462E+12
30.8	30800000	1224000	1.4982E+12
27.7	27700000	-1876000	3.5194E+12
29.7	29700000	124000	1.5376E+10
TOTAL SUM	1478800000		2.4079E+14
MEAN VALUE	29576000	Lb/in²	
STD DEV	2194498.58	Lb/in²	

$\therefore \bar{X} = 29.5760$ Mpsi = mean value

$s_X = 2.1945$ Mpsi = standard deviation

1.2

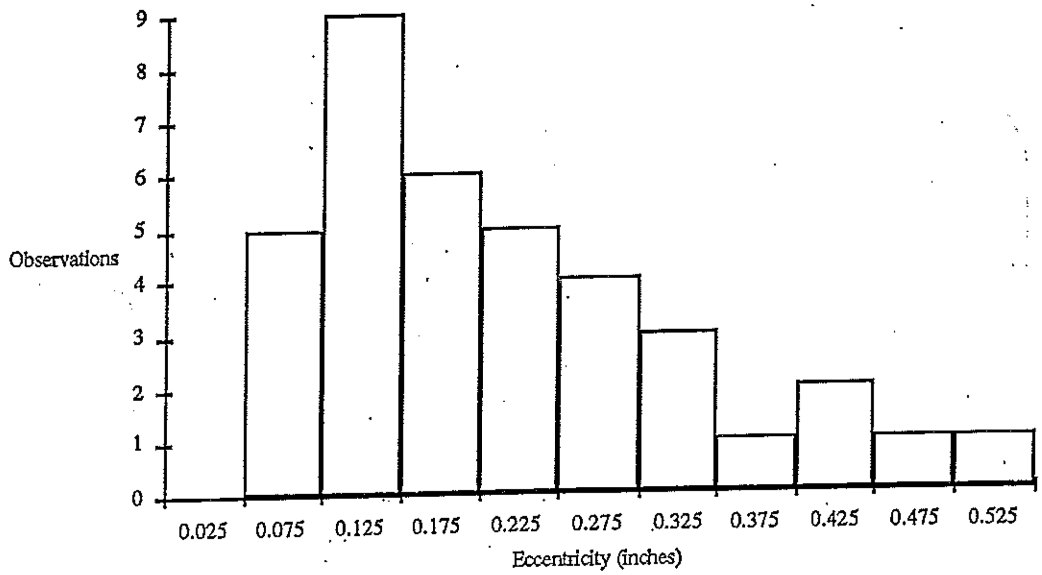
Observations of eccentricity of applied load

n	e inches	Σe inches	Σe^2 inches^2
1	0.410	0.410	0.1681
2	0.050	0.460	0.1706
3	0.090	0.550	0.1787
4	0.195	0.745	0.216725
5	0.345	1.090	0.33575
6	0.155	1.245	0.359775
7	0.320	1.565	0.462175
8	0.120	1.685	0.476575
9	0.290	1.975	0.560675
10	0.065	2.040	0.5649
11	0.275	2.315	0.640525
12	0.230	2.545	0.693425
13	0.140	2.685	0.713025
14	0.265	2.950	0.78325
15	0.215	3.165	0.829475
16	0.070	3.235	0.834375
17	0.115	3.350	0.8476
18	0.305	3.655	0.940625
19	0.435	4.090	1.12985
20	0.130	4.220	1.14675
21	0.535	4.755	1.432975
22	0.110	4.865	1.445075
23	0.205	5.070	1.4871
24	0.085	5.155	1.494325
25	0.135	5.290	1.51255
26	0.125	5.415	1.528175
27	0.185	5.600	1.5624
28	0.480	6.080	1.7928
29	0.175	6.255	1.823425
30	0.145	6.400	1.84445
31	0.380	6.780	1.98885
32	0.165	6.945	2.016075
33	0.255	7.200	2.0811
34	0.180	7.380	2.1135
35	0.240	7.620	2.1711
36	0.220	7.840	2.2195
37	0.105	7.945	2.230525

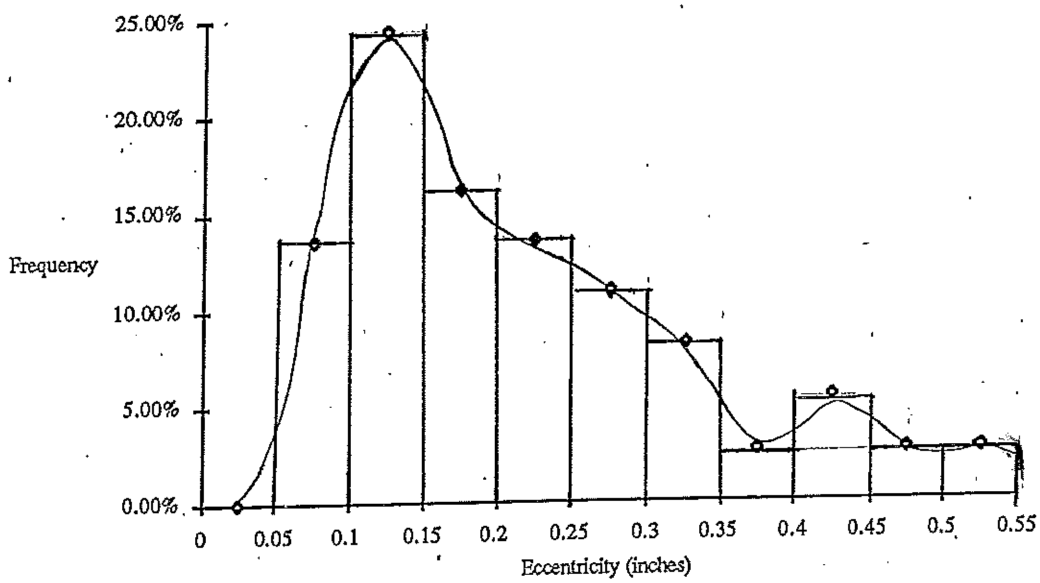
Number of observations in each interval

From:	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
To:	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
Midpoint	0.025	0.075	0.125	0.175	0.225	0.275	0.325	0.375	0.425	0.475	0.525
Total:	0	5	9	6	5	4	3	1	2	1	1
Frequency:	0.00%	13.51%	24.32%	16.22%	13.51%	10.81%	8.11%	2.70%	5.41%	2.70%	2.70%

Column Load Eccentricity Histogram



Column Load Eccentricity Relative Frequency



Mean value of eccentricity $= \bar{E} \simeq \mu_E = \frac{1}{N}\sum_{i=1}^{N} E_i = \frac{7.945}{37}$

$= 0.215$ inch

Standard deviation of eccentricity $= \left\{\frac{1}{N}\sum_{i=1}^{N}(E_i - \bar{E})^2\right\}^{\frac{1}{2}}$

$= \left\{\frac{1}{N}\sum_{i=1}^{N} E_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} E_i\right)^2\right\}^{\frac{1}{2}} = 0.119$ inch

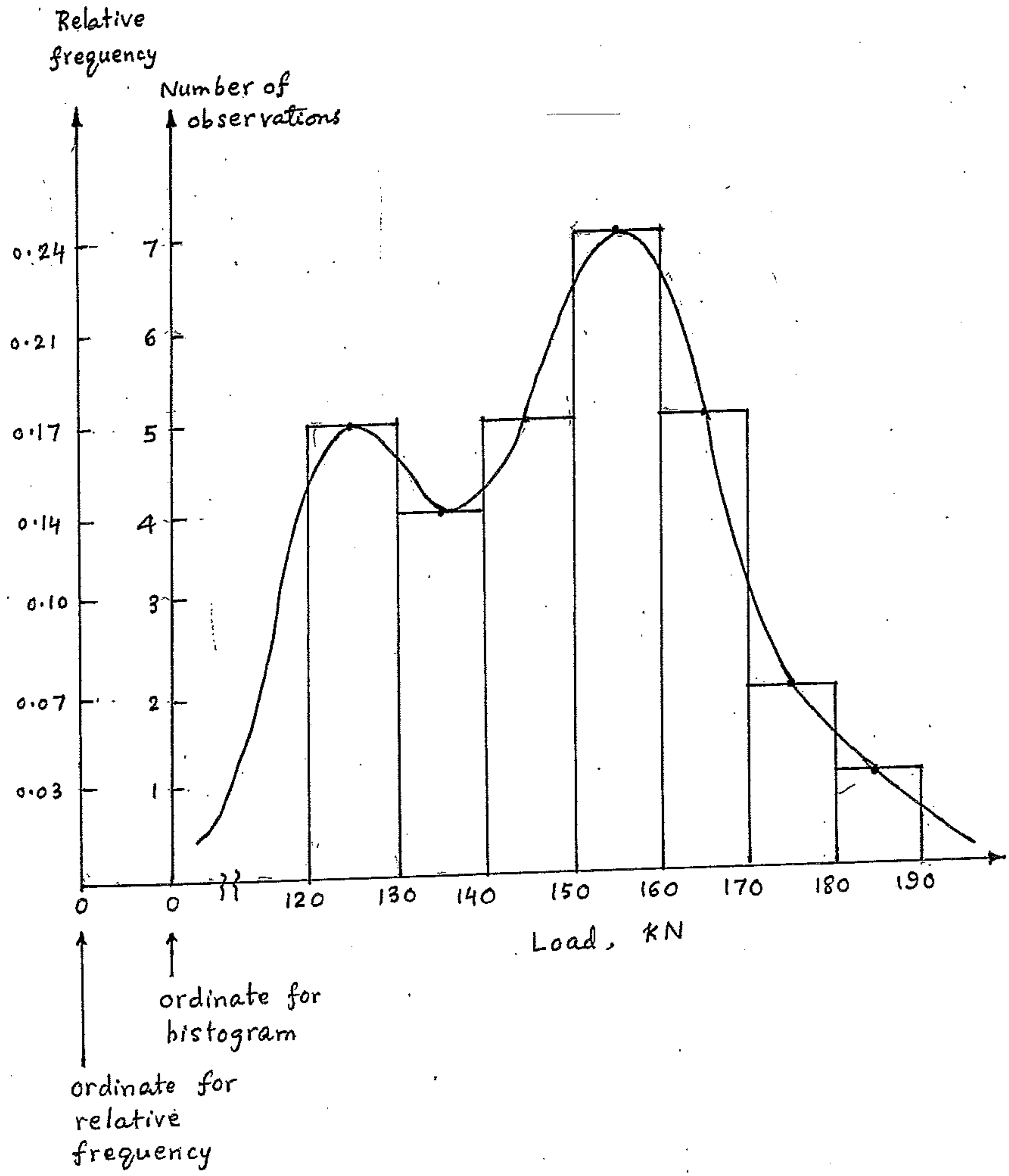
1.3 29 values of maximum load carried by welded beams given.

