Questions for Chapter 2

For each of the pairs of sets in 1-3 determine whether the Þrst is a subset of the second, the second is a subset of the Þrst, or neither is a subset of the other.
The set of people who were born in the U.S., the set of people who are U.S. citizens.

The set of students studying a programming language, the set of students studying Java.

The set of animals living in the ocean, the set of Þsh.

Prove or disprove: $A - (B \cap C) = (A - B) \cup (A - C)$.

5. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving an element table proof.

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a proof using logical equivalence.

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a Venn diagram proof.

Prove that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

Prove that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) by giving an element table proof.

Prove that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) by giving a proof using logical equivalence.

Prove that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) by giving a Venn diagram proof.

Prove or disprove: if A, B, and C are sets, then A − (B ∩ C) = (A − B) ∩ (A − C).

Prove or disprove A ⊕ (B ⊕ C) = (A ⊕ B) ⊕ C.

In questions 15Ð18 use a Venn diagram to determine which relationship, ⊆, =, or ⊇, is true for the pair of sets.

A ∪ B, A ∪ (B − A).

A ∪ (B ∩ C), (A ∪ B) ∩ C.

(A − B) ∪ (A − C), A − (B ∩ C).

(A − C) − (B − C), A − B.

In questions 19Ð23 determine whether the given set is the power set of some set. If the set is a power set, give the set of which it is a power set.

{∅, {∅}, {a}, {{a}}, {{{a}}}, {∅, a}, {∅, {a}}, {∅, {{a}}}, {a, {a}}, {a, {{a}}}, {{a}, {{a}}},
{∅, a, {a}}, {∅, a, {{a}}}, {∅, {a}, {{a}}}, {a, {a}, {{a}}}, {∅, a, {a}, {{a}}}}.

{∅, {a}}.

{∅, {a}, {∅, a}}.

{∅, {a}, {∅}, {a, ∅}}.

{∅, {a, ∅}}.

Prove that $\overline{\overline{S} \cup \overline{T}} = S \cap T$ for all sets S and T .

In 25Ð35 mark each statement TRUE or FALSE. Assume that the statement applies to all sets.

A − (B − C) = (A − B) − C.

(A − C) − (B − C) = A − B.

A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C).

A ∩ (B ∪ C) = (A ∪ B) ∩ (A ∪ C).

$\overline{A \cup \overline{B}} \cup \overline{A} = \overline{A}$.

If A ∪ C = B ∪ C, then A = B.

If A ∩ C = B ∩ C, then A = B.

If A ∩ B = A ∪ B, then A = B.

If A ⊕ B = A, then B = A.

There is a set A such that |P(A)| = 12.

A ⊕ A = A.

Find three subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9} such that the intersection of any two has size 2 and the intersection of all three has size 1.

Find $+!_{i=1}^{\infty}$ [−1/i, 1/i].

38. Find $+"_{i=1}^{\infty}$ $(1 - \frac{1}{i}, 1)$.

39. Find $+"_{i=1}^{\infty}$ $[1 - \frac{1}{i}, 1]$.

Find $+"_{i=1}^{\infty}$ (i, ∞).

Suppose U = {1, 2, . . . , 9}, A = all multiples of 2, B = all multiples of 3, and C = {3, 4, 5, 6, 7}. Find C − (B − A).

Suppose S = {1, 2, 3, 4, 5}. Find |P(S)|.

In questions 43Ð46 suppose A = {x, y} and B = {x, {x}}. Mark the statement TRUE or FALSE.

x ⊆ B.

¯ ∈ P(B).

{x} ⊆ A − B.

|P(A)| = 4.

In questions 47Ð54 suppose A = {a, b, c}. Mark the statement TRUE or FALSE.

{b, c} ∈ P(A).

{{a}} ⊆ P(A).

¯ ⊆ A.

{ } ⊆ P(A).

¯ ⊆ A × A.

{a, c} ∈ A.

{a, b} ∈ A × A.

(c, c) ∈ A × A.

In questions 55Ð62 suppose A = {1, 2, 3, 4, 5}. Mark the statement TRUE or FALSE.

{1} ∈ P(A).

{{3}} ⊆ P(A).

¯ ⊆ A.

{ } ⊆ P(A).

¯ ⊆ P(A).

{2, 4} ∈ A × A.

{ } ∈ P(A).

(1, 1) ∈ A × A.