Smallest value: 123.1 kN, Largest value: 186.9 kN

Range chosen: 120 kN – 190 kN

All data points are grouped into 7 intervals of 10 kN each.

Range of load:	Frequency of load values falling in the range
$\geq 120 < 130$ kN	5
$\geq 130 < 140$ kN	4
$\geq 140 < 150$ kN	5
$\geq 150 < 160$ kN	7
$\geq 160 < 170$ kN	5
$\geq 170 < 180$ kN	2
$\geq 180 < 190$ kN	1


Relative frequency
Number of observations
0.24
0.21
0.17
0.14
0.10
0.07
0.03
7
6
5
4
3
2
1
0
0
120
130
140
150
160
170
180
190
Load, KN
ordinate for histogram
ordinate for relative frequency

1.4 Compressive strength of concrete cylinders (in kpsi):

Data:

5.9	6.2	5.8	7.8	6.5	→ Σ = 32.2
6.3	8.9	5.3	3.7	1.4	→ Σ = 25.6
2.1	6.8	9.1	4.3	3.2	→ Σ = 25.5
7.2	6.1	5.7	4.9	2.6	→ Σ = 26.5
3.4	6.8	8.3	5.1	7.3	→ Σ = 30.9
8.2	7.7	5.4	3.7	4.5	→ Σ = 29.5
4.1	5.6	6.4	6.7	7.9	→ Σ = 30.7
6.9	7.5	5.2	4.3	6.6	→ Σ = 30.5
5.4	6.4				→ Σ = 11.8

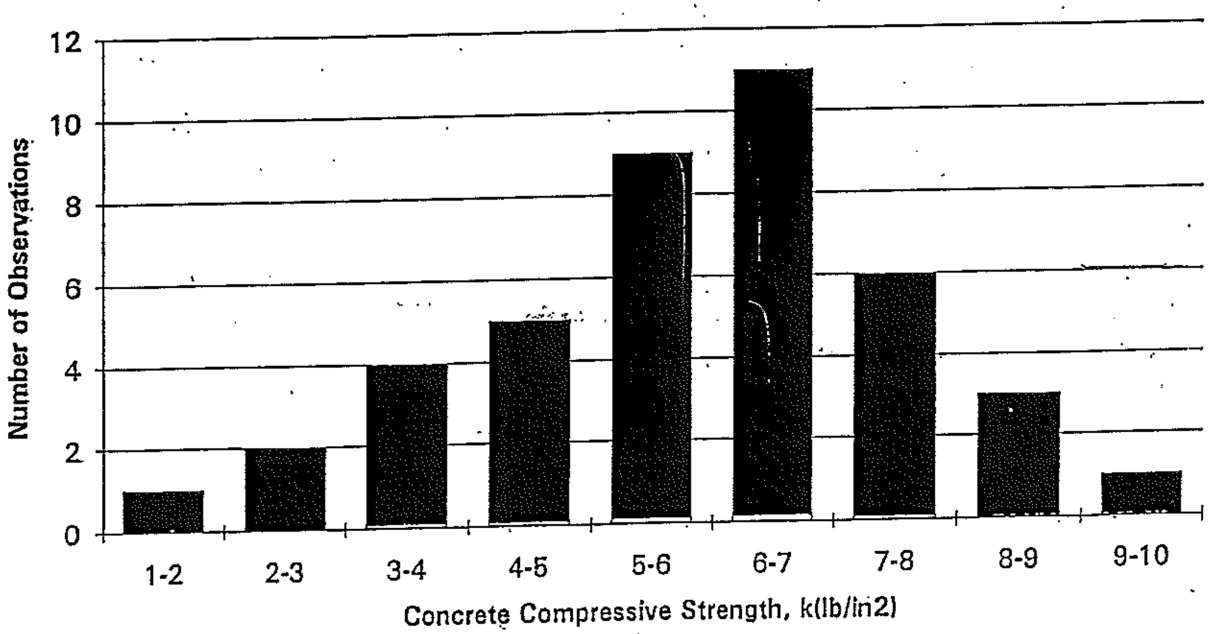
Total sum = 243.2

$$\text{mean value} = \bar{x} = \frac{243.2}{42} = 5.79 \text{ kpsi}$$

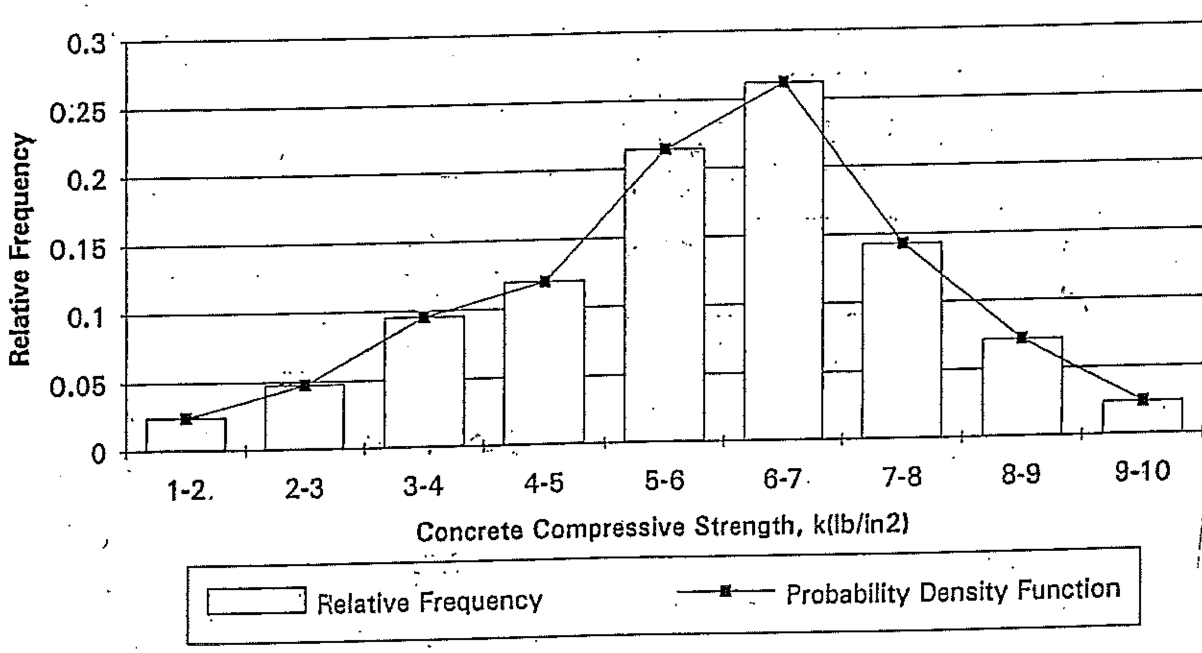
$$\text{Standard deviation} = s_x = \left\{ \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \right\}^{\frac{1}{2}} = 1.81 \text{ kpsi}$$

Range of compressive strength (kpsi)	Number of occurrances	Relative frequency
1 – 2	1	1/42 = 0.024
2 – 3	2	2/42 = 0.048
3 – 4	4	4/42 = 0.095
4 – 5	5	5/42 = 0.119
5 – 6	9	9/42 = 0.214
6 – 7	11	11/42 = 0.262
7 – 8	6	6/42 = 0.143
8 – 9	3	3/42 = 0.071
9 – 10	1	1/42 = 0.024

Histogram



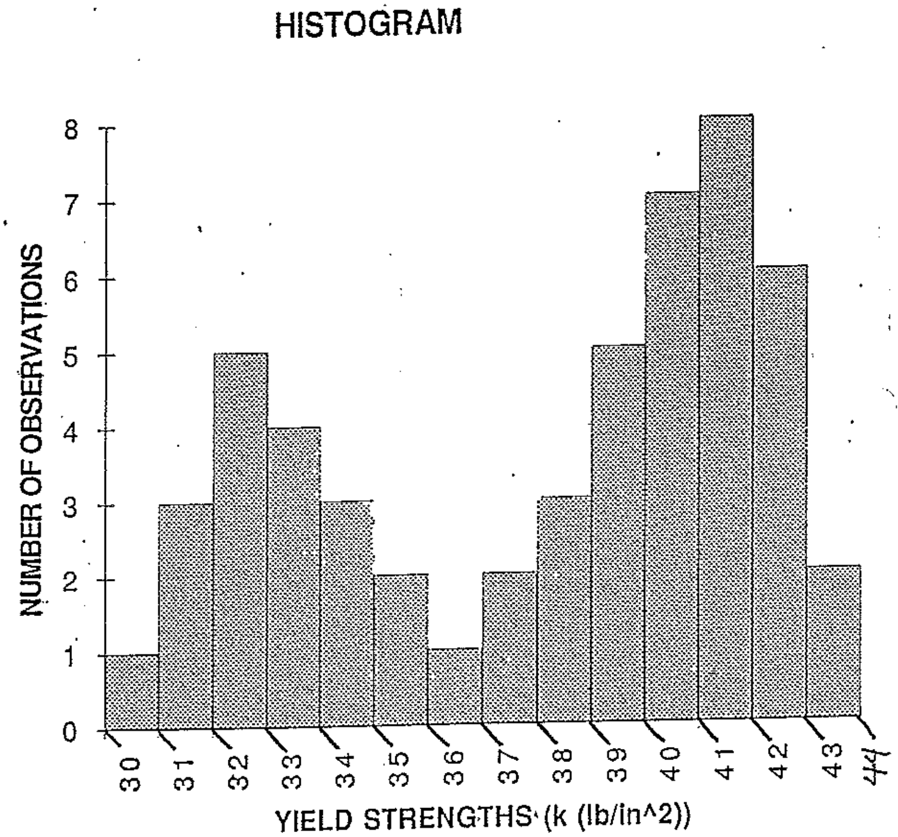
Relative frequency diagram



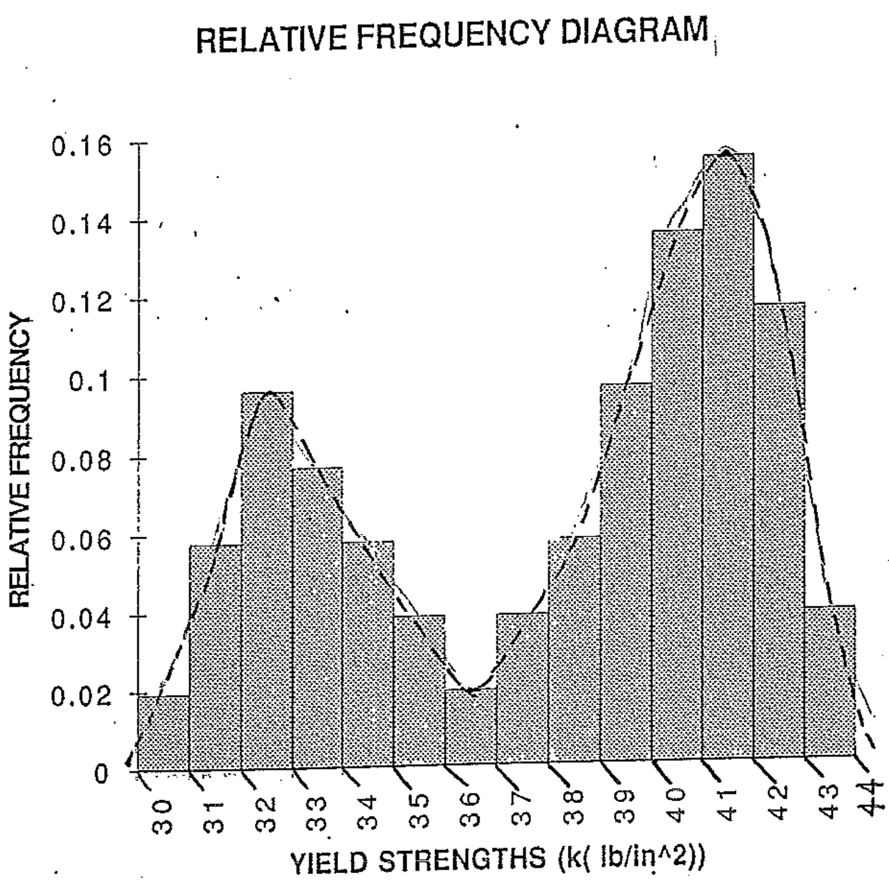
1.5 yield strength data of reinforcing bars (in kpsi) made of two different grades of steel:

Range	# of Occourances	Relative Frequency
30.0 - 30.9	1	0.019
31.0 - 31.9	3	0.058
32.0 - 32.9	5	0.096
33.0 - 33.9	4	0.077
34.0 - 34.9	3	0.058
35.0 - 35.9	2	0.038
36.0 - 36.9	1	0.019
37.0 - 37.9	2	0.038
38.0 - 38.9	3	0.058
39.0 - 39.9	5	0.096
40.0 - 40.9	7	0.135
41.0 - 41.9	8	0.154
42.0 - 42.9	6	0.115
43.0 - 43.9	2	0.038

(a)


HISTOGRAM
NUMBER OF OBSERVATIONS
0
1
2
3
4
5
6
7
8
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
YIELD STRENGTHS (k (lb/in^2))

(b)


RELATIVE FREQUENCY DIAGRAM
RELATIVE FREQUENCY
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
YIELD STRENGTHS (k(lb/in^2))

(c) The relative frequency diagram has two distinct peaks which shows that the two grades of steel have two different average yield strengths which are approximately 32.5 and 41.5 k(lb/in^2).

(d)

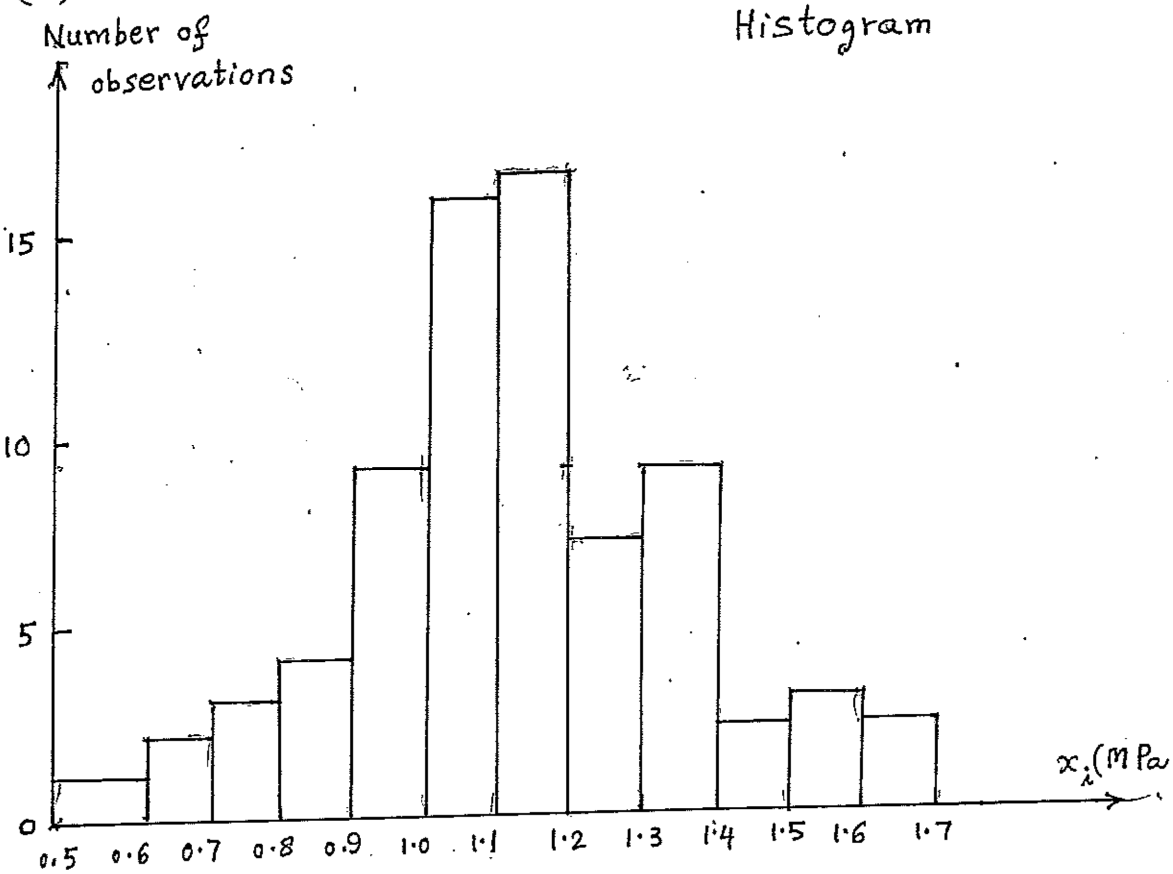
DATA	Y.STRENGTHS	Xi - Xave	(Xi - Xave)^2
35.7	35700	-2315.38462	5361005.92
31.1	31100	-6915.38462	47822544.4
33.2	33200	-4815.38462	23187929
42.5	42500	4484.61538	20111775.1
41.2	41200	3184.61538	10141775.1
42.8	42800	4784.61538	22892544.4
37.5	37500	-515.384615	265621.302
40.7	40700	2684.61538	7207159.76
42.3	42300	4284.61538	18357929
42.2	42200	4184.61538	17511005.9
34.1	34100	-3915.38462	15330236.7
40.9	40900	2884.61538	8321005.92
43.3	43300	5284.61538	27927159.8
38.8	38800	784.615385	615621.302
40.4	40400	2384.61538	5686390.53
42.9	42900	4884.61538	23859467.5
38.4	38400	384.615385	147928.994
41.7	41700	3684.61538	13576390.5
42.7	42700	4684.61538	21945621.3
40.1	40100	2084.61538	4345621.3
41.4	41400	3384.61538	11455621.3
39.2	39200	1184.61538	1403313.61
43.4	43400	5384.61538	28994082.8
40.8	40800	2784.61538	7754082.84
39.6	39600	1584.61538	2511005.92
33.8	33800	-4215.38462	17769467.5
36.6	36600	-1415.38462	2003313.61
39.9	39900	1884.61538	3551775.15
32.3	32300	-5715.38462	32665621.3
32.6	32600	-5415.38462	29326390.5
32.9	32900	-5115.38462	26167159.8
34.5	34500	-3515.38462	12357929
30.2	30200	-7815.38462	61080236.7
38.1	38100	84.6153846	7159.76331
41.5	41500	3484.61538	12142544.4
31.2	31200	-6815.38462	46449467.5
31.7	31700	-6315.38462	39884082.8
34.6	34600	-3415.38462	11664852.1
41.1	41100	3084.61538	9514852.07

37.2	37200	-815.384615	664852.071
39.5	39500	1484.61538	2204082.84
39.3	39300	1284.61538	1650236.69
35.5	35500	-2515.38462	6327159.76
33.7	33700	-4315.38462	18622544.4
32.5	32500	-5515.38462	30419467.5
40.3	40300	2284.61538	5219467.46
41.8	41800	3784.61538	14323313.6
32.2	32200	-5815.38462	33818698.2
40.6	40600	2584.61538	6680236.69
33.4	33400	-4615.38462	21301775.1
41.6	41600	3584.61538	12849467.5
41.3	41300	3284.61538	10788698.2
TOTAL SUM	1976800	LB/IN^2	816187692
MEAN VALUE	38015.3846	LB/IN^2	
STD. DEV.	3961.80731	LB/IN^2	