In questions 63Ð65 suppose the following are fuzzy sets:

= {0.7 Ann, 0.1 Bill, 0.8 Fran, 0.3 Olive, 0.5 Tom}, R

= {0.4 Ann, 0.9 Bill, 0.9 Fran, 0.6 Olive, 0.7 Tom}.

Find $\overline{F}$ and $\overline{R}$.

Find F ∪ R.

Find F ∩ R.

In questions 66Ð75, suppose A = {a, b, c} and B = {b, {c}}. Mark the statement TRUE or FALSE.

c ∈ A − B.

|P(A × B)| = 64.

¯ ∈ P(B).

B ⊆ A.

{c} ⊆ B.

{a, b} ∈ A × A.

{b, c} ∈ P(A).

{b, {c}} ∈ P(B).

¯ ⊆ A × A.

{{{c}}} ⊆ P(B).

Find A^2 if A = {1, a}.

In questions 77Ð89 determine whether the set is Þnite or inÞnite. If the set is Þnite, Þnd its size.

{ x | x ∈ Z and $x^2 < 10$ }.

P({a, b, c, d}), where P denotes the power set.

{1, 3, 5, 7, . . .}.

A × B, where A = {1, 2, 3, 4, 5} and B = {1, 2, 3}.

{ x | x ∈ N and $9x^2 - 1 = 0$ }.

P(A), where A is the power set of {a, b, c}.

A × B, where A = {a, b, c} and B = ¯.

{ x | x ∈ N and $4x^2 - 8 = 0$ }.

{ x | x ∈ Z and $x^2 = 2$ }.

P(A), where A = P({1, 2}).

{1, 10, 100, 1000, . . .}.

S × T , where S = {a, b, c} and T = {1, 2, 3, 4, 5}.

$\{ x \mid x \in Z \text{ and } x^2 < 8 \}$.

Prove that between every two rational numbers a/b and c/d
(a) there is a rational number. (b) there are an inÞnite number of rational numbers.

Prove that there is no smallest positive rational number.

Consider these functions from the set of licensed drivers in the state of New York. Is a function one-to-one if it assigns to a licensed driver his or her
birthdate
motherÕs Þrst name
drivers license number?

In 93Ð94 determine whether each of the following sets is countable or uncountable. For those that are countably inÞnite exhibit a one-to-one correspondence between the set of positive integers and that set.
The set of positive rational numbers that can be written with denominators less than 3.

The set of irrational numbers between $\sqrt{2}$ and $\pi/2$.

Adapt the Cantor diagonalization argument to show that the set of positive real numbers less than 1 with decimal representations consisting only of 0s and 1s is uncountable.

Show that (0, 1) has the same cardinality as (0, 2).
Show that (0, 1] and R have the same cardinality.

In questions 98Ð106 determine whether the rule describes a function with the given domain and codomain.

f: N → N where $f(n) = \sqrt{n}$.

h: R → R where $h(x) = \sqrt{x}$.
g: N → N where g(n) = any integer > n.

101. F : R → R where $F(x) = \frac{1}{x - 5}$.

102. F : Z → R where $F(x) = \frac{1}{x^2 - 5}$.

103. F : Z → Z where $F(x) = \frac{1}{x^2 - 5}$.

104. G: R → R where $G(x) = \begin{cases} x - 1 & \text{if } x \le 4. \\ x + 2 & \text{if } x \ge 0 \end{cases}$

105. f: R → R where $f(x) = \begin{cases} x^2 - 1 & \text{if } x \le 2 \\ & \ge \end{cases}$

G: Q → Q where G(p/q) = q.

Give an example of a function f: Z → Z that is 1-1 and not onto Z.

Give an example of a function f: Z → Z that is onto Z but not 1-1.

Give an example of a function f: Z → N that is both 1-1 and onto N.

Give an example of a function f: N → Z that is both 1-1 and onto Z.

Give an example of a function f: Z → N that is 1-1 and not onto N.

Give an example of a function f: N → Z that is onto Z and not 1-1.

Suppose f: N → N has the rule f(n) = 4n + 1. Determine whether f is 1-1.

Suppose f: N → N has the rule f(n) = 4n + 1. Determine whether f is onto N.

Suppose f: Z → Z has the rule f(n) = 3n2 − 1. Determine whether f is 1-1.

Suppose f: Z → Z has the rule f(n) = 3n − 1. Determine whether f is onto Z.

Suppose f: N → N has the rule f(n) = 3n2 − 1. Determine whether f is 1-1.