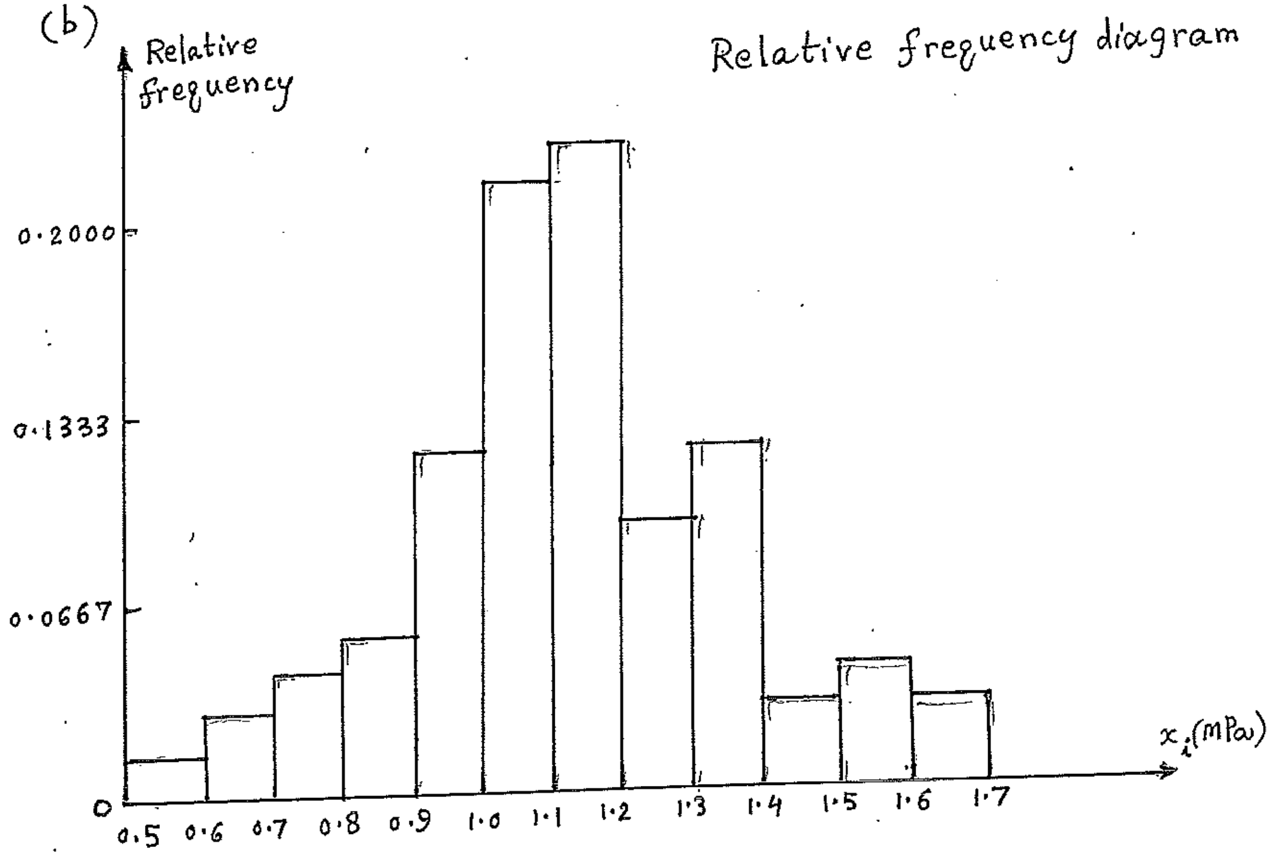
1.6 Data on compressive strength of aluminum-lithium specimens (in MPa):

Interval of compressive strength (MPa)	Number of observed values	Relative frequency Value
0.5001 – 0.6000	1	0.0133
0.6001 – 0.7000	2	0.0267
0.7001 – 0.8000	3	0.0400
0.8001 – 0.9000	4	0.0533
0.9001 – 1.0000	9	0.1200
1.0001 – 1.1000	16	0.2133
1.1001 – 1.2000	17	0.2267
1.2001 – 1.3000	7	0.0933
1.3001 – 1.4000	9	0.1200
1.4001 – 1.5000	2	0.0267
1.5001 – 1.6000	3	0.0400
1.6001 – 1.7000	2	0.0267
Total	75	1.0000

(a)


Number of observations
Histogram
15
10
5
0
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
x_i (MPa

(b)


Relative frequency
Relative frequency diagram
0.2000
0.1333
0.0667
0
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
x_i (MPa)

(c) Mean value $= \bar{x} = \frac{1}{75} \sum_{i=1}^{75} x_i = \frac{1}{75}(1.0335 + 0.9302 + \cdots + 1.3091)$

$= 1.1227$ MPa

Standard deviation $= s_X = \left\{ \frac{1}{75} \sum_{i=1}^{75} (x_i - \bar{x})^2 \right\}^{\frac{1}{2}}$

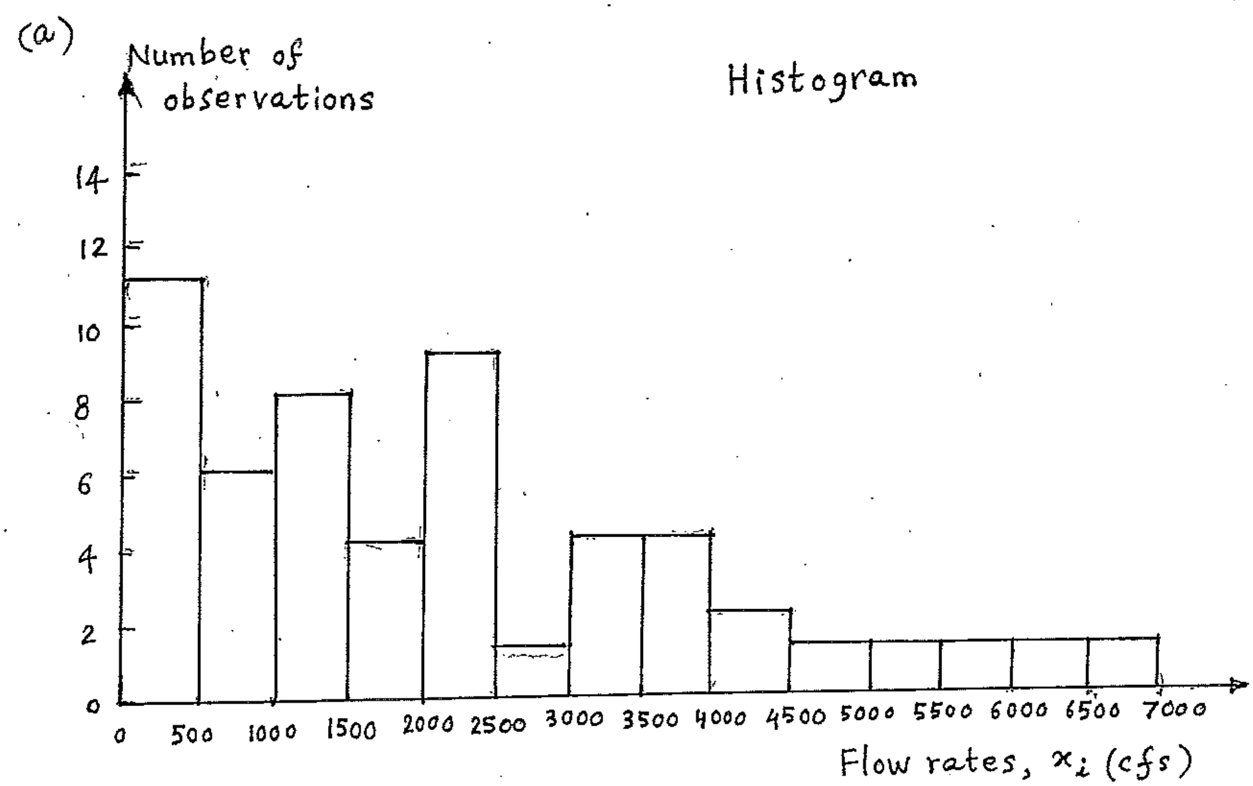
$= \left\{ \frac{1}{75} \left[(1.0335 - 1.1227)^2 + \cdots + (1.3091 - 1.1227)^2 \right] \right\}^{\frac{1}{2}}$

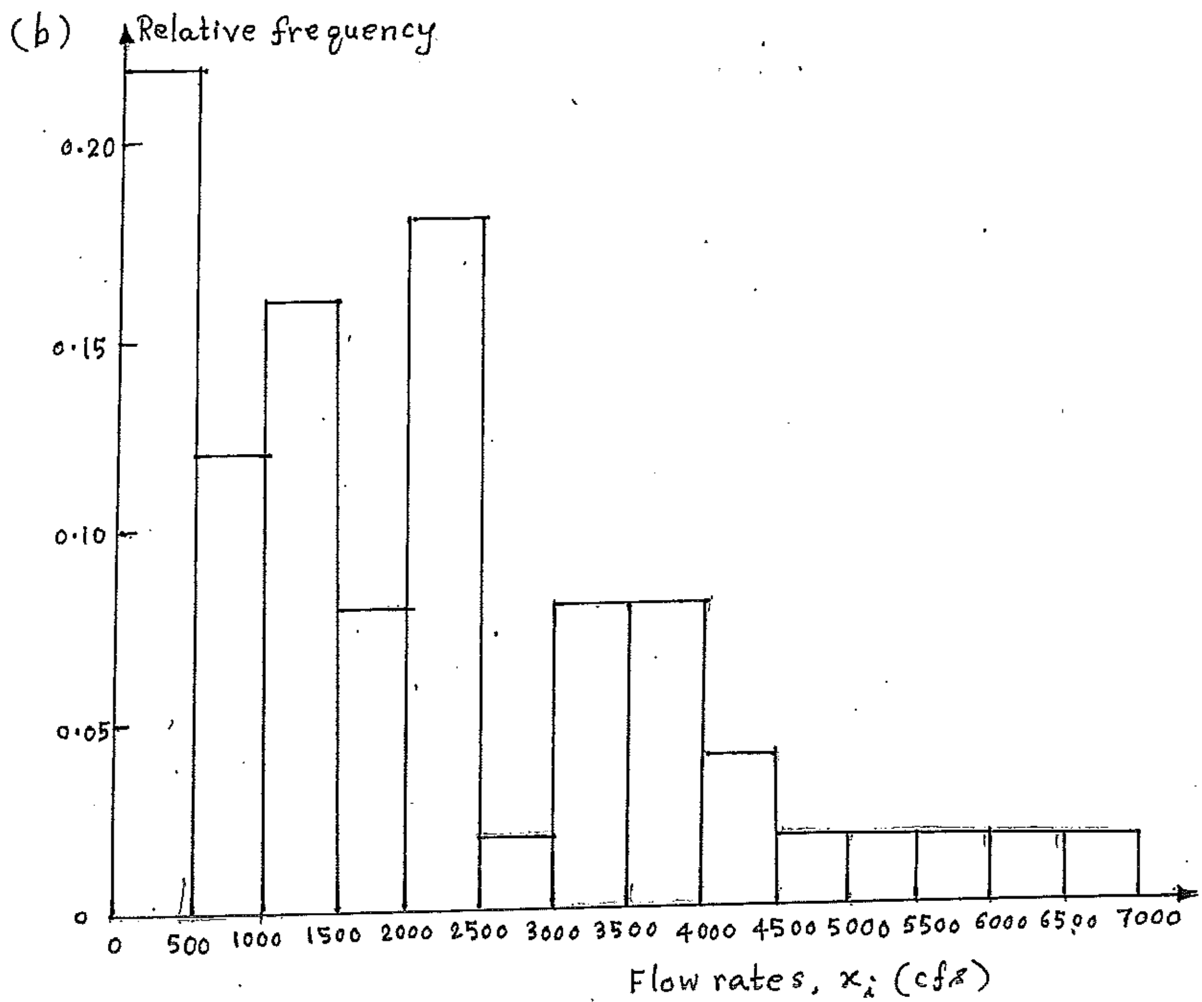
$= 0.0227$ MPa

(d) From the given data, number of specimens that gave a value of x_i below 1 MPa $=$ 19 out of 75 $= \frac{19}{75} = 0.2533$ or 25.33 %.