Suppose f: N → N has the rule f(n) = 4n2 + 1. Determine whether f is onto N.

Suppose f: R → R where f(x) = *x/2+.

(a) Draw the graph of f. (b) Is f 1-1? (c) Is f onto R?

Suppose f: R → R where f(x) = *x/2+.

If S = { x | 1 ≤ x ≤ 6 }, Þnd f(S). (b) If T = {3, 4, 5}, Þnd f^{-1}(T).

Determine whether f is a function from the set of all bit strings to the set of integers if f(S) is the position of a 1 bit in the bit string S.

Determine whether f is a function from the set of all bit strings to the set of integers if f(S) is the number of 0 bits in S.

Determine whether f is a function from the set of all bit strings to the set of integers if f(S) is the largest integer i such that the ith bit of S is 0 and f(S) = 1 when S is the empty string (the string with no bits).

Let f(x) = *x^3/3+. Find f(S) if S is:

(a) {−2, −1, 0, 1, 2, 3}. (b) {0, 1, 2, 3, 4, 5}. (c) {1, 5, 7, 11}. (d) {2, 6, 10, 14}.

Suppose f: R → Z where f(x) = 82x − 19.

(a) Draw the graph of f. (b) Is f 1-1? (Explain) (c) Is f onto Z? (Explain)

Suppose f: R → Z where f(x) = 82x − 19.

(a) If A = {x | 1 ≤ x ≤ 4}, Þnd f(A). (b) If B = {3, 4, 5, 6, 7}, Þnd f(B).

(c) If C = 9, 8 , Þnd f^{-1}(C). (d) If D = 0.4, 0.5, 0.6 , Þnd f^{-1} (D).

{− → − } $ 2 % { }

127. Suppose g: R R where g(x) = x − 1 .

(a) Draw the graph of g. (b) Is g 1-1? (c) Is g onto R?

→ $ 2 %

128. Suppose g: R R where g(x) = x − 1 .

(a) If S = x 1 x 6 , Þnd g(S). (b) If T = 2 , Þnd g^{-1}(T).

{ | ≤ ≤ } { }

Show that 8x9 = −*−x+.

Prove or disprove: For all positive real numbers x and y, *x á y+ ≤ *x+ á *y+.

Prove or disprove: For all positive real numbers x and y, 8x á y9 ≤ 8x9 á 8y9.

Suppose g: A → B and f: B → C where A = {1, 2, 3, 4}, B = {a, b, c}, C = {2, 7, 10}, and f and g are deÞned by g = {(1, b), (2, a), (3, a), (4, b)} and f = {(a, 10), (b, 7), (c, 2)}. Find f ◦ g.

Suppose g: A → B and f: B → C where A = {1, 2, 3, 4}, B = {a, b, c}, C = {2, 7, 10}, and f and g are deÞned by g = {(1, b), (2, a), (3, a), (4, b)} and f = {(a, 10), (b, 7), (c, 2)}. Find f^{-1}.

In questions 134Đ137 suppose that g: A → B and f: B → C where A = B = C = {1, 2, 3, 4}, g = {(1, 4), (2, 1), (3, 1), (4, 2)}, and f = {(1, 3), (2, 2), (3, 4), (4, 2)}.

Find f ◦ g.

Find g ◦ f.

Find g ◦ g.

Find g ◦ (g ◦ g).

In questions 138Đ141 suppose g: A → B and f: B → C where A = {1, 2, 3, 4}, B = {a, b, c}, C = {2, 8, 10}, and g and f are deÞned by g = {(1, b), (2, a), (3, b), (4, a)} and f = {(a, 8), (b, 10), (c, 2)}.

Find f ◦ g.

Find f^{-1}.

Find f ◦ f^{-1}.

Explain why g^{-1} is not a function.

In questions 142Đ143 suppose g: A → B and f: B → C where A = {a, b, c, d}, B = {1, 2, 3}, C = {2, 3, 6, 8}, and g and f are deÞned by g = {(a, 2), (b, 1), (c, 3), (d, 2)} and f = {(1, 8), (2, 3), (3, 2)}.

Find f ◦ g.

Find f^{-1}.

For any function f: A → B, deÞne a new function g: P(A) → P(B) as follows: for every S ⊆ A, g(S) = { f(x) | x ∈ S }. Prove that f is onto if and only if g is onto.