1.7 Data on annual flow rates in a river (in cfs):
50 data points:

Interval of flow rate (cfs)	Number of observed values	Relative frequency
0 - 500	11	0.22
501 – 1000	6	0.12
1001 – 1500	8	0.16
1501 – 2000	4	0.08
2001 – 2500	9	0.18
2501 – 3000	1	0.02
3001 – 3500	4	0.08
3501 – 4000	4	0.08
4001 – 4500	2	0.04
4501 – 5000	0	0
5001 – 5500	0	0
5501 – 6000	0	0
6001 – 6500	0	0
6501 – 7000	1	0.02
Total	50	1.00


(a)
Number of observations
Histogram
14
12
10
8
6
4
2
0
0 500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000 6500 7000
Flow rates, x_i (cfs)


(b)
Relative frequency
0.20
0.15
0.10
0.05
0
0 500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000 6500 7000
Flow rates, x_i (cfs)

(c) Mean value = $\bar{X} = \frac{1}{50} \sum_{i=1}^{50} x_i = \frac{88579}{50}$

$= 1,771.58$ cfs

Standard deviation = $s_X = \left\{ \frac{1}{50} \sum_{i=1}^{50} (x_i - 1771.58)^2 \right\}^{\frac{1}{2}}$

$= 1,382.9$ cfs

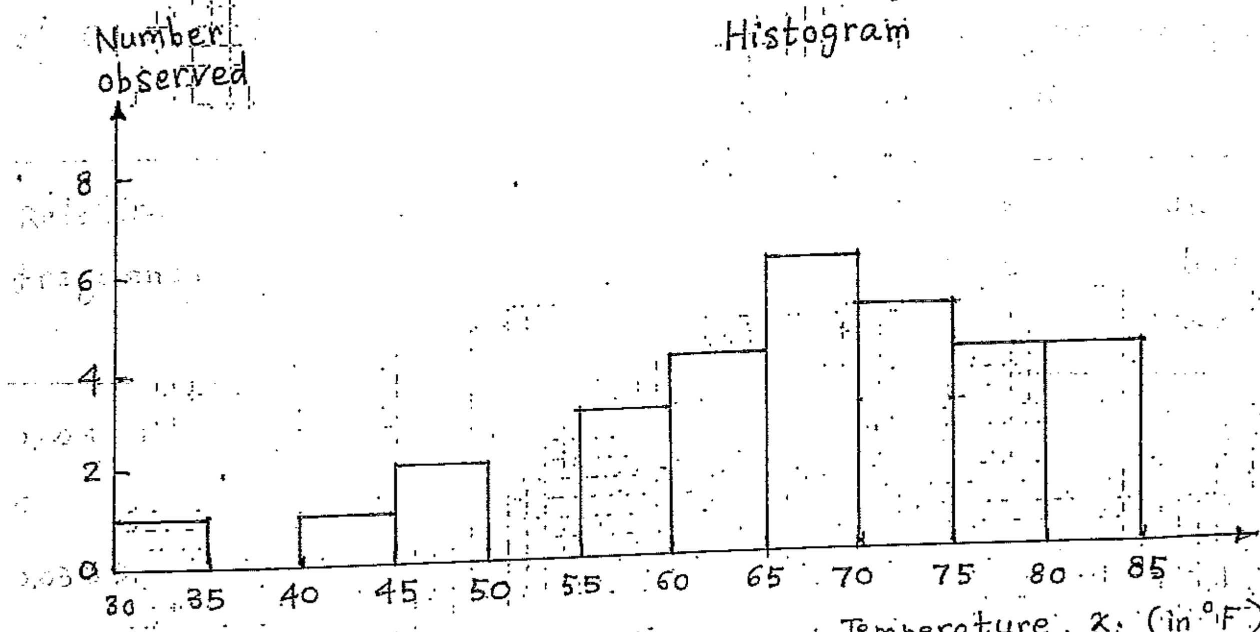
(d) Percentage of flow rates exceeding a value of 4000 cfs = 3 out of 50 = 0.06

or 6%

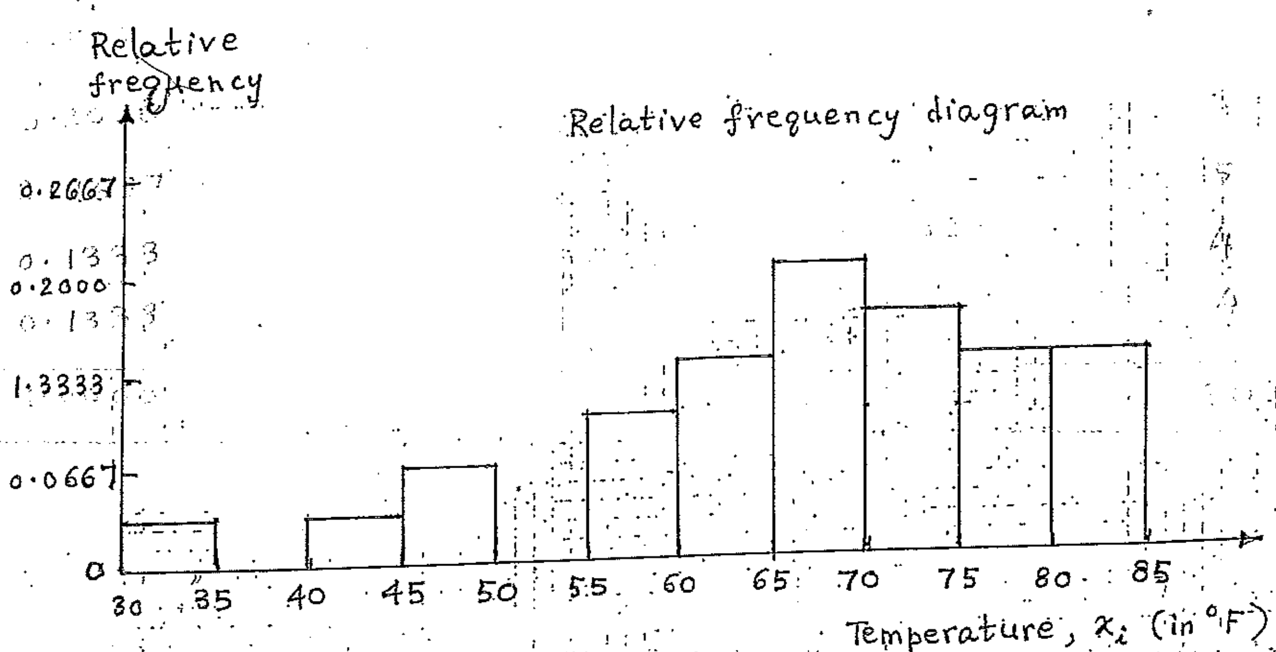
1.8 Data on joint temperatures of O-rings (in °F); 30 data points.

Interval of temperature (°F)	Number of observed values	Relative frequency
30 – 35	1	0.0333
35 – 40	0	0
40 – 45	1	0.0333
45 – 50	2	0.0667
50 – 55	0	0
55 – 60	3	0.1000
60 – 65	4	0.1333
65 – 70	6	0.2000
70 – 75	5	0.1667
75 – 80	4	0.1333
80 – 85	4	0.1333
Total	30	1.0000

(a)


Number observed
Histogram
8
6
4
2
0
30
35
40
45
50
55
60
65
70
75
80
85
Temperature, x_i (in °F)

(b)


Relative frequency
Relative frequency diagram
0.2667
0.2000
1.3333
0.0667
0
30
35
40
45
50
55
60
65
70
75
80
85
Temperature, x_i (in °F)

(c) Mean value $= \bar{X} = \frac{1}{30} \sum_{i=1}^{30} x_i = \frac{1976}{30} = 65.8667\,^\circ F$

Standard deviation $= s_X = \left\{ \frac{1}{30} \sum_{i=1}^{30} (x_i - 65.8667)^2 \right\}^{\frac{1}{2}}$