In questions 145Đ149 Þnd the inverse of the function f or else explain why the function has no inverse.

f: Z → Z where f(x) = x mod 10.

f: A → B where A = {a, b, c}, B = {1, 2, 3} and f = {(a, 2), (b, 1), (c, 3)}.

f: R → R where f(x) = 3x − 5.

f: R → R where f(x) = *2x+.

→ #x + 1 if x ≤ 4.

149. f: Z Z where f(x) = x − 2 if x ≥ 5

Suppose g: A → B and f: B → C, where f ◦ g is 1-1 and g is 1-1. Must f be 1-1?

Suppose g: A → B and f: B → C, where f ◦ g is 1-1 and f is 1-1. Must g be 1-1?

Suppose f: R → R and g: R → R where g(x) = 2x + 1 and g ◦ f(x) = 2x + 11. Find the rule for f.

In questions 153Đ157 for each partial function, determine its domain, codomain, domain of deÞnition, set of values for which it is undeÞned or if it is a total function:

f: Z → R where f(n) = 1/n.

f: Z → Z where f(n) = 8n/29.

f: Z × Z → Q where f(m, n) = m/n.

f: Z × Z → Z where f(m, n) = mn.

f: Z × Z → Z where f(m, n) = m − n if m > n.

158. For the partial function f: Z×Z → R deÞned by $f(m, n) = \frac{1}{n^2 - m^2}$, determine its domain, codomain, domain of deÞnition, and set of values for which it is undeÞned or whether it is a total function.

159. Let f: {1, 2, 3, 4, 5} → {1, 2, 3, 4, 5, 6} be a function.

(a) How many total functions are there?
(b) How many of these functions are one-to-one?

In questions 160Ð166 Þnd a formula that generates the following sequence $a_1, a_2, a_3 \ldots$.

5, 9, 13, 17, 21, ...

3, 3, 3, 3, 3, ...

15, 20, 25, 30, 35, ...

1, 0.9, 0.8, 0.7, 0.6, ...

1, 1/3, 1/5, 1/7, 1/9, ...

2, 0, 2, 0, 2, 0, 2, ...

0, 2, 0, 2, 0, 2, 0, ...

In questions 167Ð178, describe each sequence recursively. Include initial conditions and assume that the sequences begin with a_1.

$a_n = 5^n$.

The Fibonacci numbers.

0, 1, 0, 1, 0, 1, ...

$a_n = 1 + 2 + 3 + \cdots + n$.

3, 2, 1, 0, −1, −2, ...

$a_n = n!$.

1/2, 1/3, 1/4, 1/5, ...

0.1, 0.11, 0.111, 0.1111, ...

$1^2, 2^2, 3^2, 4^2$, ...

1, 111, 11111, 1111111, ...

a_n = the number of subsets of a set of size n.

1, 101, 10101, 1010101, ...

Verify that $a_n = 6$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

Verify that $a_n = 3^n$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

Verify that $a_n = 3^{n+4}$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

Verify that $a_n = 3^n + 1$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

Verify that $a_n = 7 \cdot 3^n - \pi$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

In questions 184Ð188 Þnd a recurrence relation with initial condition(s) satisÞed by the sequence. Assume a_0 is the Þrst term of the sequence.

$a_n = 2^n$.

$a_n = 2^n + 1$.

$a_n = (-1)^n$.

$a_n = 3n - 1$.

$a_n = \sqrt{2}$.

You take a job that pays $25,000 annually.

How much do you earn n years from now if you receive a three percent raise each year?
How much do you earn n years from now if you receive a Þve percent raise each year?
How much do you earn n years from now if each year you receive a raise of $1000 plus two percent of your previous yearÕs salary.

Suppose inßation continues at three percent annually. (That is, an item that costs $1.00 now will cost $1.03 next year.) Let a_n = the value (that is, the purchasing power) of one dollar after n years.

Find a recurrence relation for a_n.
What is the value of $1.00 after 20 years?
What is the value of $1.00 after 80 years?
If inßation were to continue at ten percent annually, Þnd the value of $1.00 after 20 years.
If inßation were to continue at ten percent annually, Þnd the value of $1.00 after 80 years.

Find the sum 1/4 + 1/8 + 1/16 + 1/32 + ááá .

Find the sum $2 + 4 + 8 + 16 + 32 + \cdots + 2^{28}$.

Find the sum $2 - 4 + 8 - 16 + 32 - \cdots - 2^{28}$.

Find the sum 1 − 1/2 + 1/4 − 1/8 + 1/16 − ááá .

Find the sum 2 + 1/2 + 1/8 + 1/32 + 1/128 + ááá .

Find the sum 112 + 113 + 114 + ááá + 673.

Find $\sum_{i=1}^{6}((-2)^i - 2^i)$.

Find $\sum_{j=1}^{3}\sum_{i=1}^{j} ij$.

Rewrite $\sum_{i=-3}^{4}(i^2 + 1)$ so that the index of summation has lower limit 0 and upper limit 7.

200. Find a 2 × 2 matrix $A \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ such that $A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

201. Suppose A is a 6 × 8 matrix, B is an 8 × 5 matrix, and C is a 5 × 9 matrix. Find the number of rows, the number of columns, and the number of entries in A(BC).

202. Let $A = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$. Find A^n where n is a positive integer.

203. Suppose $A = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ 0 & 6 \end{pmatrix}$. Find a matrix B such that AB = C or prove that no such matrix exists.

204. Suppose $B = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ 0 & 6 \end{pmatrix}$. Find a matrix A such that AB = C or prove that no such matrix exists.

205. Suppose $B = \begin{pmatrix} 6 & 2 \\ 3 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ 0 & 6 \end{pmatrix}$. Find a matrix A such that AB = C or prove that no such matrix exists.

If AB = AC, then B = C.

207. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, then $A^{-1} = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$.

208. If $A = \begin{pmatrix} -5 & 2 \\ 1 & 3 \end{pmatrix}$, then $A^2 = \begin{pmatrix} 25 & 4 \\ 1 & 9 \end{pmatrix}$.

If A is a 6 × 4 matrix and B is a 4 × 5 matrix, then AB has 16 entries.

210. If A and B are 2 × 2 matrices such that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, then $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ or $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

If A and B are 2 × 2 matrices, then A+B=B+A.

AB=BA for all 2 × 2 matrices A and B.

213. Suppose $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Find

(a) the join of A and B. (b) the meet of A and B. (c) the Boolean product of A and B.

Suppose A is a 2 × 2 matrix with real number entries such that AB=BA for all 2 × 2 matrices. What relationships must exist among the entries of A?

Answers for Chapter 2

The Þrst is a subset of the second, but the second is not a subset of the Þrst.

The second is a subset of the Þrst, but the Þrst is not a subset of the second.

Neither is a subset of the other.

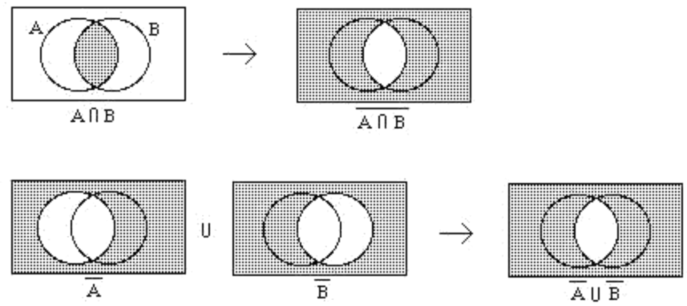
True, since $A - (B \cap C) = A \cap \overline{B \cap C} = A \cap (\overline{B} \cup \overline{C}) = (A \cap \overline{B}) \cup (A \cap \overline{C}) = (A - B) \cup (A - C)$.

$\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$: Let $x \in \overline{A \cap B}$. $\therefore x \notin A \cap B$, $\therefore x \notin A$ or $x \notin B$, $\therefore x \in \overline{A}$ or $x \in \overline{B}$, $\therefore x \in \overline{A} \cup \overline{B}$. Reversing the steps shows that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

The columns for $\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$ match: each entry is 0 if and only if A and B have the value 1.

$\overline{A \cap B} = \{x \mid x \in \overline{A \cap B}\} = \{x \mid x \notin A \cap B\} = \{x \mid \neg(x \in A \cap B)\} = \{x \mid \neg(x \in A \wedge x \in B)\} = \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$
$= \{x \mid x \notin A \vee x \notin B\} = \{x \mid x \in \overline{A} \vee x \in \overline{B}\} = \{x \mid x \in \overline{A} \cup \overline{B}\} = \overline{A} \cup \overline{B}$.

8.