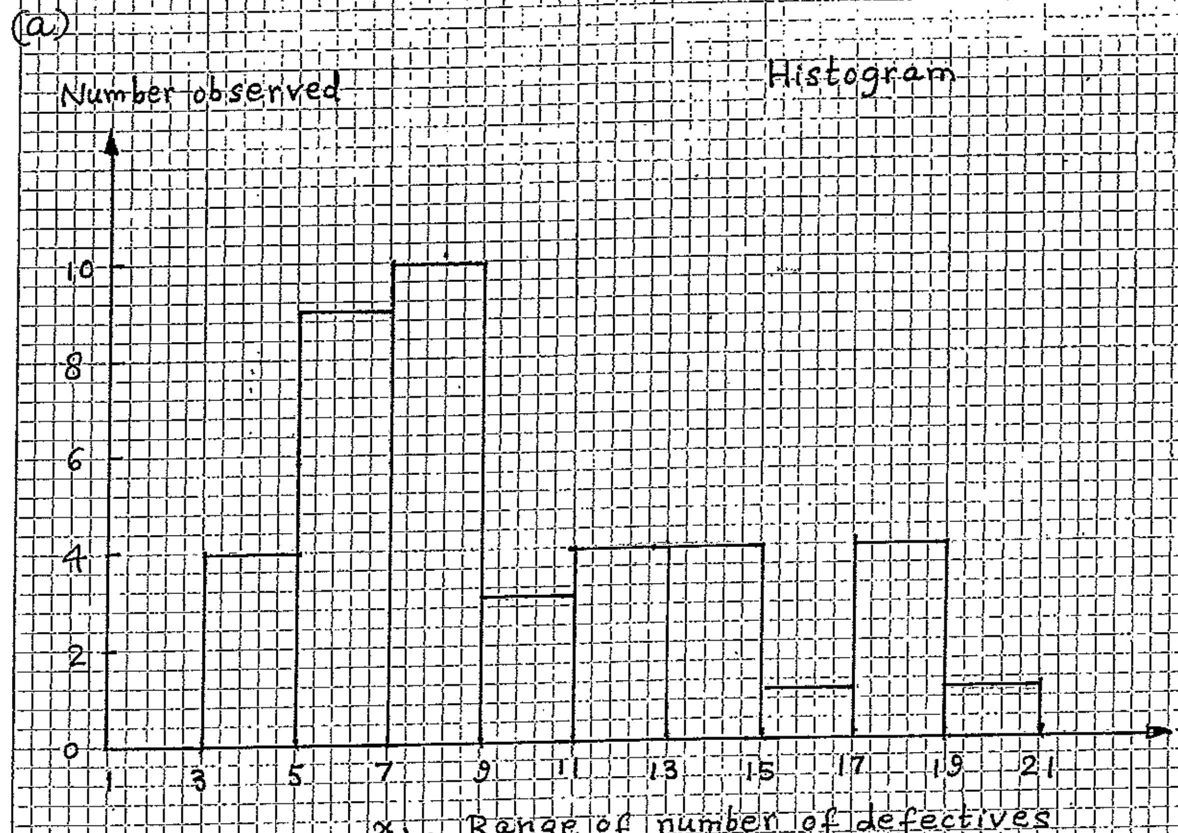
$= 2.5498\,^\circ F$

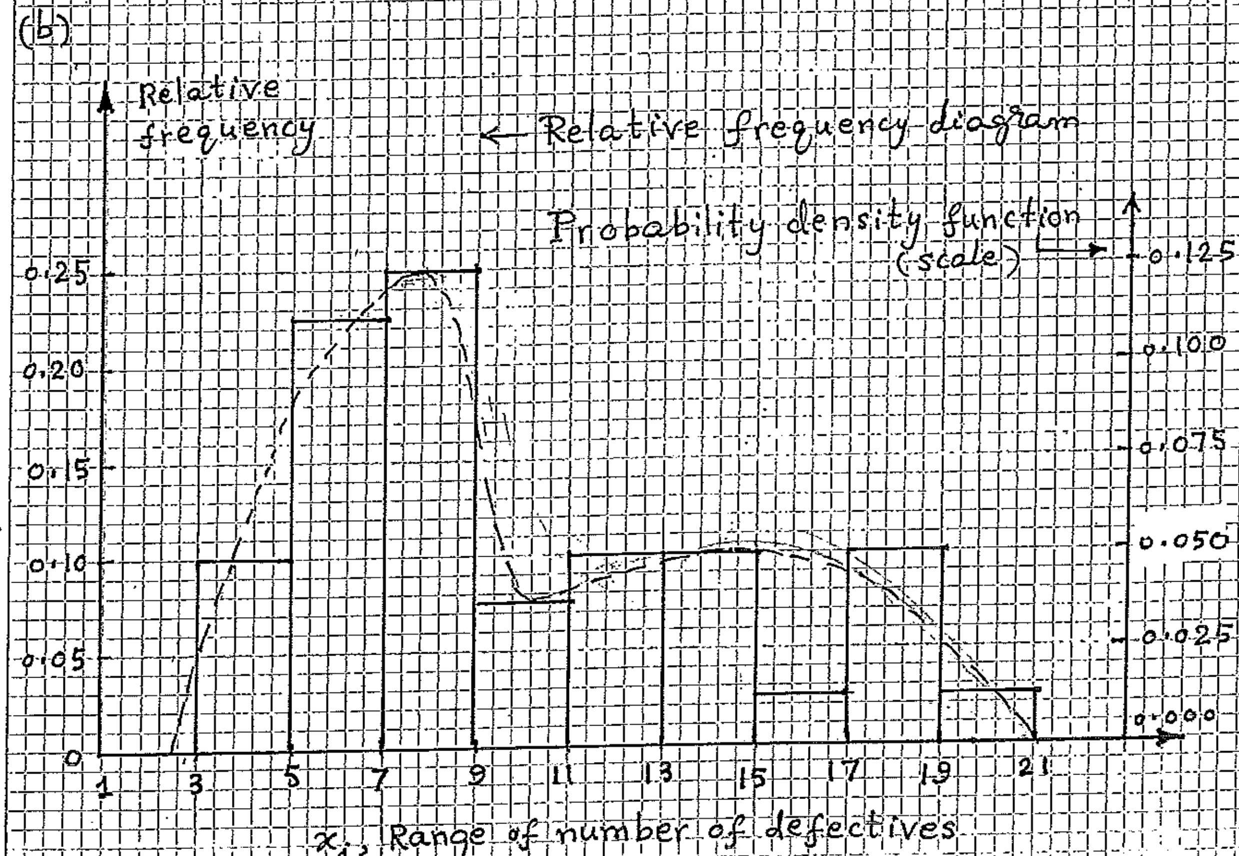
(d) Percentage of joint temperatures falling below freezing point of water ($32^\circ F$) is, from the observed data, 0 out of 30, i.e. 0%.

1.9 Data on number of defective leaf springs in samples of size 50:

Total data points = 40

Range of number of defective springs	Number of observed values	Relative frequency
3 – 4.9	4	0.100
5 – 6.9	9	0.225
7 – 8.9	10	0.250
9 – 10.9	3	0.075
11 – 12.9	4	0.100
13 – 14.9	4	0.100
15 – 16.9	1	0.025
17 – 18.9	4	0.100
19 – 20.9	1	0.025
Total:	40	1.000


(a)
Number observed
Histogram
0
2
4
6
8
10
1
3
5
7
9
11
13
15
17
19
21
x_i, Range of number of defectives


(b)
Relative frequency
← Relative frequency diagram
Probability density function (scale)
0.25
0.20
0.15
0.10
0.05
0
0.125
0.100
0.075
0.050
0.025
0.000
1
3
5
7
9
11
13
15
17
19
21
x_i, Range of number of defectives

(c) Mean value = $\bar{X} = \frac{1}{40} \sum_{i=1}^{40} x_i = 9.375$

Standard deviation $= s_X = \left\{ \frac{1}{40} \sum_{i=1}^{40} (x_i - 9.375)^2 \right\}^{\frac{1}{2}}$

$= 4.4761$

(d) Percentage of defective springs that fall outside the band defined by (mean ± 3 standard deviations)

i.e., $9.375 \pm 3(4.4761) = 9.375 \pm 13.4283$

i.e., -4.0533 to 22.8033

i.e., 0 to 22.8033

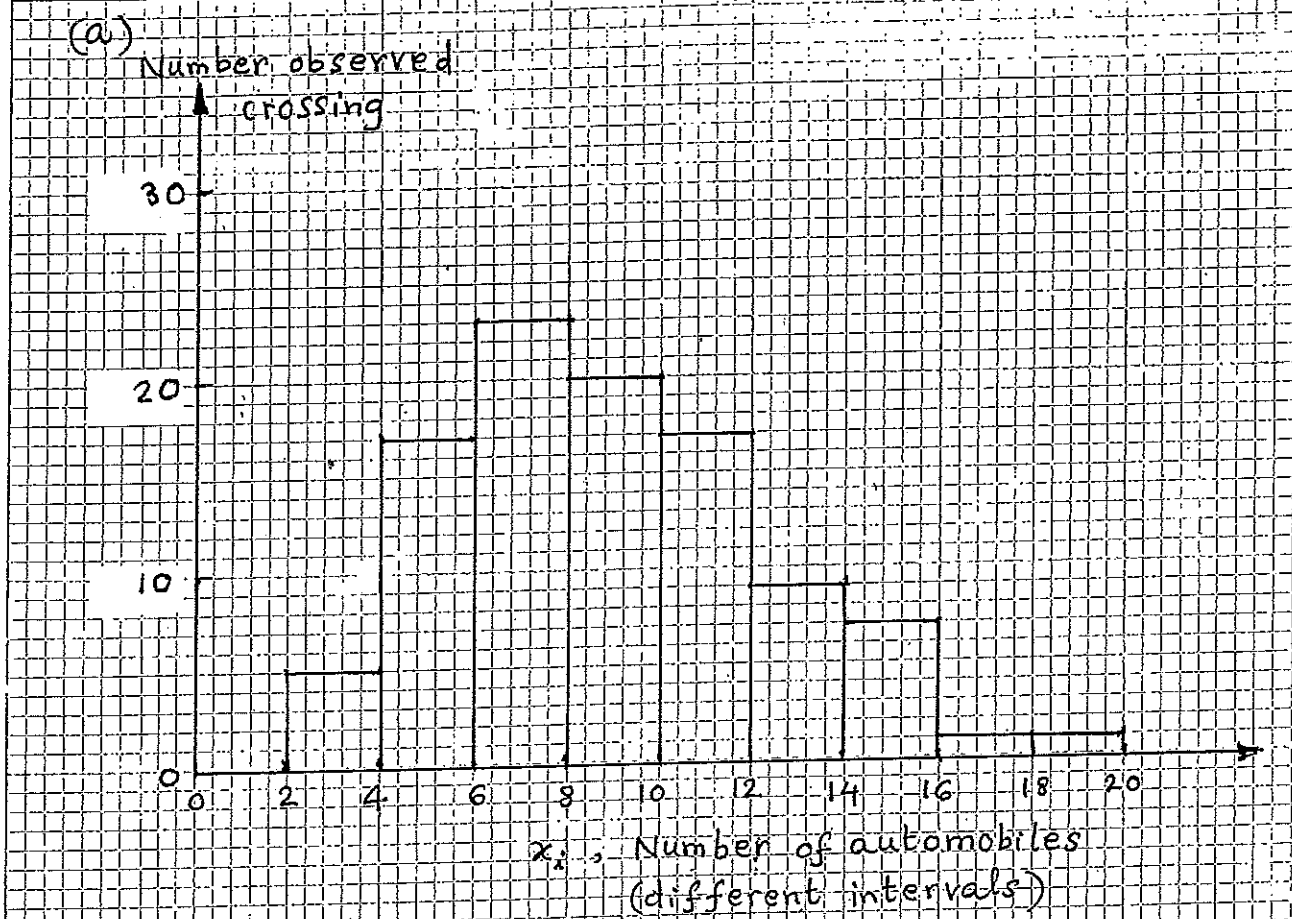
From the given data, we find that none of the data falls outside 0 – 22.8033