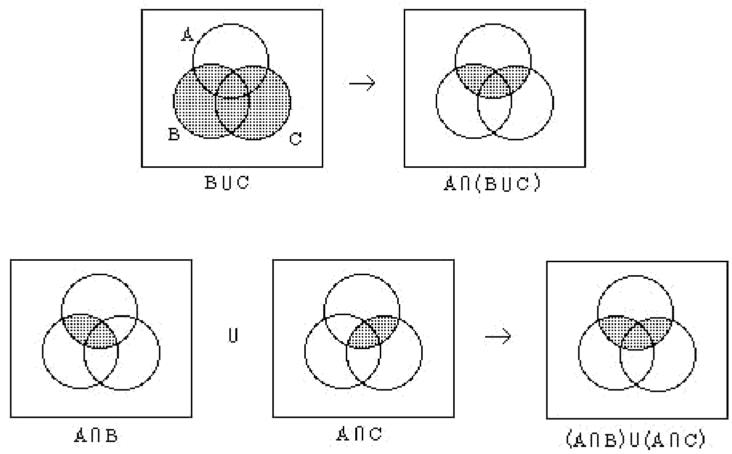


A ∩ (B ∪ C) ⊆ (A ∩ B) ∪ (A ∩ C): Let x ∈ A ∩ (B ∪ C). ∴ x ∈ A and x ∈ B ∪ C, ∴ x ∈ A and x ∈ B, or x ∈ A and x ∈ C, ∴ x ∈ (A ∩ B) ∪ (A ∩ C). Reversing the steps gives the opposite containment.

Each set has the same values in the element table: the value is 1 if and only if A has the value 1 and either B or C has the value 1.

A∩(B ∪C) = {x | x ∈ A∩(B ∪C)} = {x | x ∈ A ∧ x ∈ (B ∪C)} = {x | x ∈ A ∧ (x ∈ B ∨ x ∈ C)} = {x | (x ∈ A ∧ x ∈ B) ∨ (x ∈ A ∧ x ∈ C)} = {x | x ∈ A∩B ∨ x ∈ A∩C} = {x | x ∈ (A∩B) ∪ (A∩C)} = (A∩B) ∪ (A∩C).

12.



False. For example, let A = {1, 2}, B = {1}, C = {2}.

True, using either a membership table or a containment proof, for example.

=.

⊇.

=.

⊆.

Yes {∅, a, {a}, {{a}}}.

Yes, {a}.

No, it lacks {∅}.

Yes, {{a, ∅}}.

No, it lacks {a} and {∅}.

Since $\overline{S} \cup \overline{T} = \overline{S \cap T}$ (De MorganÕs law), the complements are equal.

False.

False.

True.

False.

True.

False.

False.
True.
False.
False.
False.
For example, {1, 2, 3}, {2, 3, 4}, {1, 3, 4}.
[−1, 1].
‾.
{1}.
‾.
{4, 5, 6, 7}.
32.
False.
True.
False.
True.
True.
True.
True.
True.
True.
True.
False.
True.
True.
True.
True.
True.
True.
False.
False.
True.
$\overline{F}$ = {0.3 Ann, 0.9 Bill, 0.2 Fran, 0.7 Olive, 0.5 Tom},
$\overline{R}$ = {0.6 Ann, 0.1 Bill, 0.1 Fran, 0.4 Olive, 0.3 Tom}
{0.7 Ann, 0.9 Bill, 0.9 Fran, 0.6 Olive, 0.7 Tom}.
{0.4 Ann, 0.1 Bill, 0.8 Fran, 0.3 Olive, 0.5 Tom}.
True.
True.
True.
False.
False.
False.
True.

73. True.
74. True.
75. True.
76. $A^2 = \{(1, 1), (1, a), (a, 1), (a, a)\}$
77. 7.
78. 16.
79. InÞnite.
80. 15.
81. 0.
82. 256.
83. 0.
84. 0.
85. 0.
86. 16.
87. InÞnite.
88. 15.
89. 5.
90. (a) Assume $\frac{a}{b} < \frac{c}{d}$. Then $\frac{a}{b} < \frac{\frac{a}{b} + \frac{c}{d}}{2} = \frac{ad+bc}{2bd} < \frac{c}{d}$.

(b) Assume $\frac{a}{b} < \frac{c}{d}$. Let m_1 be the midpoint of $\left(\frac{a}{b}, \frac{c}{d}\right)$. For $i > 1$ let m_i be the midpoint of $\left(\frac{a}{b}, m_{i-1}\right)$.

If $0 < \frac{a}{b}$, then $0 < \ldots < \frac{a}{4b} < \frac{a}{3b} < \frac{a}{2b} < \frac{a}{b}$.

(a) No (b) No (c) Yes

Countable. To Þnd a correspondence, follow the path in Example 4 in Section 2.5, using only the Þrst three lines.

Uncountable

Assume that these numbers are countable, and list them in order $r_1, r_2, r_3, \ldots$. Then form a new number r, whose i-th decimal digit is 0, if the i-th decimal digit of r_i is 1, and whose i-th decimal digit is 1, if the i-th decimal digit of r_i is 0. Clearly r is not in the list $r_1, r_2, r_3, \ldots$, therefore the original assumption is false.

The function f(x) = 2x is one-to-one and onto.

Example 2.5.6 shows that |(0, 1]| = |(0, 1)|, and Exercise 2.5.34 shows that |(0, 1]| =R .

Not a function; f(2) is not an integer.

Function.

Not a function; g(1) has more than one value.

Not a function; F (5) not deÞned.

Function.

Not a function; F (1) not an integer.

Not a function; the cases overlap. For example, G(1) is equal to both 3 and 0.

Not a function; f(3) not deÞned.

Not a function; f(1/2) = 2 and f(2/4) = 4.

f(n) = 2n.

f(n) = *n/2+.

$$f(n) = \begin{cases} -2n, & n \le 0 \\ 2n - 1, & n > 0. \end{cases}$$

110. f(n) = −2n , n even

2

2n, n 0

\# 2 n + 1, n > 0.

111. f(n) = − n ≤

112. f(n) = 2 , n even

2 n odd.

Yes.

No.

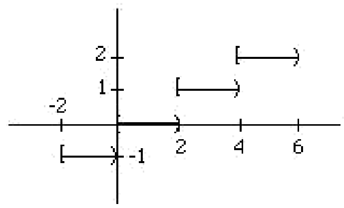
No.

No.

Yes.

No.

(a)



No.

No.

(a) {0, 1, 2, 3}

[6, 12).

No; there may be no 1 bits or more than one 1 bit.

Yes.

No; f not deÞned for the string of all 1Õs, for example S = 11111.

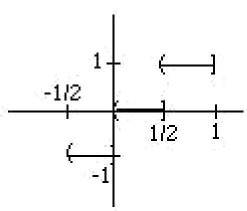
(a) {−3, −1, 0, 2, 9}.

{0, 2, 9, 21, 41}.

{0, 41, 114, 443}.

{2, 72, 333, 914}.

(a)



No.

Yes.

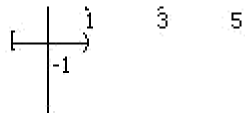
(a) {1, 2, 3, 4, 5, 6, 7}.

{5, 7, 9, 11, 13}.

(−9/2, −7/2].

¯.

(a)



(b) No.
(c) No.

128. (a) {0, 1, 2}.
(b) [5, 7).

129. Let n = 8x9, so that n − 1 < x ≤ n. Multiplying by −1 yields −n + 1 > −x ≥ −n, which means that −n = *−x+.

130. False: x = y = 1.5.

131. True: x ≤ 8x9, y ≤ 8y9; therefore xy ≤ 8x98y9; since 8x98y9 is an integer at least as great as xy, then 8xy9 ≤ 8x98y9.

132. {(1, 7), (2, 10), (3, 10), (4, 7)}.

133. {(2, c), (7, b), (10, a)}.

134. {(1, 2), (2, 3), (3, 3), (4, 2)}.

135. {(1, 1), (2, 1), (3, 2), (4, 1)}.

136. {(1, 2), (2, 4), (3, 4), (4, 1)}.

137. {(1, 1), (2, 2), (3, 2), (4, 4)}.

138. {(1, 10), (2, 8), (3, 10), (4, 8)}.

139. {(2, c), (8, a), (10, b)}.

140. {(2, 2), (8, 8), (10, 10)}.

141. $g^{-1}(a)$ is equal to both 2 and 4.

142. {(a, 3), (b, 8), (c, 2), (d, 3)}.

143. {(2, 3), (3, 2), (8, 1)}.

144. Suppose f is onto. Let T ∈ P(B) and let S = { x ∈ A | f(x) ∈ T }. Then g(S) = T , and g is onto. If f is not onto B, let y ∈ B − f(A). Then there is no subset S of A such that g(S) = {y}.

145. $f^{-1}(10)$ does not exist.

146. {(1, b), (2, a), (3, c)}.

147. $f^{-1}(x) = \frac{5 + x}{3}$.

$f^{-1}(\frac{1}{2})$ does not exist.