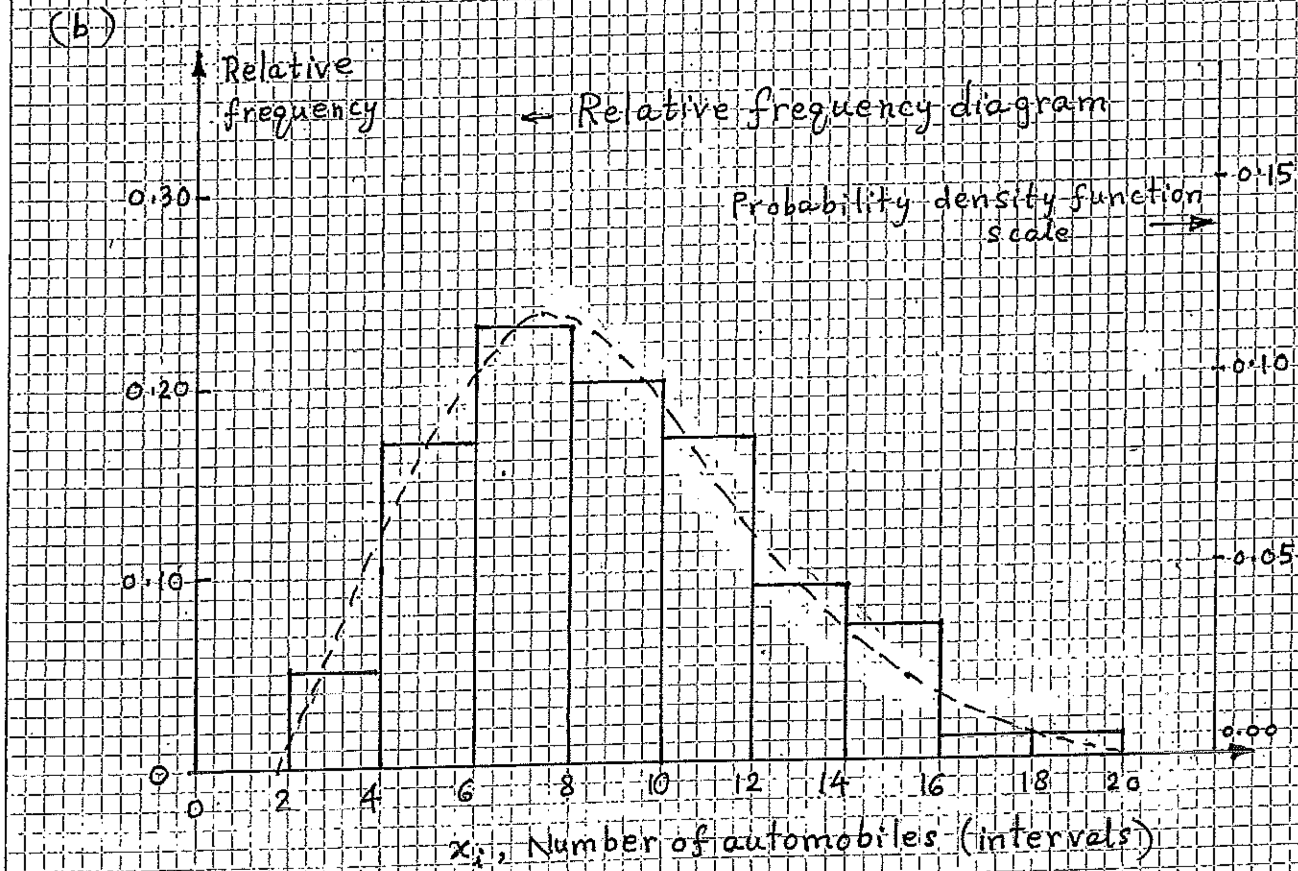
Hence the required percentage is zero.

(1.10) Data on number of automobiles crossing an intersection :

Data points: 100

Number of automobiles crossing intersection	Number observed	Relative frequency
1	0	0
2	0	0
3	2	0.02
4	3	0.03
5	7	0.07
6	10	0.10
7	11	0.11
8	12	0.12
9	14	0.14
10	6	0.06
11	7	0.07
12	10	0.10
13	6	0.06
14	3	0.03
15	6	0.06
16	1	0.01
17	1	0.01
18	0	0
19	1	0.01
20	0	0
Total:	100	1.00


(a)
Number observed crossing
30
20
10
0
0
2
4
6
8
10
12
14
16
18
20
x_i, Number of automobiles (different intervals)


(b)
Relative frequency
← Relative frequency diagram
Probability density function scale →
0.30
0.20
0.10
0
0.15
0.10
0.05
0.00
0
2
4
6
8
10
12
14
16
18
20
x_i, Number of automobiles (intervals)

(c) Sample mean $= \frac{1}{100} \sum_{i=1}^{100} x_i = 9.31 = \overline{X}$

Sample standard deviation $= \left\{ \frac{1}{99} \sum_{i=1}^{100} (x_i - 9.31)^2 \right\}^{\frac{1}{2}}$

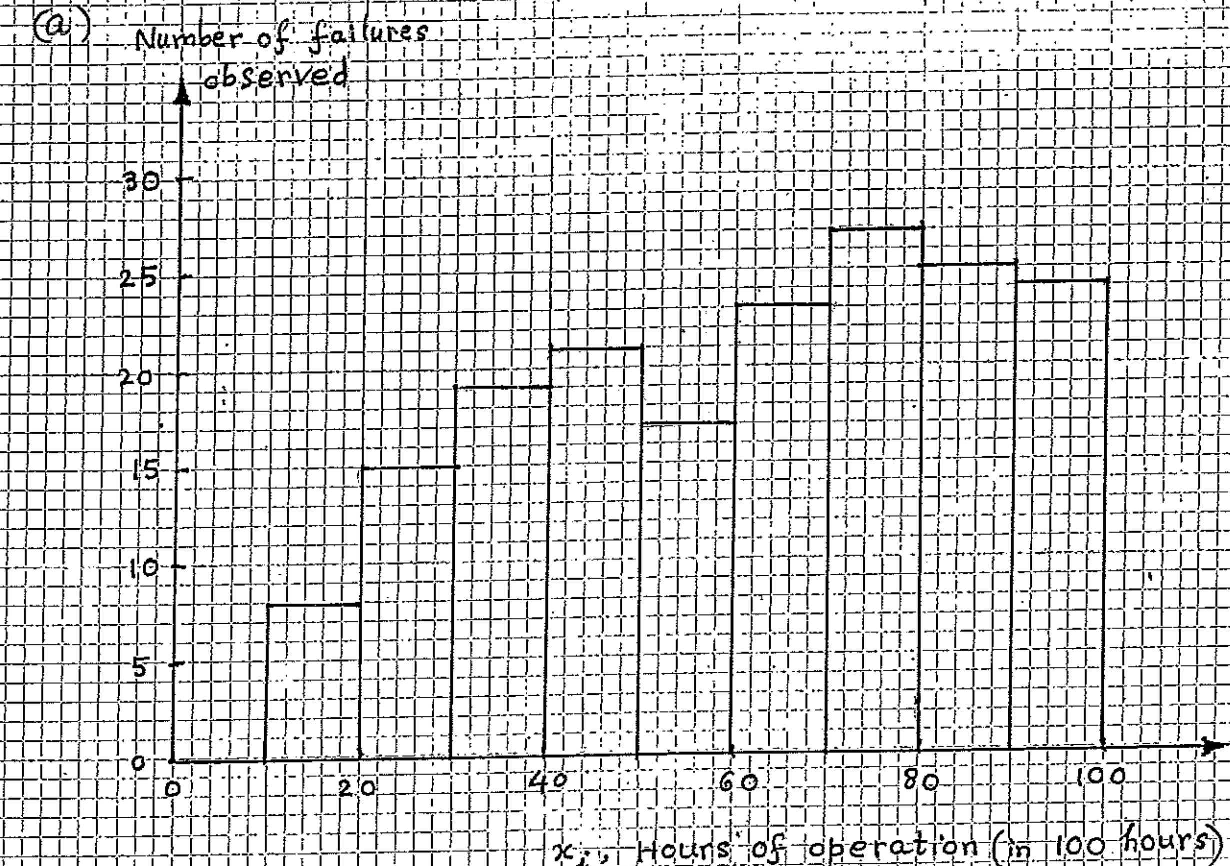
$= s_X = 3.3795$

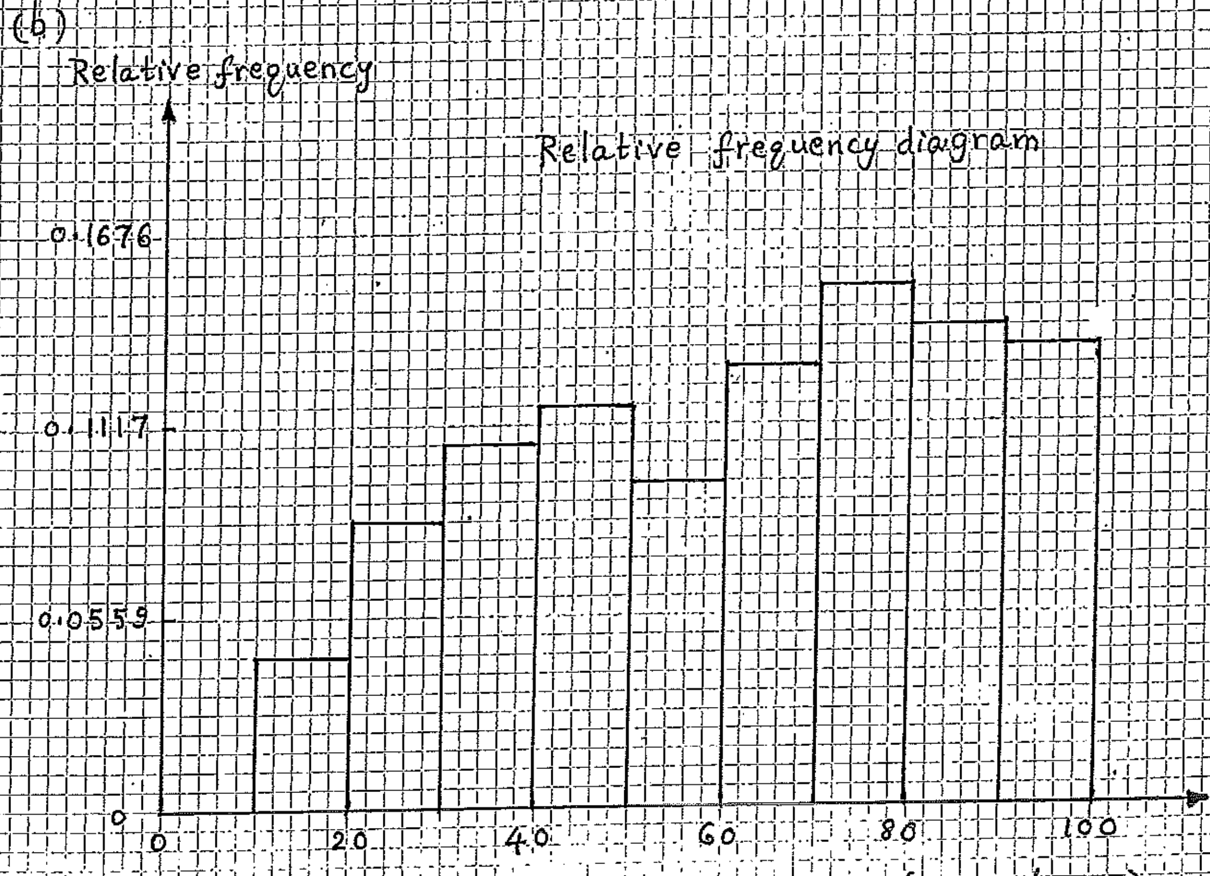
(d) Percentage of number of automobiles exceeding a value of 15 (from given data) is 3 out of 100, i.e., 3%.