$f^{-1}(5)$ is not a single value.

No.

Yes.

f(x) = x + 5.

Z, R, Z − {0}, {0}.

Z, Z, Z, total function.

Z × Z, Q, Z × (Z − {0}), Z × {0}.

Z × Z, Z, Z × Z, total function.

Z × Z, Z, { (m, n) | m > n }, { (m, n) | m ≤ n }.

Z × Z, R, { (m, n) | m =) n or m =) −n }, { (m, n) | m = n or m = −n }.

(a) 6^5 = 7,776.
(b) 6 á 5 á 4 á 3 á 2 = 720.

a_n = 4n + 1.

a_n = 3.

a_n = 5(n + 2).

a_n = 1 − (n − 1)/10.

$a_n = 1/(2n - 1)$.

$a_n = 1 + (-1)^{n+1}$.

$a_n = 1 + (-1)^n$.

$a_n = 5a_{n-1}$, $a_1 = 5$.

$a_n = a_{n-1} + a_{n-2}$, $a_1 = a_2 = 1$.

$a_n = a_{n-2}$, $a_1 = 0$, $a_2 = 1$.

$a_n = a_{n-1} + n$, $a_1 = 1$.

$a_n = a_{n-1} - 1$, $a_1 = 3$.

$a_n = na_{n-1}$, $a_1 = 1$.

173. $a_n = \frac{a_{n-1}}{1 + a_{n-1}}$, $a_1 = 1/2$.

174. $a_n = a_{n-1} + 1/10^n$, $a_1 = 0.1$.

175. $a_n = a_{n-1} + 2n - 1$, $a_1 = 1$.

176. $a_n = 100a_{n-1} + 11$.

177. $a_n = 2$ á a_{n-1}, $a_1 = 2$.

178. $a_n = 100a_{n-1} + 1$, $a_1 = 1$.

179. 4 á 6 − 3 á 6 = 1 á 6 = 6.

180. 4 á 3^{n-1} − 3 á 3^{n-2} = 4 á 3^{n-1} − 3^{n-1} = 3 á 3^{n-1} = 3^n.

181. 4 á 3^{n+3} − 3 á 3^{n+2} = 4 á 3^{n+3} − 3^{n+3} = 3 á 3^{n+3} = 3^{n+4}.

182. $4(3^{n-1} + 1) - 3(3^{n-2} + 1)$ = 4 á 3^{n-1} − 3^{n-1} + 4 − 3 = $3^{n-1}(4 - 1) + 1 = 3^n + 1$.

183. 4(7 á 3^{n-1} − π) − 3(7 á 3^{n-2} − π) = 28 á 3^{n-1} − 7 á 3^{n-1} − 4π + 3π = 7 á 3^n − π.

184. $a_n = 2a_{n-1}$, $a_0 = 1$.

185. $a_n = 2a_{n-1} - 1$, $a_0 = 2$.

186. $a_n = -a_{n-1}$, $a_0 = 1$.

187. $a_n = a_{n-1} + 3$, $a_0 = -1$.

188. $a_n = a_{n-1}$, $a_0 = \sqrt{2}$.

189. (a) 25, 000 á 1.03^n. (b) 25, 000 á 1.05^n. (c) 25, 000 á 1.02^n + 1, 000 $\left(\frac{1.02^n - 1}{0.02}\right)$.

(a) $a_n = a_{n-1}/1.03$. (b) $a_{20} = 1/1.03^{20} \approx 0.55$. (c) $a_{80} = 1/1.03^{80} \approx 0.09$. (d) $1/1.1^{20} \approx 0.15$. (e) $1/1.1^{80} \approx 0.00$.

1/2.

$2^{29} - 2$.

$\frac{2}{3} + \frac{2}{3}(2^{29})$.

2/3.

8/3.

220,585.

−84.

25.

$\sum_{i=0}^{7}((i - 3)^2 + 1)$.

' (−2a a

201. A(BC) has 6 rows, 9 columns, and 54 entries.

202. $A^n = \begin{pmatrix} 1 & mn \\ 0 & 1 \end{pmatrix}$.

' 4 −13 (. −2 8

' 3 −7/2 (. −6 9

None exists since det B = 0 and det C =) 0.

False.

False.

False.

False.

False.

True.

False.

1 1 10 0 0

1 1 0 (b) 1 0 0 (c) 1 1 0

213. (a) 0 1 1 . 0 1 1 . 0 1 1.

1 1 1

' a 0 (. 0 a