1.11 Failure data of disk drives:

Total number of failures: 179

Hours of operation (unit: 100 hours)	Number of failures observed	Relative frequency
0 – 10.0	0	0
10.1 – 20.0	8	0.0447
20.1 – 30.0	15	0.0838
30.1 – 40.0	19	0.1061
40.1 – 50.0	21	0.1173
50.1 – 60.0	17	0.0950
60.1 – 70.0	23	0.1285
70.1 – 80.0	27	0.1508
80.1 – 90.0	25	0.1397
90.1 – 100.0	24	0.1341
Total:	179	1.0000


(a)
Number of failures observed
30
25
20
15
10
5
0
0
20
40
60
80
100
x_i, Hours of operation (in 100 hours)


(b)
Relative frequency
Relative frequency diagram
0.1676
0.1117
0.0559
0
0
20
40
60
80
100
x_i, Hours of operation (in 100 hours)

(c) Sample mean = $\overline{X} = \left(\frac{1}{179} \sum_i x_i\right) = 61.257 \times 100$ hours

Sample standard deviation = $s_X = 23.99 \times 100$ hours

(d) Expected reliability of disk drives at 8,000 hours of operation (from given data)

= Probability of hours of operation ≥ 80 units

= $(25 + 24)$ out of 179 failures

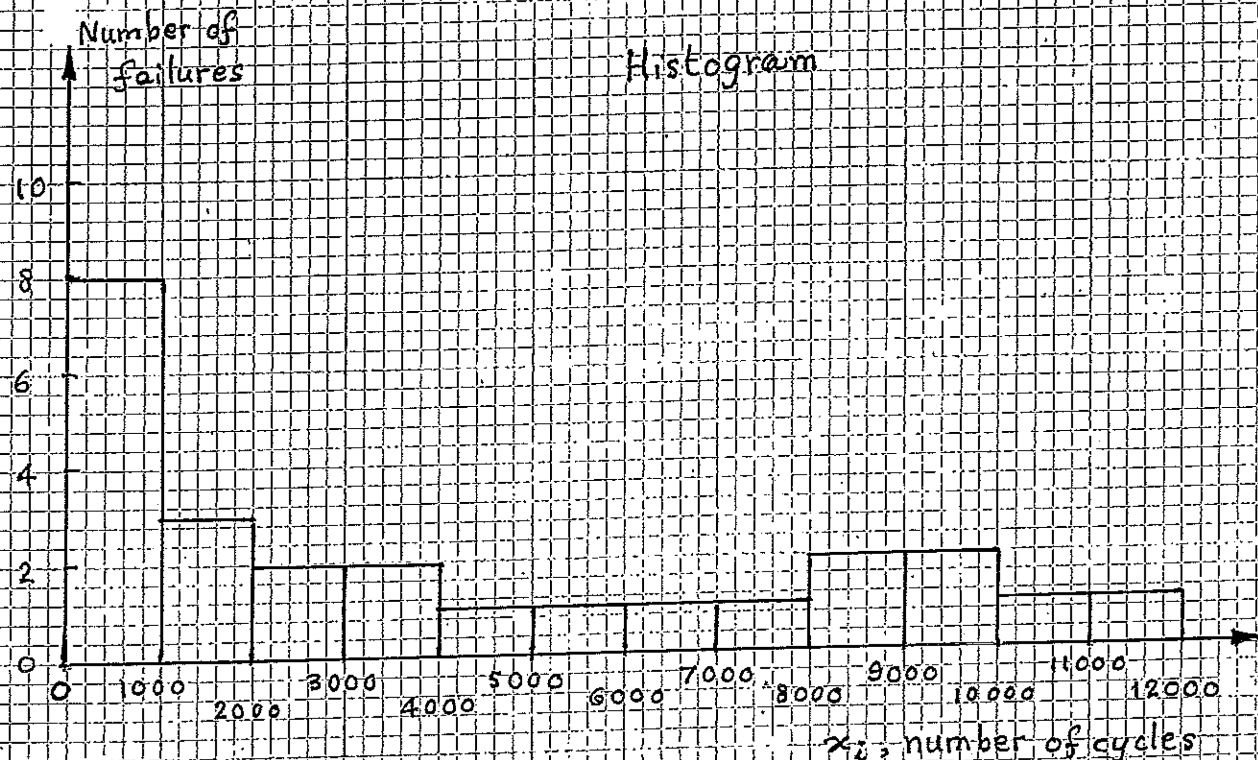
= 0.2737

1.12 Failure data of turbine blades

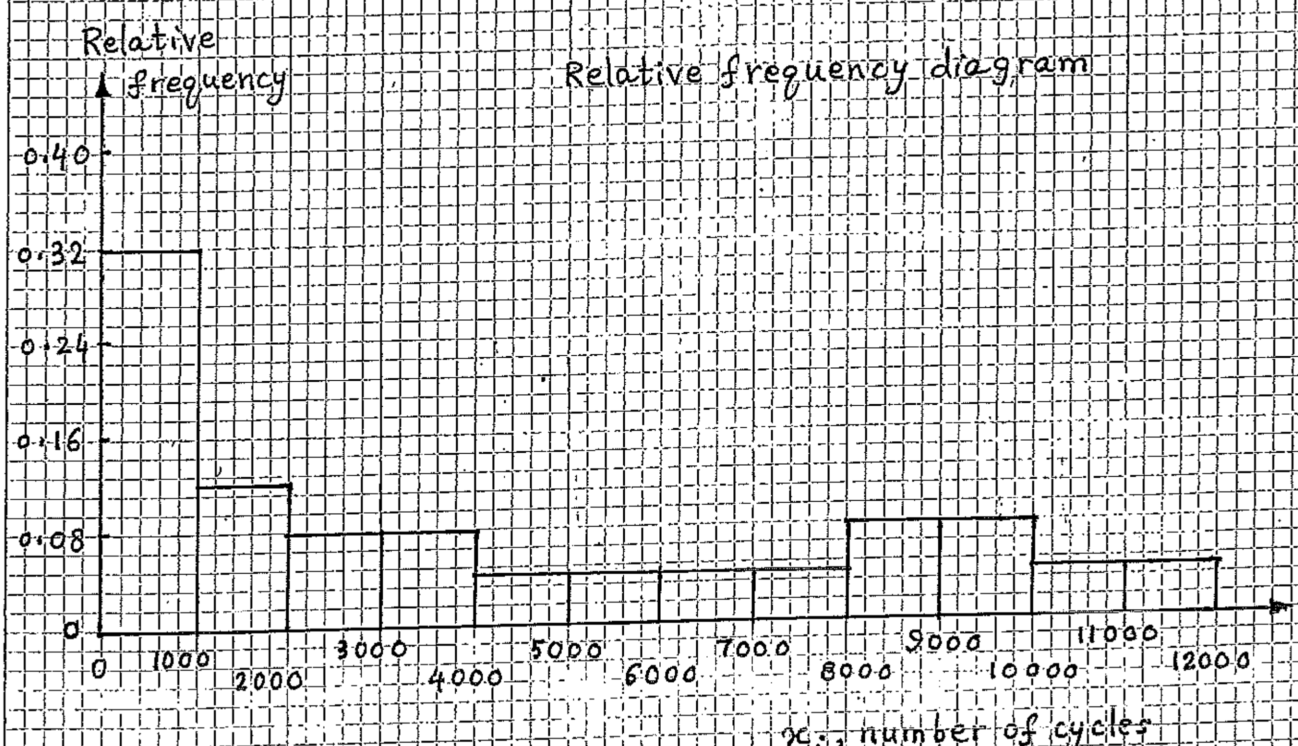
Data points: 25

Number of cycles at failure (Range)	Number of failures observed in the range	Relative frequency
0 – 1000	8	0.32
1001 – 2000	3	0.12
2001 – 3000	2	0.08
3001 – 4000	2	0.08
4001 – 5000	1	0.04
5001 – 6000	1	0.04
6001 – 7000	1	0.04
7001 – 8000	1	0.04
8001 – 9000	2	0.08
9001 – 10000	2	0.08
10001 – 11000	1	0.04
11001 – 12000	1	0.04
Total:	25	1.00

(a)


Number of failures
Histogram
10
8
6
4
2
0
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
x_i, number of cycles

(b)


Relative frequency
Relative frequency diagram
0.40
0.32
0.24
0.16
0.08
0
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
x_i, number of cycles

(c) Sample mean = $\overline{x} = \frac{1}{25} \sum_{i=1}^{25} x_i$ = 4125.16 cycles

Sample standard deviation = $s_x = \left\{ \frac{1}{25} \sum_{i=1}^{25} (x_i - \overline{x})^2 \right\}^{\frac{1}{2}}$

= 3765.42 cycles

(d) Percentage of turbine blades that have a life greater than 7500 cycles (from given data)

= 7 out of 25 = 28%.